A CHANNEL-CHANGE GAME FOR MULTIPLE INTERFERING COGNITIVE WIRELESS NETWORKS

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ABSTRACT

The proliferation of wireless networks on unlicensed communication bands is leading to coexisting networks, creating interference problems. In this paper, we present a game-theoretic model of dynamic channel change for multiple highly interfering cognitive wireless networks. The channel-change decisions depend on the number of coexisting networks and the cost of channel change. Game-theoretic analysis reflects the choices and motivations of independent, rational, selfish decision makers that do not trust one another. The channel-change probability is shown to increase with the number of coexisting networks. We also compare these decisions to idealized, socially optimal decisions that maximize the expected benefit of the coexisting networks. The difference between the two analyses gives the cost of non-cooperation. We see that this cost goes down as the number of networks increases.

I. INTRODUCTION

In recent years, there has been a proliferation of wireless networks on unlicensed communication bands, such as the ISM bands at 2.4 GHz. Multiple networks may find themselves using the same communication band at the same time, resulting in interference problems [10], [12]. One way of addressing large-scale spectrum sharing is through the use of cognitive wireless networks that are intelligent and can dynamically switch communication channels based on interference and load conditions on the current channel. We have developed and analyzed a number of models with different constraints to capture a variety of channel-change scenarios [13], [14], [15]. In this paper, we look at a channel-change scenario with multiple interfering cognitive wireless networks, and study the effect of varying the number of such networks.

We use the tools of game theory [4], [11] to make channel-change decisions. Game theory has been used extensively to model strategic interactions among people. Game-theoretic analysis reflects the choices and motivations of independent, rational, selfish decision makers who do not trust one another. Here, we also compare game-theoretic decisions to idealized, socially optimal decisions that maximize the expected benefit of all coexisting networks.

Recently, game theory has been used to model several aspects of wireless networks [1]. Mangold et al [8], [3] have used game theory to model channel-sharing decisions by coexisting wireless networks. They address the issue of how to share the current channel more effectively, whereas we look at channel change. Further, their work is concerned with real-time traffic and quality-of-service issues. Game theory has also been applied to access control in single Aloha-like networks [2], [6], [7].

In Section II, an overview of our channel-change scenario is provided. Sections III and IV describe game-theoretic modeling and analysis of the channel-change scenario, whereas Section V presents its socially optimal analysis. Finally, some conclusions are presented in Section VI.

II. CHANNEL-CHANGE SCENARIO

In our dissertation [13], we have introduced the modeling of channel-change decisions using game theory by developing five models to capture different channel-change scenarios. We have focused on simple scenarios to provide initial insight. In this paper, one of these models is presented. In all models, interference resulting from similar networks (such as IEEE 802.11) residing on the same wireless communication channel is considered. In [14], two two-network single-stage decision-making models are presented with unlimited and limited numbers of channels, and a variable *channel-sharing overhead* in terms of extra transmission time due to channel sharing with other networks. In [15], two two-network high interference scenarios are considered with unlimited and limited channels, and multi-stage decision-making. In this paper, the assumptions are the same as in [15], but multiple interfering networks are modeled instead of two networks. We have focused on a high interference scenario for multiple networks because dynamic channel change is more relevant here, and the assumption simplifies the analysis. Our work can be extended to add other complexities in the future.

In our high interference scenario, we assume channel blocking, where no network is able to make any progress as long as any other network is on the same channel. Each network has the option of changing to a new channel in order to make progress, or remaining on the current channel with the hope that the other coexisting networks will move to other channels. A time delay is associated with moving from one channel to another, known as the *channel-change overhead*, and represented as v with $v>0$. In this paper, v is treated as a simple variable, but more complex functions can be considered in the future.

We assume the availability of a potentially unlimited number of alternate channels that the networks could use. Even if all the coexisting networks simultaneously change their current channels, they do not interfere with each other again. The impact of limited channels and interference on a channel even after channel-change is analyzed in [15] for two networks.

The networks are assumed to have intelligent access points, capable of making dynamic decisions regarding which channel to use. The networks are further assumed to have a protocol whereby any network can request its member devices to dynamically switch to a new channel.[1] The intelligent access points can potentially run various decision-making algorithms. We assume that the access points make decisions expected to achieve the best payoffs for their own networks, consistent with game-theoretic assumptions. Note that interference from RF sources, such as microwave ovens, is not considered.

The question arises: can a network change channel when it is being blocked completely? For example, in an IEEE 802.11 network, changing channel would require that the *beacon* containing control information reaches all the stations of the network. We assume situations of high interference but not complete blockage. Control messages, which are typically small and more robustly modulated, are successfully transmitted even when the data transmission is blocked.

[1] This capability has already been defined in the IEEE 802.11h standard, where the access point sends information regarding channel change in its beacon and specifies the number of beacon intervals after which the change will be effective.

III. GAME MODELING

A. Multi-Stage Game

Channel change is modeled using multi-stage games. Multi-stage games capture situations of dynamic decision-making and resulting system behavior over a period of time. They take into consideration the possibility of future decisions. In real-world scenarios, channel-change decision-making is expected to be an ongoing activity as traffic conditions on different channels change over time, making multi-stage modeling more appropriate.

To model multi-stage decision-making, each network considers time as a sequence of "time slots". An example would be the time between adjacent beacons in IEEE 802.11, but the analysis is not limited to a particular network. One stage of decision-making is carried out in a time slot. We assume that at the beginning of an arbitrary time slot, n similar networks, with $n \geq 2$, find themselves coexisting on a single channel. Each network has a message to transmit, where the messages are assumed to be of equal duration. Assuming high interference and channel blocking, none of the coexisting networks can successfully transmit its message.

At the beginning of each time slot, each network can either choose to change channel (C), or to remain (R) on the current channel and attempt transmission. If a network chooses to change channel, it incurs a channel-change delay equal to v time slots. If the network chooses to remain, it either experiences a blockage or a clear channel, depending upon whether the other networks have chosen to remain or change respectively. In the case of blockage, the game must be played again in the next time slot. The number of players remaining in the game in the next time slot depends on how many have remained on the current channel.

The decision-making is considered from the view-point of network i, our network of interest. Since all networks are identical, the same reasoning applies to other networks as well. The game ends for network i when it is able to transmit its message successfully, either because it has changed to another channel, or because all other networks have moved away to other channels. Each network has the objective of minimizing its *channel acquisition cost* defined as the total expected delay incurred until achieving a clear channel for transmission.

In this paper, we use the assumption that the decision-making in each network is done simultaneously. However we believe that with channel blocking, removing the assumption of synchronization between networks does not have a very high impact on the results. A detailed study of this issue is left for future work.

B. Game Representation

We use multi-stage games with a simple structure known as *multi-stage games with observed actions*[2] [4] (p. 70). Each stage of this game is a strategic-form game with complete information [4] (p. 4). Since there are $n \geq 2$ decision makers in the game, the set of players ϑ is given by $\{1, 2, .. n\}$. The choice of actions for each player is the same: *Change* to another channel or *Remain* on the current channel. Hence the set of actions for all players is given by $A_1 = A_2 = \ldots = A_n = \{\text{Change, Remain}\}$. In each stage of the game, all players simultaneously choose their actions. An action profile a represents the combination of the actions taken by each of the players.

The goal for each player (network) is to minimize the time to acquire a clear channel. The expected cost of the multi-stage game with n identical players is represented by U_n [3] and is the same for all players. The *cost*[4] to each player for the remaining game at any stage depends on the number of networks still remaining on the current channel. Hence each network (player) is affected by the action taken by it, as well as the actions taken by the other networks (players).

Since the networks are assumed identical, we can analyze a stage of the game from the viewpoint of representative network i. At the start of a stage of the game, assume player i is co-located on a channel with n-1 other networks. Network i may choose either to *remain* on the current channel, or to *change* channel. Changing channel incurs a cost of v time slots due to channel-change delay, and ends the game for player i. The cost to network i of remaining on the current channel in this stage of the game is affected by how many other networks choose to change to some other channel, and how many remain on the current channel. The number of remaining networks determines the contention in the next stage of the game for network i. However, since all networks are identical, cost is not affected by which specific network stays and which one leaves.

With the above considerations in mind, the decision faced by network i at the start of any stage of the game is represented by the matrix in Fig. 1. The rows show the actions available to network i. Each of the columns represents the number of other networks that change to some other channel. For example, the column labeled "k" represents the case when k of the other networks have changed to some other channel, leaving n-k-1 other networks still on the current channel. Each cell represents the cost for network i under the specified actions.

Network *i* Actions	Number of Other Networks Changing Channel				
	n-1	n-2	k	1	0
Change	v	v	v	v	v
Remain	0	$1 + U_2$	$1 + U_{n-k}$	$1 + U_{n-1}$	$1 + U_n$

Fig. 1: The n-network channel-change game where v gives the channel-change overhead.

Let us look at the "Change" row of Fig. 1. If network i *changes* to another channel, it incurs a fixed channel-change cost of v time units, independent of the actions of any of the other networks. Since we assume that an adequate number of empty channels is available, network i will always be able to find one such channel where it is not in contention with any other network. Hence all costs in the "Change" row are given by v, where $v>0$.

Let us now examine the "Remain" row in Fig. 1. As already explained, the cost depends on how many other networks remain on this channel. Assuming that the number of other networks changing channel is given by the variable X, consider the case $X=k$ when k networks change to other channels, leaving n-k-1 other networks still on this channel. Counting network i, there are n-k

[2] Referred to as an *extensive game with perfect information and simultaneous moves* in [11] (p. 206).

[3] When a single "pure" action is selected by each player, the cost or utility of the action is represented by u, but when a "mixed" action is selected, which uses a probability distribution with pure actions, the expected cost or utility is represented by U. In this paper, we use mixed actions as discussed in Section IV.

[4] Note that in game theory, usually the term "utility" or "payoff" is used instead of "cost" because it represents a benefit that the player would like to maximize. In this paper, since the objective of the players is to minimize the channel acquisition time, the term "cost" is used instead. However, in a slight abuse of notation, we continue to use the symbol U to represent this cost.

networks left on this channel in the next subgame. Hence the expected delay of the remaining subgame, which consists of *n-k* players, is given by U_{n-k}. Adding a delay of 1 time unit for the current stage gives a total cost of $1+ U_{n-k}$. The value of *k* is in the range *0…(n-1)*, since there are a maximum of *n-1* other networks.

In the case *X=n-1*, all other networks change channels. We have a special case because network *i* is the only network to remain on the current channel, and hence it can transmit without any delay. Thus the cost for this case is 0.

We see that Fig. 1 represents the full game as well as the subgame from this point onwards for player *i*. The number of players in the subgame may be less than the original number of networks, *n*.

IV. GAME ANALYSIS

Let us examine how this game will be played by rational players (networks) by finding its stable operating point. For a multi-stage game, this is given by its subgame perfect equilibrium [4] (pp.72-74), which corresponds to finding the *Nash Equilibrium* for each subgame of the full game.

A *Nash Equilibrium* (NE) is a very important solution concept in game theory. It is an action profile *a** such that each player's action is an optimal response to the other players' actions [11] (p.23). There is no motivation for a player to deviate unilaterally from this action profile.

We need to find the Nash Equilibrium for each subgame of the complete multi-stage game. However, in our game, all subgames have the same structure as the complete game. The complete history of prior play is captured by the number of players in the subgame. Hence, we need to simply find a Nash Equilibrium of the *n*-player game represented in Fig. 1.

Of the two possible actions, *change* and *remain*, what action does network *i* choose? Since all networks are assumed identical (e.g. drawn from the same population), they should be able to choose the same strategy. The choice of a strategy should not depend on the identity of the network. However, if all networks choose the same action, the resulting action profile is not a Nash Equilibrium, since a better solution can be found for some network. Hence we consider using a mixed strategy by assigning probabilities to each of the two pure strategies. All networks can assign the same values to these probabilities, and hence choose the same mixed strategy.

We will determine the action of network *i* by finding the Nash Equilibrium using mixed strategies. We start this analysis by assuming that all other networks (except *i*) choose the strategy *change* with probability p_n and *remain* with probability $1- p_n$ when there are n networks in the game. Network *i* has a belief from prior experience that this is the choice that will be made by the other networks. Hence it determines its own action by first calculating its expected cost to *change* channel and to *remain* on the current channel, and then choosing the action with the lower cost.

We determine the probability $P_{n,k}$ of *k* other networks changing channels using probability theory. Given that the total number of other networks is *n-1* and C_k^{n-1} represents the number of ways of choosing *k* networks from *n-1* identical networks:

$$P_{n,k} = C_k^{n-1} {p_n}^k (1-p_n)^{n-k-1} \tag{1}$$

From Probability Theory, we also know that the following result holds:

$$\sum_{k=0}^{n-1} P_{n,k} = 1 \tag{2}$$

To find the NE, we determine the expected cost of the *n*-player subgame for network *i* if it chooses to *change* channel, as given by ${U_n}^C$:

$${U_n}^C = v(P_{n,n-1} + P_{n,n-2} + \ldots + P_{n,1} + P_{n,0}) = v \tag{3}$$

Similarly, the expected cost of the subgame for network *i* when choosing to *remain* on the current channel is given by ${U_n}^R$ as follows:

$${U_n}^R = \sum_{k=0}^{n-2} (1 + U_{n-k}) P_{n,k} \tag{4}$$

Network *i* will choose to *change* channel if ${U_n}^C < {U_n}^R$ and to *remain* on the current channel if ${U_n}^C > {U_n}^R$. When ${U_n}^C = {U_n}^R$, network *i* has no preference between the two strategies. Under this condition, network *i* has the same cost whether it chooses the pure strategy of *change* or *remain* or some mixed strategy which is a combination of the two. In particular, it can choose the mixed strategy of $(p_n, 1-p_n)$ and obtain the same expected cost. Thus, it has no motivation to deviate from the mixed strategy of $(p_n, 1-p_n)$ given that all other networks have also chosen this mixed strategy. This is the Nash Equilibrium condition. Under NE, the two costs are equal, i.e. $U_n^C = U_n^R = U_n$. Combining this with (3), we see that $U_n = v$. Thus U_n is not a function of n or k. Using this fact, (4) becomes:

$$v = (1+v)\sum_{k=0}^{n-2} P_{n,k} \quad (5)$$

Using (2) with the above, we obtain:

$$P_{n,n-1} = \frac{1}{1+v} \quad (6)$$

From (1), the probability $P_{n,n-1}$ of all other networks changing channel is given by p_n^{n-1}. Thus, the NE channel-change probability with n coexisting networks is given by:

$$p_n* = \sqrt[n-1]{\frac{1}{1+v}} \quad (7)$$

Also, we already know that the NE delay is the same as the channel-change time v:

$$U_n* = v \quad (8)$$

The NE channel-change probability p_n* is shown in Fig. 2. We see that if the cost of changing channel v goes up, the probability of changing channel p_n* comes down. Also, p_n* increases as the number of networks increases. This agrees with intuition, since larger numbers of networks lead to greater contention. Note that all networks use the same value of the channel-change probability p_n*.

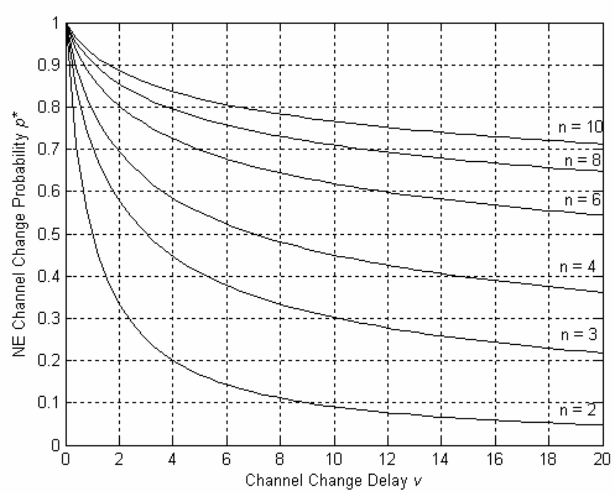

Fig. 2: The Nash Equilibrium channel-change probability $p*$ with respect to the channel-change delay v for different number of coexisting networks n.

V. SOCIALLY OPTIMAL ANALYSIS

The results of game-theoretic analysis obtained above are compared to those obtained from analyzing "socially optimal" decisions made by an abstract "centralized" decision maker. The socially optimal decision maker imposes a channel-change strategy on each network which promotes the best interest of all networks, rather than allowing each of the networks to choose a channel-change strategy from non-cooperative individual self-interest. As in the game case, a symmetric strategy is sought which treats each network equally. So we will seek a socially-optimal channel-change probability (p_n') defined to minimize the total expected cost of all networks. We will see how the socially optimal channel-change probability (p_n') compares to the game-theoretic value p_n* derived in the previous section.

As before, let U_n be the total expected delay for a network i before it can start transmission, given n networks on the same channel, each using identical channel-change probability p_n. From the symmetry of the networks, minimizing the total expected cost of all networks is equivalent to minimizing U_n, the expected cost for any individual network.

Again, as before, let U_n^C be the expected delay a network incurs if it decides to change channel, and U_n^R the expected delay if it decides to stay on the current channel. The expression for U_n is given by:

$$U_n = p_n U_n^C + (1-p_n)U_n^R \quad (9)$$

We use the values of U_n^C and U_n^R obtained in (3) and (4) respectively. Substituting these values in (9) gives:

$$U_n = p_n v + (1-p_n)\sum_{k=0}^{n-2} P_{n,k}(1+U_{n-k}) \quad (10)$$

Expressing U_n in terms of U_1, U_2,…, U_{n-1} we get:

$$U_n = \frac{p_n v + (1-p_n)^n + (1-p_n)\sum_{k=1}^{n-2} C_k^{n-1} p_n^k (1-p_n)^{n-k-1}(1+U'_{n-k})}{1-(1-p_n)^n} \quad (11)$$

Note that an iterative solution approach is assumed, so that when solving for U_n, the variables U_1, U_2,…, U_{n-1} are represented by U_1', U_2',…, U_{n-1}' because they have already been solved and the optimal values have been found. The centralized channel-change probability p_n' selected by the centralized decision maker is one that minimizes the delay

U_n. Since the expression for U_n is very complex, the value of p_n is found by numerical methods using MATLAB [9]. These values are determined iteratively, starting with *n=2*. We have found solutions for *n=2,3,4*. The expressions used for them are:

$$U_2 = \frac{p_2 v + (1-p_2)^2}{1-(1-p_2)^2} \tag{12}$$

$$U_3 = \frac{p_3 v + (1-p_3)^3 + 2p_3(1-p_3)^2(1+U_2')}{1-(1-p_3)^3} \tag{13}$$

$$U_4 = \frac{p_4 v + (1-p_4)^4 + 3p_4(1-p_4)^3(1+U_3') + 3p_4^2(1-p_4)^2(1+U_2')}{1-(1-p_4)^4} \tag{14}$$

The centralized and game-theoretic channel-change probabilities for *v = 2, 5, 10* and *n = 2, 3, 4* are shown below in Fig. 3. The centralized channel-change probability is always higher. As the cost of channel change (*v*) goes up, the channel-change probability goes down. As the number of networks goes up, the channel-change probability goes up. Also, as the number of networks goes up, the difference between the centralized and game-theoretic probabilities reduces. Both ultimately converge towards 1, since with very large numbers of networks on the current channel, there is diminishing probability that all other networks will change channels, and hence increasing probability that this network will end up changing channel.

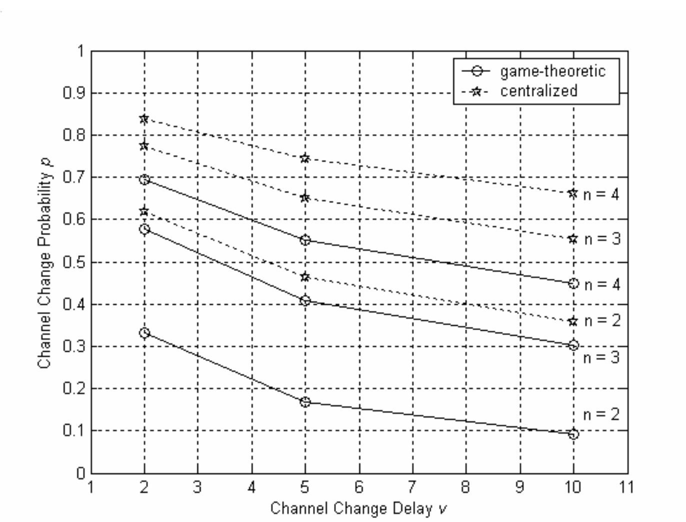

Fig. 3: Comparison of game-theoretic and socially optimal channel-change probabilities for the *n*-network many-channel game. The socially optimal decision maker forces more channel change.

The corresponding values of delay cost are shown in Fig. 4. The values of delay are lower for centralized decisions compared to game-theoretic decisions. As *v* increases, the value of delay increases proportionately, because the expected cost is bounded by *v*. The difference with game-theoretic values decreases as *n* increases. As the number of networks goes up, the centralized decision gets closer to always changing the channel (*p′ =1*), and thus the expected cost approaches *v*, which is the game-theoretic expected cost.

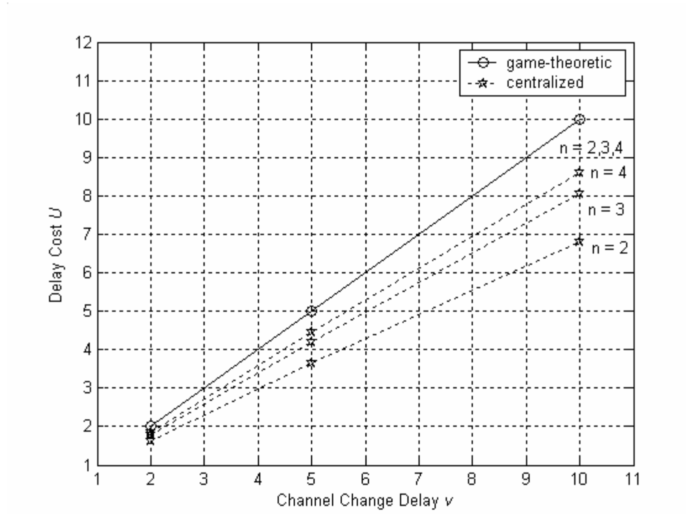

Fig. 4: Comparison of game-theoretic and socially optimal delay cost for the *n*-network many-channel game. The socially optimal decision maker gives lower delay. The difference between the two is the cost of non-cooperation.

It is interesting to note that with self-interest-based rational decision-making, all networks do less well than they would with centralized, socially optimal decisions. One may naively expect that self-interest-based decisions would be better for an individual by providing a win-lose proposition. However, they lead to the “tragedy of commons” [5] instead. Since all players are maximizing self-interest, they are in fact less effective in resource-sharing. The difference between the two outcomes is the price paid for non-cooperation resulting from lack of trust. We see that this price goes down as *n* goes up.

VI. CONCLUSION

In our recent work [13], [14], [15], we have introduced the modeling of channel-change decision-making using game theory. Several simple single-stage and multi-stage models have been developed to provide initial insights, using various assumptions to model different scenarios. In this paper, we have extended the results presented in [15] to cover multiple networks where $n > 2$. In future work, other models which relax the assumptions of this paper can be developed such as by limiting the number of available channels when $n > 2$, and removing the channel-blocking assumption in multi-stage decision-making. The results can also be extended by the use of simulations.

The issue of dynamic channel change is very relevant in unlicensed bands, since they are shared by many different networks. Further, unlicensed usage is expected to grow as more and more licensed bands start supporting unlicensed secondary users. This will make dynamic channel change even more important in the future. Incorporating channel change is also helpful in allowing the easy set up of access points, especially in large installations, because channels do not need to be manually pre-assigned.

This paper makes a contribution to the understanding of game-theoretic, self-interest-based decisions for dynamic channel change in cognitive wireless networks. Existing work in this area addresses the issue of how to share the current channel more effectively, whereas we consider the option of changing the transmission channel to address coexistence problems.

We have also compared game-theoretic decisions with centralized, socially optimal decisions. Socially optimal decisions require the presence of only trusted parties so that the common good of all can be ensured. Game-theoretic decision-making, on the other hand, reflects scenarios in which there is no trust between the players.

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