Artificial Intelligence

First-Order Logic

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and cons of propositional logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares“
    - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
• Equality	=
• Quantifiers	$\forall$, $\exists$

Atomic sentences

Atomic sentence = *predicate* (*term*$_1$,...,*term*$_n$)
or *term*$_1$ = *term*$_2$

Term = *function* (*term*$_1$,...,*term*$_n$)
or *constant* or *variable*

E.g.,

- *Brother(KingJohn,RichardTheLionheart)*
- *> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$,

E.g.

- *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

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Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for
  - constant symbols → objects
  - predicate symbols → relations
  - function symbols → functional relations
- An atomic sentence *predicate*($term_1,\ldots,term_n$) is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

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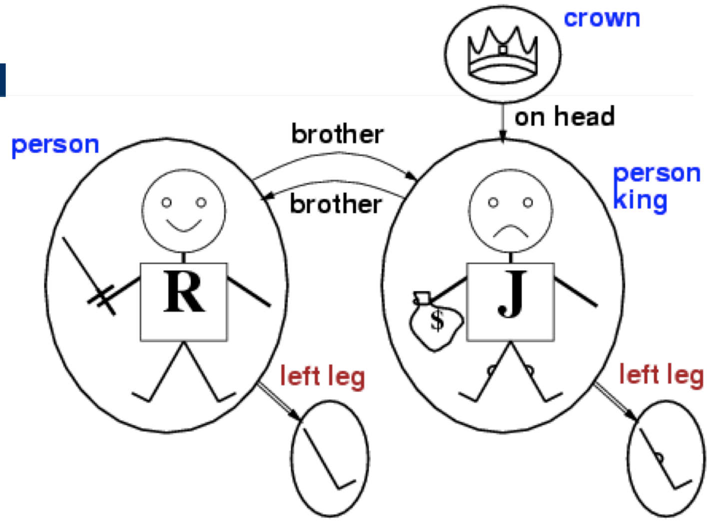
Models for FOL: Example



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Universal quantification

- ∀<*variables*> <*sentence*>
- Everyone at SMU is smart:

$\forall x\ At(x,SMU) \Rightarrow Smart(x)$

- $\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$At(KingJohn,SMU) \Rightarrow Smart(KingJohn)$
$\wedge\ At(Richard,SMU) \Rightarrow Smart(Richard)$
$\wedge\ At(Rupert,SMU) \Rightarrow Smart(Rupert)$
$\wedge\ \ldots$

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A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall$x At(x,SMU) $\wedge$ Smart(x)

  means "Everyone is at SMU and everyone is smart"

Existential quantification

- $\exists$<*variables*> <*sentence*>
- Someone at SMU is smart:
- $\exists x$ At(x,SMU) $\wedge$ Smart(x)
- $\exists x$ *P* is true in a model *m* iff *P* is true with *x* being some possible object in the model
- Roughly speaking, equivalent to the disjunction of instantiations of *P*

  At(KingJohn,SMU) $\wedge$ Smart(KingJohn)
  $\vee$ At(Richard,SMU) $\wedge$ Smart(Richard)
  $\vee$ At(Rupert,SMU) $\wedge$ Smart(Rupert)
  $\vee$ ...

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x\ At(x,SMU) \Rightarrow Smart(x)$$

is true if there is anyone who is not at SMU!

Properties of quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$
- $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$
- $\exists x\ \forall y\ Loves(x,y)$
  - "There is a person who loves everyone in the world"
- $\forall y\ \exists x\ Loves(x,y)$
  - "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other
- $\forall x\ Likes(x,IceCream) \quad \neg(\exists x\ \neg Likes(x,IceCream))$
- $\exists x\ Likes(x,Broccoli) \quad \neg(\forall x\ \neg Likes(x,Broccoli))$

Equality

- $term_1$ = $term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*:

  $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge (\exists m,f\ \neg (m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y))]$

Using FOL

The kinship domain:

- Brothers are siblings

  $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$
- One's mother is one's female parent

  $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \wedge Parent(m,c))$
- "Sibling" is symmetric

  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:
  `Tell`(KB,Percept([Smell,Breeze,None],5))
  `Ask`(KB,∃a BestAction(a,5))
- i.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution σ,
- *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Hillary,y/Bill}
  *S*σ = Smarter(Hillary,Bill)
- `Ask`(KB,S) returns some/all σ such that KB ⊨ σ

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Knowledge base for the wumpus world

- Perception
  - ∀t,s,b Percept([s,b,Glitter],t) ⇒ Glitter(t)
- Reflex
  - ∀t Glitter(t) ⇒ BestAction(Grab,t)

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Deducing hidden properties

- $\forall x,y,a,b\ Adjacent([x,y],[a,b]) \Leftrightarrow$
  $[a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

Properties of squares:

- $\forall s,t\ At(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$

Squares are breezy near a pit:

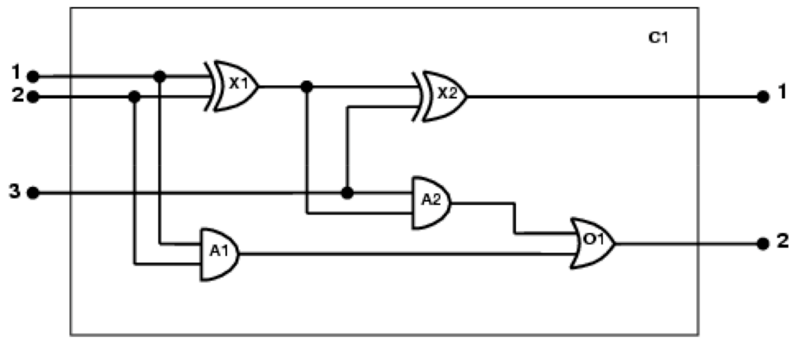
- Diagnostic rule---infer cause from effect
  ∀s Breezy(s) ⇒ \Exi{r} Adjacent(r,s) ∧ Pit(r)$
- Causal rule---infer effect from cause
  ∀r Pit(r) ⇒ [∀s Adjacent(r,s) ⇒ Breezy(s)$]

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



The electronic circuits domain

1. Identify the task
   - Does the circuit actually add properly? (circuit verification)
2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   - Irrelevant: size, shape, color, cost of gates
3. Decide on a vocabulary
   - Alternatives:
     $Type(X_1) = XOR$
     $Type(X_1, XOR)$
     $XOR(X_1)$

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Signal(t_1) = Signal(t_2)
   - $\forall t$ Signal(t) = 1 $\vee$ Signal(t) = 0
   - $1 \neq 0$
   - $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Connected(t_2, t_1)
   - $\forall g$ Type(g) = OR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 1
   - $\forall g$ Type(g) = AND $\Rightarrow$ Signal(Out(1,g)) = 0 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 0
   - $\forall g$ Type(g) = XOR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ Signal(In(1,g)) $\neq$ Signal(In(2,g))
   - $\forall g$ Type(g) = NOT $\Rightarrow$ Signal(Out(1,g)) $\neq$ Signal(In(1,g))

The electronic circuits domain

5. Encode the specific problem instance

Type(X_1) = XOR
Type(X_2) = XOR
Type(A_1) = AND
Type(A_2) = AND
Type(O_1) = OR

Connected(Out(1,X_1),In(1,X_2))
Connected(Out(1,X_1),In(2,A_2))
Connected(Out(1,A_2),In(1,O_1))
Connected(Out(1,A_1),In(2,O_1))
Connected(Out(1,X_2),Out(1,C_1))
Connected(Out(1,O_1),Out(2,C_1))

Connected(In(1,C_1),In(1,X_1))
Connected(In(1,C_1),In(1,A_1))
Connected(In(2,C_1),In(2,X_1))
Connected(In(2,C_1),In(2,A_1))
Connected(In(3,C_1),In(2,X_2))
Connected(In(3,C_1),In(1,A_2))

The electronic circuits domain

6. Pose queries to the inference procedure: What are the possible sets of values of all the terminals for the adder circuit?

   $\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = $i_1 \wedge$ Signal(In(2,C_1)) = $i_2 \wedge$ Signal(In(3,C_1)) = $i_3 \wedge$ Signal(Out(1,C_1)) = $o_1 \wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

   May have omitted assertions like $1 \neq 0$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world