
Teaching with the STL

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Part 1

Introduction to STL Concepts

STL: What and Why

- | Generic data structures (containers) and algorithms for operating upon them
- | Part of the C++ definition
- | Developed by Stepanov, Lee, and Musser
- | Generic programming != OOP (no encapsulation)
- | Extensible -- you can add your own elements

Templates are not Classes

- | These are not cookies
- | You can't eat them
- | They can be used to make cookies



Templates are not Classes

- | These are cookies
- | They are made with a cookie cutter
- | You can eat them



Templates are not Cookies

- | Templates are used to create classes
- | You can't compile them
- | You can instantiate them
 - » This gives you a class
- | The instantiations are compiled
- | The instantiations are strongly typed like other classes

Templates are not Classes

```
template <class E>
class stack
{
    ...
    void push(E e){...}
}
```

<- A class template

```
stack <int> S;
```

<- a template class

```
S.push(55);
```

Templates are not Functions

```
template <class E>
E& min(E& a, E& b)
{
    if(a < b) return a;
    return b;
}
```

<- a function template

```
abox = min(box1, box2);
```

<- a template function

The Standard Template Library

| Containers

- » array, vector, deque, list, set, map, multiset, multimap

| Algorithms

- » sort, search, and nearly everything else

| Iterators

- » generalize pointers and pointer arithmetic

| Adaptors

- » change the behavior of other components

| Allocators

- » memory management

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The Major Dimensions

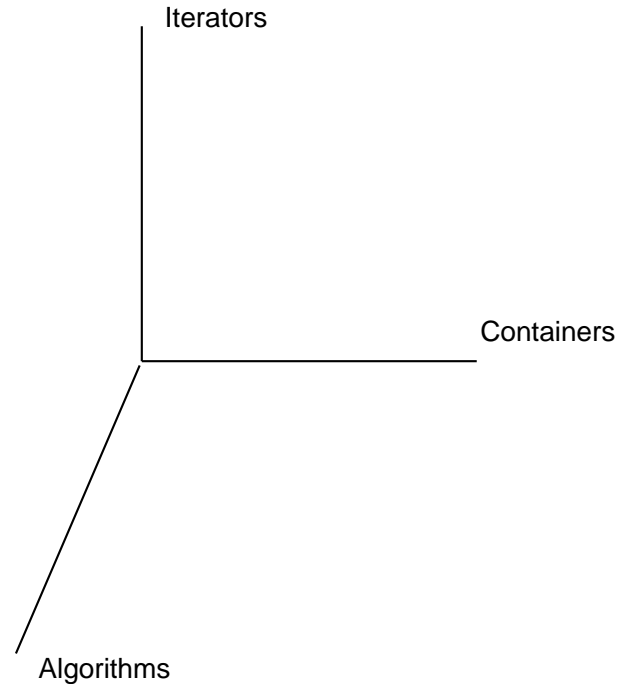
Independent Development of:

- | Containers
 - » contain values
- | Algorithms
 - » operate on containers
- | Iterators
 - » interface between containers and algorithms

The Major Dimensions

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 - » contain values
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Simple Examples

```
// Summing a list -- typical early assignment
int main()
{
    int sum(0);
    cout << "Enter your integers:\n";
    while(1) {
        int entry;
        cin >> entry;
        if (cin.eof()) break;
        sum += entry;
    }
    cout << "The sum was " << sum << endl;
    return 0;
}
```

... just add STL!

```
#include <algo.h>
#include <iostream.h>

int main()
{
    cout << "Enter your integers:" << endl;
    // set up an input stream iterator for cin
    istream_iterator < int, ptrdiff_t > cinlter(cin), eos;
    // accumulate the sum using the input iterator
    int sum = accumulate(cinlter, eos, 0);
    cout << "The sum was " << sum << endl;
    return 0;
}
```

Vector Manipulation

```
vector<int> v;  
v.push_back(3);  
v.push_back(4);  
v.push_back(5);  
v.push_back(6);
```

```
vector<int>::iterator i;
```

```
for(i = v.begin(); i != v.end(); ++i)  
    cout << *i << endl;  
sort(v.begin(); v.end());  
for(i = v.begin(); i != v.end(); ++i)  
    cout << *i << endl;
```

Sorting a file of strings

```
cout << "Enter name of file to sort: ";
string sortFileName;
cin >> sortFileName;
ifstream sortFile(sortFileName.c_str());
istream_iterator < string, ptrdiff_t >
    sortFileIter(sortFile), eos;
vector < string > sortVector;
copy(sortFileIter, eos, back_inserter(sortVector));
cout << "Sorting " << sortVector.size() << " words.\n";
sort(sortVector.begin(), sortVector.end());
ostream_iterator < string > coutIter(cout, "\n");
cout << "Sorted file is: \n";
copy(sortVector.begin(), sortVector.end(), coutIter);
```


Iterators are the Key

Containers +
Iterators +
Algorithms
= STL Programs

Iterator Flavors

- | Forward Iterators (operator++)
 - » Input Iterators
 - » Output Iterators
- | Bidirectional Iterators (operator --)
- | Random Access Iterators (operator +=)

Iterator Flavors

- | Forward Iterators (operator++)
 - » Input Iterators
 - » Output Iterators
- | Bidirectional Iterators (operator --)
- | Random Access Iterators (operator +=)

All Iterators have operator*

All Containers produce iterators begin() and end()
begin references first. end is “after” last

Slouching Toward Iterators

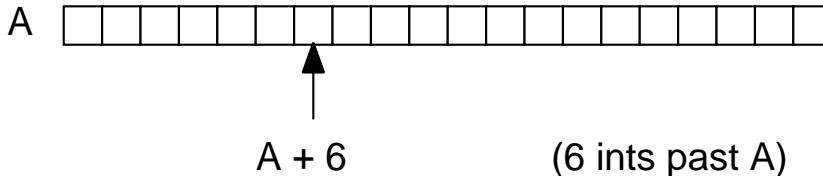
```
template < class T >
void selectionSort(T elements[ ], int length)
{
    for(int i = 0; i < length - 1; ++i)
    {
        int s = i;
        T small = elements[s] ;
        for(unsigned j = i + 1; j < length; ++j)
            if(elements[j] < small)
            {
                s = j;
                small = elements[s] ;
            }
        elements[s] = elements[i] ;
        elements[i] = small;
    }
}
```

Pt. 1: Dependent on Arrays

Pointer Duality Law

```
int * A = new int [20];
```

$A[i]$ is equivalent to $*(A + i)$



Slouching Towards Iterators



```
int elements [20] = ...  
selectionSort(elements, 20)
```

```
int * start = elements;  
int * end = elements + 20; // or &elements[20]
```

```
selectionSort(start, end);
```

Pt. 2: The Goal

The Replacements

```
template < class T >
void selectionSort(T elements[ ], int length)
{
    for(int i = 0; i < length - 1; ++i)
    {
        int s = i;
        T small = elements[s] ;
        for(unsigned j = i + 1; j < length; ++j)
            if(elements[j] < small)
            {
                s = j;
                small = elements[s] ;
            }
        elements[s] = elements[i] ;
        elements[i] = small;
    }
}
```

start = elements
end = elements + length
loc = & elements[s]
where = & elements[i]
inner = & elements[j]

Slouching Towards Iterators

```
template < class T >
void selectionSort(T* start, T* end)
{
    for(T* where = start ; where < end - 1 ; ++where)
    {
        T* loc = where;
        T small = *loc;
        for(T* inner = where + 1; inner < end; ++inner)
            if(*inner < *loc)
            {
                loc = inner;
                small = *loc;
            }
        *loc = *where;
        *where = small;
    }
}
```

Pt 3: The Result (almost)

Slouching Towards Iterators

```
template < class Iterator , class value_type >
void selectionSort_aux(Iterator start, Iterator end, value_type)
{
    for(Iterator where = start ; where < end - 1 ; ++where)
    {
        Iterator loc = where;
        value_type small = *loc;
        for(Iterator inner = where + 1; inner < end;
            ++inner)
            if(*inner < *loc)
            {
                loc = inner;
                small = *loc;
            }
        *loc = *where;
        *where = small;
    }
}
```

Pt 3: The Result (...)

Slouching Towards Iterators

```
template < class Iterator >
inline void selectionSort (Iterator start, Iterator end)
{
    selectionSort_aux(start, end, *start);
}
```

Pt 3: The Result !

The Advantages

| This version will sort more than arrays.

» All we need is a structure referenced by a datatype like a pointer that implements

- operator *
- operator++
- operator+
- operator-
- operator=
- operator<

With care we could
reduce this list

Such datatypes are called iterators

The Lesson

- | Implement containers separate from algorithms
- | Use pointer-like structures as an interfacing mechanism

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To Gain 

Advantages

- | Generality
- | A framework for thinking about containers and algorithms
- | Smaller written code
- | Smaller compiled code

Advantages

- | Generality
- | A framework for thinking about containers and algorithms
- | Smaller written code
- | Smaller compiled code

But... 

Disadvantages

- | Students must become thoroughly familiar with all aspects of pointers including
 - » The pointer duality law
 - » Pointer arithmetic
 - » Pointer “gotchas”

Iterators, Pro and Con

- | Iterators make it possible for the STL to be efficient and flexible, but...
- | As an iterator-based library, the STL provides *no safety* for the user
- | Basically, if you like C++, you will probably like the STL!

Exercise

Rewrite this search function using Iterators:

```
template <class T>
int linearSearch(T a[], int length, T target)
{
    for (int i = 0; i < length; i++)
        if (target == a[i])
            return i; // return position in array
    return -1; //use -1 to indicate item not found
}
```

Sample solution

```
template <class Iterator, class T>
Iterator linearSearch(Iterator begin, Iterator end,
                    T target)
{
    for (Iterator i = begin; i < end; i++)
        if (target == *i)
            return i;
    return end; //use end to indicate item not found
}
```

```
....
int a[] = {1, 2, 3, 4, 5};
if (int * pos = linearSearch(a, a+5, 4) < a+5)
    cout << "found " << *p << endl;
else
    cout << "not found\n";
```

Iterators for STL Containers

- | Work like a pointer
- | To access an individual member of a container, you dereference an iterator (*)
- | Advance iterator with ++
- | Special past-the-end iterator used to mark end of container (improper to dereference)
- | STL typically does not bounds check iterators
- | Containers provide an iterator constructor

Extended Example: Using Vectors

- | Vectors are a generic container provided by STL
- | Similar to built-in arrays, but grow dynamically at the end

Vector example, p. 1

```
#include <iostream.h>
#include <vector.h>
#include <algo.h>
template <class T>
void vectorPrint(char * label, vector<T> v)
{
    cout << label << endl;
    for (int i=0; i < v.size(); i++)
        cout << v[i] << '\t';
    cout << "\n\n";
}
```

Vector example, part 2

```
int main()
{
    // create a vector containing ints from 1 to 10
    vector<int> v1;
    for (int i = 0; i < 10; i++)
        v1.push_back(i+1);
    vectorPrint("v1 initial state", v1);

    // create a second vector of the same size
    vector<int> v2(v1.size());

    // copy vector v1 to v2
    copy(v1.begin(), v1.end(), v2.begin());
    vectorPrint("v2 should be a copy of v1", v2);
}
```

Vector example, p.3

```
cout << "v1 == v2? " << (v1 == v2) << endl;
```

```
// create another vector and fill it  
vector<double> v3(20);  
fill(v3.begin(), v3.begin() + 20, 3.14);  
vectorPrint("vector v3", v3);
```

```
// rotate the first vector  
rotate(v1.begin(), v1.begin()+5, v1.end());  
vectorPrint("v1 rotated", v1);
```

```
// now, sort it  
sort(v1.begin(), v1.end());  
vectorPrint("v1 sorted", v1);
```


Vector example, p. 4

```
// find the location of 5 in v2
vector<int>::iterator loc;
loc = find(v1.begin(), v1.end(), 5);
cout << "this should be 5: " << *loc << endl;

}
```

Exercise

Write a program that reads in an arbitrary number of double-precision floating point numbers > 0.0 , terminated by 0.0 , puts them into a vector, sorts them, and finds the median.

Sample solution

```
int main()
{
    vector<double> v;
    double x;
    cin >> x;
    while (x > 0.0)
    {
        v.push_back(x);
        cin >> x;
    }
    sort(v.begin(), v.end());
    if (v.size() % 2 == 0) // even case
        cout << (v[v.size()/2 - 1] +
                 v[(v.size()/2)])/2 << endl;
    else
        cout << v[v.size()/2] << endl;
}
```

Five classes of iterators

Iterators classified by the operations they support

I input

- » can be compared (`==`, `!=`), incremented, dereferenced as rvalue (assign from)
- » usually used with input streams (`cin`)
- » single pass

I output

- » compared, dereferenced as lvalue (assign to)
- » usually used with output streams (`cout`)
- » single pass

Iterator Classes

- | forward
 - » union of features of input/output iterators
 - » plus, can be traversed more than once
- | bidirectional
 - » does everything a forward iterator does
 - » plus, can traverse in reverse, using "--"

Iterator Classes

- | random access
 - » does everything a bidirectional iterator does
 - » plus can access any location in constant time
 - » supports pointer-like arithmetic, e.g. $i + 7$
 - » can compare relationally: $<$, $>$, $<=$, $>=$

Relationship among algorithms, iterators, containers

| Iterators form a hierarchy



| Algorithms classified by the iterators they *require*; containers by the iterators they *support*

| If an algorithm requires a particular iterator, it can also use those higher in the hierarchy

Sample Relationships

- | Lists provide bidirectional iterators (*not* random access) so you can't use the sort algorithm, nor can you use a list as a heap. (But lists have a special sort method.)
- | The `next_permutation` algorithm requires a bidirectional iterator, so you can use it on a list (or a vector...) but not on the input stream.

Containers

- | Objects that store other objects
- | All STL containers are generic (templated) but homogeneous
- | Built-in C-style arrays work as containers
- | Sequence Containers: *vector*, *deque*, *list*
- | Associative Containers: *set*, *multiset*, *map*, *multimap*

Containers

- | Ordinary Arrays (I.e. regions of memory)
- | Vectors -- expandable array
- | Deques -- expandable at both ends
- | Lists -- doubly linked circular with header
- | Sets and Multisets -- red-black tree
- | Maps and Multimaps -- dictionary like

Note: Implementation is not specified
but efficiency is specified.

All Containers Provide

- | A Storage Service
 - » insert and erase...
- | An Associated Iterator type
 - » The type of iterator determines what can be done with the container.
- | `begin()` and `end()` iterators - - - `[b, e)`
- | A collection of types: `vector::value_type...`
- | constructors, assignment, cast, equality...

Sequence Containers

| vector

- » $O(1)$ access to any element, $O(1)$ (amortized) to add to end, $O(n)$ to add elsewhere
- » grows dynamically as objects added to end; vector handles storage management
- » use `[]` syntax to access elements
- » use `push_back()` to add to end of vector (and grow size), `insert()` to add within (also `pop_back()`)
- » Fastest (average) container for most purposes.

Vector Example

```
vector < int > v;  
v.push_back(47);  
v.push_back(17);  
cout << v.size() << '\t' <<  
    v[0] << '\t' << v[1] <<  
    endl;
```

```
2  47  17
```

Sequence Containers

| deque

- » Expandable “array” at both ends
- » `push_front`, `pop_front`
- » Average $O(1)$ insert at both ends
- » Linear insert in middle
- » Random Access Iterators
- » Good choice for queues & such.

Deque Example

```
deque < char > dc;  
dc.push_back('h');  
dc.push_back('o');  
dc.push_front('i');  
dc.push_front('h');  
cout << dc.size() << '\t' <<  
dc[0] << dc[1] << dc[2] << dc[3]  
    endl;  
2 hiho
```

Sequence Containers

| list

- » doubly-linked list
- » $O(1)$ insertions, *no random access*
- » Slower on average than vector or deque
- » special functions for splicing, merging, etc.
- » can use `push_front` , `push_back` , and `insert`
- » access items using bidirectional iterators

Associative Containers: sets and multisets

- | two template parameters: a key type and a comparison relation (sets are ordered)
- | insert and find operations are $O(\log n)$
- | so really, a set is a balanced binary tree that stores just keys (no associated data)
- | only difference between set and multiset is duplicate keys in multiset

Set example

```
set < int, greater < int > > intSet;  
intSet.insert(2);  
intSet.insert(2);  
intSet.insert(7);  
cout << intSet.count(2) << '\t' <<  
      intSet.count(17) << endl;
```

1 0

Associative Containers: Maps and Multimaps

- | Just like a set, except key/data pairs
- | e.g. a symbol table, a dictionary
- | Performance is tree-like, i.e. $O(\log n)$
- | Can retrieve the members in order, using iterators
- | Uses the parameterized type `pair < keyType, dataType >`

Iterator classes for STL containers

- | random access: vector, deque, C array
- | bidirectional: list, set, multiset, map, multimap
- | input: istream, const C array
- | output: ostream

Algorithms

- | Defined in terms of a specific iterator type
 - » e.g. sort requires random access iterators
- | Work with all containers that provide that iterator type -- including user written.
- | Combine good generality w/ good efficiency
- | Do not appear within container classes (generally)
 - » This is important to generality & efficiency

Classification of Algorithms

- | Non-mutating sequence algorithms
 - » do not modify contents of container
 - » most require a pair of iterators, specifying a range
 - » e.g.: `find`, `for_each`, `count`
- | Mutating sequence algorithms
 - » modify contents
 - » e.g.: `copy`, `fill`, `reverse`, `rotate`

Example: find

```
int a[] = {1,2,3,14,4};  
vector<int> v(a, a+5); // construct vector from array  
vector<int>::iterator v_loc;  
v_loc = find(v.begin(), v.end(), 3);  
// v_loc is now an iterator pointing to 3 in v
```

```
list<int> l(a, a+5); // construct a list from array  
list<int>::iterator list_loc;  
list_loc = find(l.begin(), l.end(), 4);  
// list_loc is an iterator pointing to 4 in l  
list_loc = find(l.begin(), l.end(), -1);  
// list_loc is an iterator pointing to l.end()  
// to indicate failure
```

Example: for_each

```
void print_char(char c)
{
    cout << c << endl;
}
int main()
{
    char s[] = "ABCDEFGH";
    vector<char> vc(s, s + strlen(s));
    set<char> sc(s, s + strlen(s));
    for_each(vc.begin(), vc.end(), print_char);
    for_each(s, s+strlen(s), print_char);
    for_each(sc.begin(), sc.end(), print_char);
}
```


Classification of Algorithms

- | Sorting and related
 - » sort, nth_element, binary_search, min/max
- | Numeric
 - » accumulate, inner_product, partial_sum

Function Objects 1

- | Predicates

- » A function of one argument returning bool

- | Comparisons

- » A function of two arguments returning bool

- | Unary Operator, Binary Operator

- » A function of one or two arguments returning a value

Function Objects 2

- | Can be functions or template functions
- | Can be objects implementing an appropriate operator()
- | Many are built in
 - » less..., plus..., and...,...
- | Function adaptors too
 - » not1, not2, bind1st, bind2nd,...

Function Object Example

```
class stringLess
{
    bool operator()(char* s1, char* s2)
    {
        return strcmp(s1, s2) < 0;
    }
    . . .
} // Defines a function object.

vector< char* > stringVec;
. . .
sort (stringVec.begin(), stringVec.end(), stringLess());
// Note the constructor call in the last argument ^^^^
```

Adaptors

- | Change the interface of a component
- | Example: stack adaptor lets you use a list or deque with push and pop
- | insert iterator adaptors: used to grow a container. Most common is `back_inserter()`
- | `istream_iterator`, `ostream_iterator` let you use input and output streams as containers

Stack/Queue example (part 1)

```
typedef list < int > intList;
stack < intList > s;
queue < intList > q;
cout << "Enter some numbers for the stack & queue:\n";
// set up an input stream iterator for cin
istream_iterator < int, ptrdiff_t > cinIter(cin), eos;
// put items onto stack and queue
for ( ; cinIter != eos; cinIter++) {
    s.push(*cinIter);
    q.push(*cinIter);
}
```

Stack/Queue Example

(Part 2)

```
// pop items from the stack and print them out
cout << "Here's what you get when "
        << " you pop the stack: ";
while(!s.empty()) {
    cout << s.top() << " ";
    s.pop();
}
cout << "\n\nHere's what you get when"
        << " you pop the queue: ";
while(!q.empty()) {
    cout << q.front() << " ";
    q.pop();
}
cout << "\n\n";
```

Extended example: Counting words

- | Read in a file, identify all words that appear more than once, list them with number of appearances
- | Algorithm:
 - » read all words into a vector of strings
 - » sort the vector
 - » look for repeated words, count them, put them into a map
 - » index in map is # of appearances; data is list of words

word map data structure

count

word lists

4

(and, or, of)

3

(we, can)

2

(STL, template, Utica, pistachio)

Acknowledgment: example inspired by the “anagram” program of Musser and Saini.

word count (part 1)

```
// First, open input file, set up iterator for reading
cout << "Enter name of file to process: ";
string inFileName;
cin >> inFileName;
ifstream inFile(inFileName.c_str());
istream_iterator < string, ptrdiff_t >
    inFileIter(inFile), eos;
// Set up wordVector to hold the string input, and get
// it from input file,
// then sort it to prepare for processing
typedef vector < string > stringVector;
stringVector wordVector;
copy(inFileIter, eos, back_inserter(wordVector));
cout << "Read in " << wordVector.size() << " words.\n";
sort(wordVector.begin(), wordVector.end());
```

word count (part 2)

```
// Now set up a map to hold the duplicated words.
// The key field is the number of times the word occurs,
// and the data field is a list of all words that occurred
// that number of times.
// The map is organized from largest to smallest.
typedef map < int, list < string >, greater < int > >
    stringCountMap;
stringCountMap wordMap;

// create an iterator "current" for wordVector,
// then process the vector,
// looking for duplicated words and entering them into wordMap
stringVector::iterator current = wordVector.begin();
while (current != wordVector.end()) {
    // set current to next occurrence of a duplicated word
    current = adjacent_find(current, wordVector.end());
    if (current == wordVector.end()) break;
}
```

word count (part 3)

```
// find the next word that *doesn't* match the current word,  
// thus setting up an open interval [current, nextWordPos)  
// that marks all matching words  
stringVector::iterator nextWordPos =  
    find_if(current+1, wordVector.end(),  
            not1(bind1st(equal_to<string>(), *current)));  
// add the word to the list at the appropriate map entry,  
// determined by the number of times it's in the vector  
wordMap[nextWordPos-current].push_back(*current);  
// continue processing the vector at the next word  
current = nextWordPos;  
}
```

word count (part 4)

```
// set up an output iterator and present results
ostream_iterator < string > coutIter(cout, "\n");
cout << "map file is: \n";

// iterate through the map, print the key
// (which indicates the number of
// occurrences of the word), then copy the list
// of words to output
stringCountMap::const_iterator wordMapIter =
    wordMap.begin();
for (; wordMapIter != wordMap.end(); ++wordMapIter) {
    cout << "Count " << (*wordMapIter).first << ":\n";
    copy((*wordMapIter).second.begin(),
         (*wordMapIter).second.end(), coutIter);
    cout << "----\n\n";
}
}
```

Sample output

Enter name of file to process: cx6.cpp

Read in 441 words.

map file is:

Count 26:

//

Count 23:

the

Count 12:

of

Count 9:

<<

Count 8:

#include

to

Count 7:

<

and

string

Count 6:

in

words

etc.

Extending the STL

- | Not standardized but available
 - » `hash_set`
 - » `hash_map`
 - » `hash_multiset`
 - » `hash_multimap`
- | Like `set...` but have a (self reorganizing) hashed implementation
- | Constant average time for insert/erase

STL in Java

- | ObjectSpace has developed an equivalent library for Java
- | (JGL) Java Generic Library
- | Public domain, available on internet.
- | Depends on run-time typing instead of compile time typing, but is otherwise equivalent.

Resources

- | <http://csis.pace.edu/~bergin>
- | <http://www.objectspace.com>
- | <http://www.cs.rpi.edu/~musser/stl.html>
- | http://weber.u.washington.edu/~bytewave/bytewave_stl.html
- | <ftp.cs.rpi.edu/pub/stl>
- | <http://www.sgi.com/Technology/STL/>
- | <http://www.cs.brown.edu/people/jak/programming/stl-tutorial/home.html>

Books

- | Data Structures Programming with the STL, Bergin, Springer-Verlag (to appear)
- | STL Tutorial and Reference Guide, Musser and Saini, Addison-Wesley, 1996
- | The STL <primer>, Glass and Schuchert, Prentice-Hall, 1996
- | The Standard Template Library, Plauger, Stepanov, and Musser, Prentice-Hall, 1996

map and multimap

- | Ordered set (multiset) of key-value pairs
- | Kept in key order
- | $O(\lg n)$ inserts and deletions
- | Bidirectional iterators
- | Good choice for dictionaries, property lists, & finite functions as long as keys have comparison operation

vector

- | Expandable array -- operator[]
- | push_back, pop_back
- | Average $O(1)$ insert at end.
- | $O(n)$ insert in middle
- | Random Access Iterators
- | Fastest (average) container for most purposes.

deque

- | Expandable “array” at both ends
- | `push_front`, `pop_front`
- | Average $O(1)$ insert at both ends
- | Linear insert in middle
- | Random Access Iterators
- | Good choice for queues & such.

list

- | Doubly linked list
- | $O(1)$ inserts everywhere, but slower on average than vector and deque
- | Bidirectional iterators
- | Some specialized algorithms (sort).

set and multiset

- | Sorted set (multiset) of values
- | $O(\lg n)$ inserts and deletions
 - » Balanced binary search tree
- | Sorted with respect to operator $<$ or any user defined comparison operator
- | Bidirectional iterators
- | Good choice if elements must stay in order.

All Iterators Provide

- | operator*
 - » may be readonly or read/write
- | copy constructor
- | operator++ and operator++(int)
- | operator== and operator!=
- | Most provide operator=

Specialized Iterators

- | Forward
 - » provide operator=
- | Bidirectional (extend forward)
 - » provide operator-- and operator--(int)
- | Random Access (extend bidirectional)
 - » provide operator<..., operator+=..., operator-