Simple Channel-Change Games for Spectrum-Agile Wireless Networks

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Abstract—**The proliferation of wireless networks on unlicensed communication bands is leading to coexisting networks, creating interference problems. In this paper, we present simple game-theoretic models of dynamic channel-change decision-making for intelligent, spectrum-agile wireless networks to address interference problems. The channel-change decisions depend on the cost of channel change and the level of interference on the current and new channels. Game-theoretic analysis reflects the choices and motivations of independent, rational, selfish decision makers that do not trust one another. We also compare these decisions to idealized, socially optimal decisions that maximize the expected benefit of the coexisting networks. While game-theoretic decisions are more suited to an untrusted environment, socially optimal decisions give better performance in a trusted environment.**

Index Terms—**channel change, game theory, spectrum agile, wireless networks.**

I. INTRODUCTION

IN recent years, there has been a proliferation of wireless networks on unlicensed communication bands, such as the ISM bands at 2.4 GHz. Multiple networks may find themselves using the same communication band at the same time, resulting in interference problems [9], [11]. One way of addressing large-scale spectrum sharing is through the use of smart, spectrum-agile networks that can dynamically switch communication channels based on interference and load conditions on the current channel.

In this paper, we use the tools of game theory [4], [10] to make channel-change decisions in spectrum-agile wireless networks. Game theory has been used extensively to model strategic interactions among people. Game-theoretic analysis reflects the choices and motivations of independent, rational, selfish decision makers who do not trust one another. Here, we also compare game-theoretic decisions to idealized, socially optimal decisions that maximize the expected benefit of all coexisting networks.

Recently, game theory has been used to model several aspects of wireless networks [1]. Mangold et al [8], [3] have

used game theory to model channel-sharing decisions by coexisting wireless networks. They address the issue of how to share the current channel more effectively, whereas we look at channel change. Further, their work is concerned with real-time traffic and quality-of-service issues. Game theory has also been applied to access control in single Aloha-like networks [2], [6], [7].

In Section II, an overview of channel-change scenarios is provided. In Sections III and IV, two channel-change models are presented that model two different decision-making scenarios. Finally, some conclusions are presented in Section V.

II. CHANNEL-CHANGE SCENARIOS

We model two similar networks such as IEEE 802.11 that reside on the same wireless communication channel and wish to transmit messages. Each network's message transmission time is longer than if it was alone on the channel. We assume that the message size for each network is identical. The message transmission time for each network, if it is alone on a channel, has been normalized to 1 time unit. When both networks are on the same channel, the *increase* in message transmission time for each network is given by the *channel-sharing overhead*, and represented by m, with $m>0$. In this paper, we vary the channel-sharing overhead to model a range of interference scenarios.

Instead of sharing a channel with another network, each network has the option of changing to a new channel in order to make faster progress. However, there is a time delay in moving from one channel to another, known as the *channel-change overhead* and represented as v with $v>0$.

Two channel-change scenarios are presented based on the number of available channels. Channel-change decision-making is expected to be affected by the number of wireless channels available for transmission. In order to first focus on the simplest models, we consider a *many-channel* scenario in which a potentially unlimited number of alternate channels are available for switching, and a *two-channel* scenario that is more constrained. In the many-channel case, even if the coexisting networks simultaneously change their channels, they do not interfere with each other again. However, in the two-channel scenario, the networks will interfere again after

simultaneous channel change. More complex cases are left for future work.

The transmission environment is assumed to consist of two or more transmission channels. The networks are assumed to have intelligent access points, capable of making dynamic decisions regarding which channel to use. The networks are further assumed to have a protocol whereby any network can request its member devices to dynamically switch to a new channel.[1] The intelligent access points can potentially run various decision-making algorithms. We assume that the access points make decisions expected to achieve the best payoffs for their own networks, consistent with game-theoretic assumptions. Note that interference from RF sources, such as microwave ovens, is not considered.

III. TWO-NETWORK MANY-CHANNEL GAME

A. Game Representation

We represent the decision-making of two coexisting networks as a two-player strategic-form game with complete information [4] (p. 4). The strategic game is a model of interacting decision makers for static, one-stage decisions applicable to a wide range of situations. Since there are two decision makers in the game, Network 1 and Network 2, the set of players ϑ is given by $\{1, 2\}$. The choice of actions for each player is the same: *Change* to another channel or *Remain* on the current channel. Hence the set of actions for player 1 and player 2 is given by $A_1 = A_2 = \{Change, Remain\}$.

An action profile a is a combination of an action taken by player 1 and an action taken by player 2. An example is (Change, Remain), in which player 1 (Network 1) chooses the action *Change* and player 2 chooses the action *Remain*. Since there are two choices of actions for each of the players, four combinations of choices are possible. These four choice combinations constitute the set of all possible action profiles. This set is given by $\{(C, C), (C, R), (R, C), (R, R)\}$, where C stands for *Change* and R for *Remain*.

The goal for each player is to minimize the expected time taken for its own message transmission. Hence the expected message transmission time is used as each player's "utility" or "payoff". Since the objective is to minimize the expected transmission time, we will refer to it as "cost" rather than "utility", but use u to represent it. Each player (network) is affected by the action taken by it, as well as the action taken by the other player. Hence the *cost* depends on the selected action profile. For example, $u_1(C, C)$ represents the cost of the action profile (C, C) for network 1.

The game played by the two networks can be represented as a matrix as shown in Fig. 1. Each cell of the matrix represents an action profile. It shows two quantities, such that the first and second correspond to the costs incurred for networks 1

and 2 respectively for the action profile represented.

		Network 2	
		Change	Remain
Network 1	Change	$v+1, v+1$	$v+1, 1$
	Remain	$1, v+1$	$m+1, m+1$

Fig. 1: The two-network channel-change game with many channels, where v gives the channel-change overhead, and m the channel-sharing overhead.

The value of $u_1(C, C)$ is given by $v+1$ (Fig. 1). In this scenario, when both networks change channels, they always switch to *different* channels. The transmission cost for network 1 is v time units for changing channel and 1 time unit for transmitting when alone on the new channel. For similar reasons, $u_1(C, R)$ is also given by $v+1$. However, $u_1(R, C) = 1$ because network 2 changes to another channel, and network 1 is left alone on the current channel. In the last case, $u_1(R, R) = m+1$, since both networks share the same channel, and they each incur the channel-sharing overhead m in addition to the transmission time without interference. We have assumed that the two networks are similar, hence the costs for network 2 can also be obtained using similar reasoning.

B. Game-Theoretic Analysis

Let us now examine how this game will be played by rational players (networks) using the solution concept of *Nash Equilibrium* (NE). A Nash Equilibrium is an action profile a^* such that each player's action is an optimal response to the other players' actions [10] (p.23). There is no motivation for a player to deviate unilaterally from this action profile.

In Fig. 1, if network 2 changes channel, network 1 will prefer to remain on the current channel. Since $v+1 > 1$ (we know that $v > 0$), the transmission time on the current channel will always be lower. If network 2 remains on the current channel, network 1 will make one of two decisions: *change* channel if $v < m$, and *remain* on the current channel if $v \geq m$ (there is no motivation to change channel if $v = m$). Thus, we have two cases to analyze, depending on whether the channel-change overhead v is greater than or less than the channel-sharing overhead m.

Case $v \geq m$

When $v \geq m$, network 1 always chooses to *remain* on the current channel, irrespective of whether network 2 chooses *change* or *remain*. Since the two networks are similar, network 2 will apply the same reasoning as network 1 and also always choose *remain*. Hence we have a *dominant action profile* of (R, R) in this game [10] (p.45-46). This action profile is also the *Nash Equilibrium* of this game. The NE cost u^* of the game for each network is given by:

$$u^* = m + 1 \tag{1}$$

Case $v < m$

[1] This capability has already been defined in the IEEE 802.11h standard, where the access point sends information regarding channel change in its beacon and specifies the number of beacon intervals after which the change will be effective.

When $v < m$, if network 2 chooses *change*, network 1 will choose *remain* since $v+1 > 1$. However, if network 2 chooses to *remain* on the current channel, network 1 will prefer to *change* to another channel. Thus there are two stable action profiles in this game, depending on the action chosen by the other player: (R, C) and (C, R). Given that the other player does not change its action, there is no motivation for a player to deviate unilaterally from either of these two action profiles. Hence, these two action profiles are the two Nash Equilibria (NE) of this game, using pure strategies.

We notice that the two NE in pure strategies, (R, C) and (C, R) are not symmetric, because when one network chooses *change*, the other chooses *remain*, and vice-versa. Since the two networks are assumed identical (e.g. drawn from the same population), they should be able to choose the same strategy. The choice of a strategy should not depend on whether a network is Network 1 or Network 2. Hence we consider using a mixed strategy by assigning probabilities to each of the two pure strategies. Since the networks are identical, both will assign the same values to these probabilities.

We are interested in determining the stable operating point for our game using mixed strategies, given by the Nash Equilibrium. We start this analysis by assuming that network 2 chooses the strategy *change* with probability p and *remain* with probability $1-p$. Network 1 has a belief from prior experience that this is the choice that will be made by network 2. Hence it determines its own action by first calculating its expected cost to *change* channel and to *remain* on the current channel, and then choosing the action with the lower cost.

Network 1's expected cost to change channel, U_C, is found by considering the first row of Fig. 1, with p and $1-p$ as weights for the $u_1(C, C)$ and $u_1(C, R)$ costs:

$$U_C = p(v+1) + (1-p)(v+1) = v+1 \qquad (2)$$

Similarly, by considering the second row of Fig. 1, the expected cost to *remain* on the current channel is given by U_R as:

$$U_R = p \times 1 + (1-p)(m+1) = m+1-mp \qquad (3)$$

Network 1 will choose to *change* channel if $U_C < U_R$ and to *remain* on the current channel if $U_C > U_R$. When $U_C = U_R$, network 1 has no preference between the two strategies. Under this condition, network 1 has the same cost whether it chooses the pure strategy of *change* or *remain* or some mixed strategy which is a combination of the two. In particular, it can choose the mixed strategy of *(p, 1-p)* and obtain the same expected cost. Thus, it has no motivation to deviate from the mixed strategy of *(p, 1-p)* given that network 2 has chosen this mixed strategy. This is the Nash Equilibrium (NE) condition. Thus, under NE, $U_C = U_R$, and using (1)-(3), we get:

$$p^* = \begin{cases} 1 - \dfrac{v}{m} & if\ v < m \\ 0 & if\ v \geq m \end{cases} \qquad (4)$$

$$U^* = \begin{cases} v+1 & if\ v < m \\ m+1 & if\ v \geq m \end{cases} \qquad (5)$$

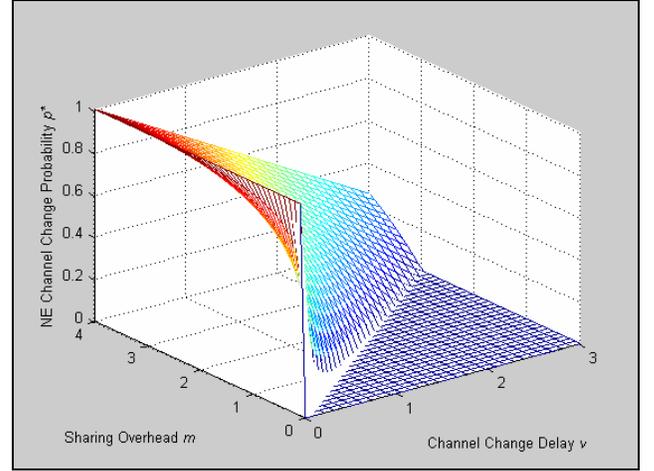These quantities are plotted below in Fig. 2-3.

Fig. 2: The Nash Equilibrium channel-change probability p^* with respect to the channel-change delay v and the channel-sharing overhead m.
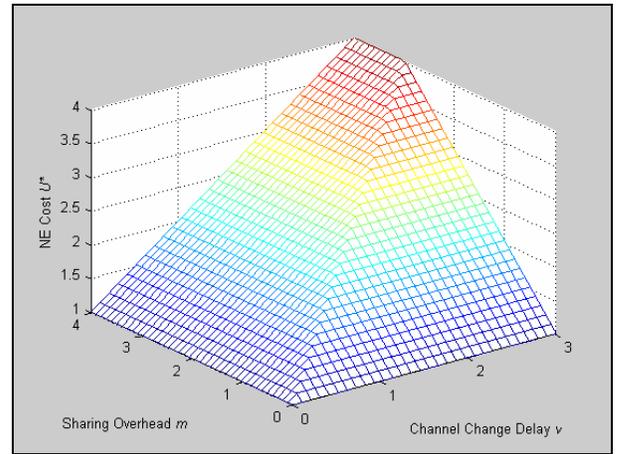

Fig. 3: The Nash Equilibrium message transmission delay U^* with respect to the channel-change delay v and the channel-sharing overhead m.

C. Socially Optimal Analysis

The results of game-theoretic analysis obtained above are compared to those obtained from analyzing "socially optimal" decisions made by an abstract "centralized" decision maker. The socially optimal decision maker promotes the best interest of all networks, rather than the self-interest of any individual network. It assigns a channel-change probability of p to each of the two networks, instead of allowing each of them to non-cooperatively choose their own channel-change probabilities.

We will see how the socially optimal channel-change probability (p') compares to the game-theoretic value $p*$ derived in the previous section.

Let U_C and U_R be the expected *change* and *remain* transmission costs for both networks as before. Then, U is given by:

$$U = pU_C + (1-p)U_R \qquad (6)$$

Substituting U_C and U_R using (2) and (3) respectively, and minimizing U with respect to p gives:

$$p' = \begin{cases} 1 - \dfrac{v}{2m} & if \ v < 2m \\ 0 & if \ v \geq 2m \end{cases} \qquad (7)$$

$$U' = \begin{cases} 1 + v - \dfrac{v^2}{4m} & if \ v < 2m \\ m+1 & if \ v \geq 2m \end{cases} \qquad (8)$$

The values of p' and $p*$ are compared in Fig. 4, while U' and $U*$ are compared in Fig. 5.
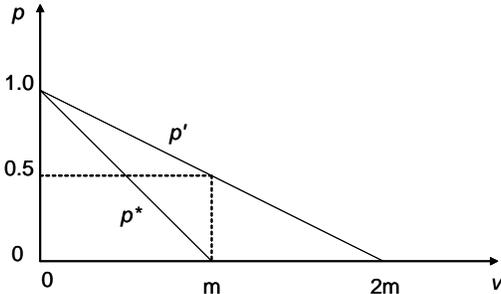

Fig. 4: Comparison of game-theoretic and socially optimal channel-change probabilities for the two-network many-channel game. The socially optimal decision maker forces more channel change.
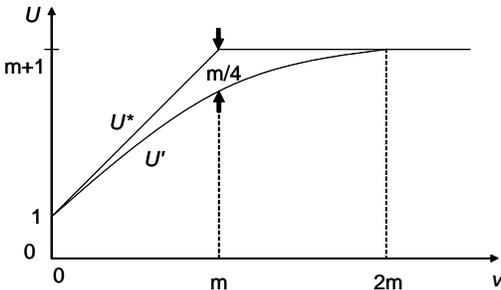

Fig. 5: Comparison of game-theoretic and socially optimal message transmission delay cost for the two-network many-channel game. The socially optimal decision maker gives lower transmission delay. The maximum cost of game-theoretic decisions in this case is m/4 or v/4.

It is interesting to note that with self-interest-based rational decision-making, both networks do less well than they would with centralized, socially optimal decisions. One may naively expect that self-interest-based decisions would be better for an individual by providing a win-lose proposition. However, they lead to the "tragedy of commons" [5] instead. Since both players are maximizing self-interest, they are in fact less effective in resource-sharing. The difference between the two outcomes is the price paid for non-cooperation resulting from lack of trust.

IV. TWO-NETWORK TWO-CHANNEL GAME

In the previous game, a large enough number of alternate channels was available, so that every channel change resulted in a network finding an unused channel. We now limit the total number of available channels to two. All other aspects of the decision-making scenario are the same as before. With only two channels available, a simple scenario of channel contention is created when both networks change their channels simultaneously. This scenario introduces the more realistic expectation that changing a channel does not always solve the interference problem.

The two-channel game is shown in Fig. 6. The only difference between Fig. 1 and Fig. 6 is in the cost for action profile (C, C). When both networks change channel, they find themselves sharing the new channel as well.

		Network 2	
		Change	Remain
Network 1	Change	v+m+1, v+m+1	v+1, 1
	Remain	1, v+1	m+1, m+1

Fig. 6: Two-network two-channel game. The main difference with many channels in Fig. 1 is that when both networks change channels, they share the new channel as well.

Using the same procedure as before, we determine the stable operating point for the game by finding the Nash Equilibrium (NE) for this game, which is given by:

$$p* = \begin{cases} \dfrac{1}{2}\left(1 - \dfrac{v}{m}\right) & if \ v < m \\ 0 & if \ v \geq m \end{cases} \qquad (9)$$

$$U* = \begin{cases} 1 + \dfrac{v+m}{2} & if \ v < m \\ m+1 & if \ v \geq m \end{cases} \qquad (10)$$

The analysis for centralized, "socially optimal" decisions with two channels, carried out using the same approach as for many-channels, gives the following results:

$$p' = \begin{cases} \dfrac{1}{2}\left(1 - \dfrac{v}{2m}\right) & if \ v < 2m \\ 0 & if \ v \geq 2m \end{cases} \qquad (11)$$

$$U' = \begin{cases} 1 + \dfrac{v+m}{2} - \dfrac{v^2}{8m} & \text{if } v < 2m \\ m+1 & \text{if } v \geq 2m \end{cases} \qquad (12)$$

Notice that the game-theoretic and socially optimal channel-change probabilities for two channels given by (9) and (11) respectively are exactly one-half the respective values for many channels given by (4) and (6). Hence, the comparison of (9) and (11) is as shown in Fig. 4, with all values on the p axis reduced by a factor of 0.5. The comparison of game-theoretic and socially optimal message transmission delay for the two-channel scenario is shown in Fig. 7.
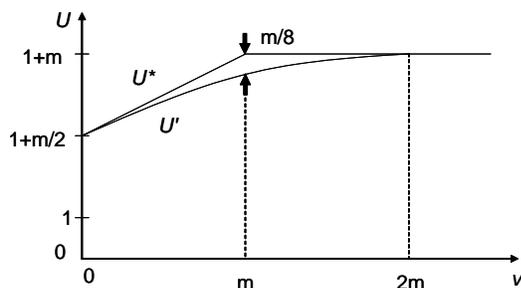

Fig. 7: Comparison of message transmission delay for game-theoretic and socially optimal decisions with two channels. The socially optimal decision maker always has a lower transmission delay. The maximum cost of game-theoretic decisions in this case is m/8 or v/8.

V. Conclusion

We introduce the idea of modeling dynamic channel change decisions as games to handle the problem of interference from coexisting spectrum-agile wireless networks. In this paper, we make a start by presenting two simple, single-stage channel-change games. We have also developed several multi-stage models in [12]. Many other models could also be considered by other researchers.

The issue of dynamic channel change is very relevant in unlicensed bands, since they are shared by many different networks. Further, unlicensed usage is expected to grow as more and more licensed bands start supporting unlicensed secondary users. This will make dynamic channel change even more important in the future. Incorporating channel change is also helpful in allowing the easy set up of access points, especially in large installations, because channels do not need to be manually pre-assigned.

This paper makes a contribution to the understanding of game-theoretic, self-interest-based decisions for dynamic channel change in spectrum-agile coexisting networks. Existing work in this area addresses the issue of how to share the current channel more effectively, whereas we consider the option of changing the transmission channel to address coexistence problems.

We have also compared game-theoretic decisions with centralized, socially optimal decisions. Socially optimal decisions require the presence of only trusted parties so that the common good of all can be ensured. Game-theoretic decision-making, on the other hand, reflects scenarios in which there is no trust between the players.

Existing networks, such as Ethernet, use centrally-imposed strategies through the use of standards. These strategies attempt to provide fairness for all users, and can be seen as approximations of socially optimal strategies. In such an environment, trusted access is enforced by using the same network standard, and allowing only those devices on the network that conform to the standard. Centralized, socially beneficial decision-making works very well here. However, when many different networks share a channel, there is no common standard being used that can restrict access to trusted networks only. In such an environment, self-interest based game-theoretic decisions are a better reflection of reality even though an additional transmission delay cost is incurred.

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