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Shape Matching with Ordered Boundary Points Using a Least-Cost Diagonal Method

Carl E. Abrams, Sung-Hyuk Cha, and Charles C. Tappert

Computer Science Department, Pace University
861 Bedford Road, Pleasantville, New York, 10570 USA

Abstract – The *"Shape Context" is a recently developed shape descriptor. It is an information rich descriptor that is invariant to translation, scale and rotation. It assumes that the boundary points which represent the shape are in no particular order. The main algorithm employed is the Hungarian algorithm that solves the point matching problem on $O(n^3)$. In this paper the workings of the shape context in the case when the boundary points are in order is examined. A more efficient method called," the least cost diagonal" , which solves the matching problem on $O(n^2)$ is introduced.*

Introduction

Shape matching is a central challenge in the field of computer vision. The shape descriptors used in shape matching can generally be classified into either external or internal [7]. Internal descriptors abstract the shape using the points contained by the shape boundary. External descriptors use the boundary points to create the shape abstraction. The shape context was introduced by Belongie [2]. It is a novel, information rich external shape descriptor that is translation, scale and rotation invariant.

The notion of a shape context is to represent the relationship of each point on the boundary of a shape to all the other points on the shape and then convert that representation to a histogram. With each point represented as a histogram matching one shape to another is simplified to matching histograms and finding the best overall fit. The best over all fit is the least cost assignment of points of one shape to the points of the other. The shape context assumes that there is no order to the points that represent the boundary. The matching of the points is a form of the classic bi-partite graph matching "Assignment" problem. The assignment problem itself is solved by the application of the Hungarian algorithm which runs in $O(n^3)$ [6].

By ordering the boundary points a simpler and faster matching technique can be employed. The technique named, "the least cost diagonal", finds the best fit which must occur along a diagonal of the shape context cost matrix. In this paper, the shape context is examined and compared to the least cost diagonal method for accuracy and speed.

Shape Contexts

The first step in the creation of a shape context, once the boundary has been detected, is to draw a vector from each point on the boundary to every other point on the boundary. (Figure 1) shows the boundary of a shape, a piece of pottery, where the numbered x's denote the actual sub-set of points used to define the boundary. In this case only 6 are shown for clarity.

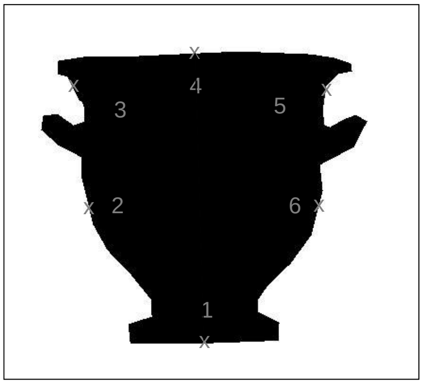

Figure 1. Pottery Shape

The next step is to draw a vector from one of the points on the boundary, say point # 1 shown in (Figure 2) to all of the remaining points on the boundary. The set of vectors form the basis of the shape context for point 1 of the boundary.

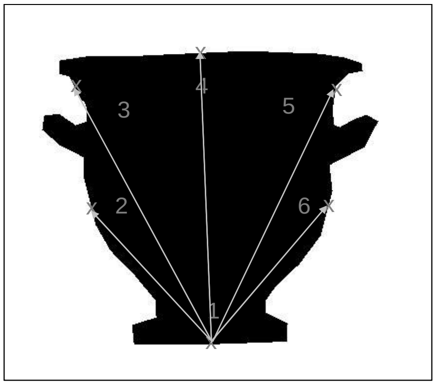

Figure 2. Pottery Shape Context Point 1

The same procedure is followed for the other 5 points as shown in (Figure 3)

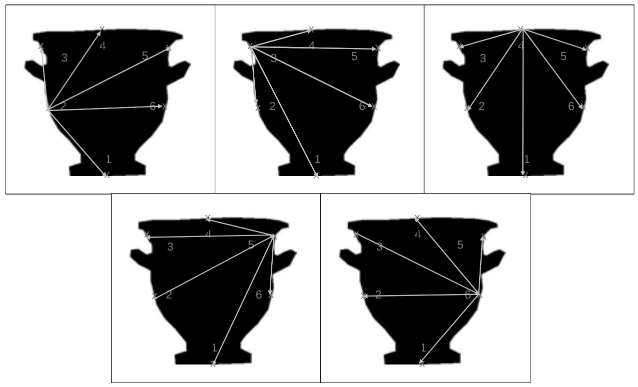

Figure 3. Shape Context Vectors for Pottery Shape

The set of six sets of vectors for each point represent the shape context for the boundary. They vectors must be transformed into a form that is useful for the computation of the comparison of shapes: compact, comparable and computable. The transformation is accomplished by converting the vectors at each point to a polar histogram. That is, since a vector is defined by its angle with respect to a reference point and its distance, quantizing both dimensions will yield a set of six histograms, one at each point. The bins spacing selected for the histogram is 30 degree increments for the angles yielding 12 angle bins. The distance r is divided into 5 increments. The log r is used to make the shape histogram more sensitive to points closer in to the reference point [2]. The total number of bins is 60.

Rotational invariance is achieved by the proper selection of the "reference point" for the vector angles. By using the turning angle at a point as the reference angle from which the angles to all the other points on the shape are compared, shape contexts that are insensitive to rotational changes in the shape are created [3].

Once again, a similar operation is performed at the other points resulting in a compact representation of the boundary as a set of six shape context histograms. Assuming that another shape has gone through the same procedure and has been abstracted into a set of histograms a quantitative comparison of the two shapes is easily obtained using the simple $\chi 2$ histogram comparison: $C_{i,j} = 1/2\sum_{k=1}^{K}[g(k) - h(k)]^2 / (g(k) + h(k))$

$C_{i,j}$ is the cost of the ith point on the first shape and the jth point on the second shape. K is the number of bins, in this case 60. g is the histogram at point i on the first shape and h is the histogram at point j on the second shape. The method of comparison assumes that the points on the shape boundary are in no particular order [5]. The resulting cost matrix derived from the above is, using our specific example, a 6 by 6 matrix. The comparison of the two shapes is now performed by solving the assignment problem based on the two 6 by 6 cost matrices.

The Shape Context in Action

To fully appreciate the shape context it is instructive to observe its characteristics as it is applied to a relatively simple shape. Consider the two right triangles shown (Figure 4). They are identical except for scale. The points are labeled from 1 to 3 for ease of comparison. . Computationally, the two triangles are shown to be equivalent using the 3 by 3 cost matrix generated using the $\chi 2$ histogram distance measure and the Hungarian algorithm.

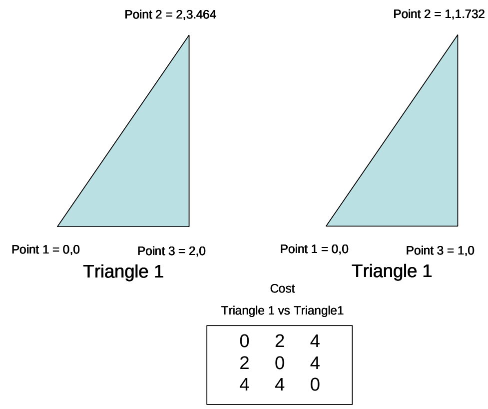

Figure 4. 30,60 , 90 Degree Triangles

The three shape contexts for the two triangles, full size and scaled down, are represented by the log polar histograms as shown in (Figure 5). The shades of gray indicate the population density of each bin with white being unpopulated.

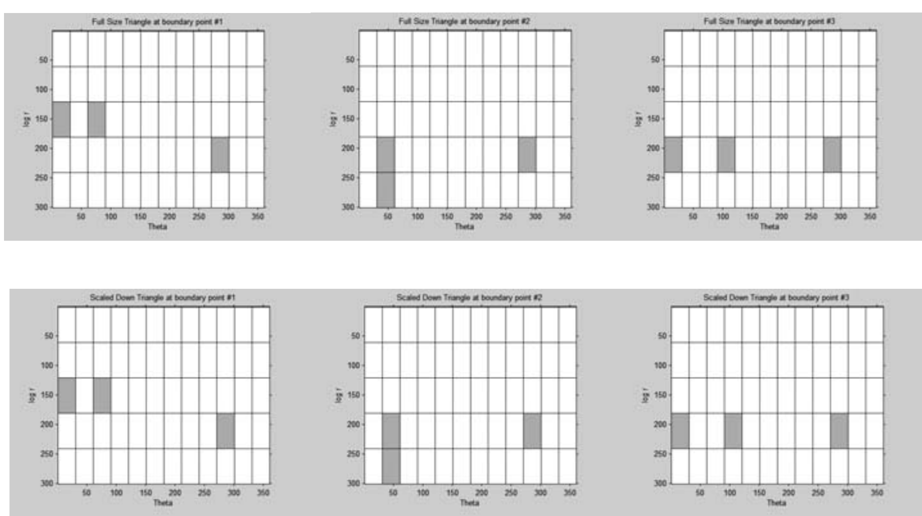

Figure 5. Triangle Shape Context Histograms

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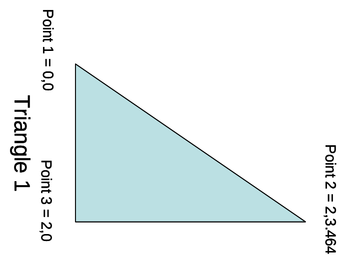
Rotation invariance is demonstrated by simply rotating the original triangle by 90 degrees, computing the shape contexts and the $\chi 2$ distance matrix. (Figure 6)



Cost

4	0	2
4	2	0
0	4	4

Figure 6. Rotated Triangle

The best match again occurs but it is shifted away from the main diagonal and confirmed by the Hungarian least cost path assignment.

The Least Cost Diagonal

As noted above, when using ordered point boundaries, the best match occurs along a diagonal of the cost matrix. Such a result is both visually and intuitively appealing. Looking at (Figure 6) the rotated triangle, we observe that the best match should always be in order since the points on the boundary are in order. And because the cost matrix is n^2 in size, the matching algorithm must do a maximum of n^2 comparisons. Therefore, finding the best match of one shape to another when both shapes have ordered boundary points is a simple matter of finding the diagonal that represents the least cost, hence the name “the least cost diagonal”.

Experimental Results

The least cost method was compared using a database of pottery figures. The shapes of the pottery had been previously classified by Bishop [4]. Each pottery class was made up of between 8 and 11 pottery shapes. The shapes of class 2 are shown in (Figure 7). Nineteen experiments were conducted. Each compared the shapes of one shape class to the other shape classes. Each comparison was in turn run with varying numbers of points in the boundary of the shape: 3,4,5,10,20 and 100 points. The cost matrix was analyzed using NN (nearest neighbor) techniques.

Figure 7. Pottery Shapes for Class 2

The above experiments were performed using the Hungarian method and the “Least Cost Diagonal” methods. The correctly classified shapes for each number of boundary points was then calculated, plotted and compared. The results a displayed in (Figure 8)

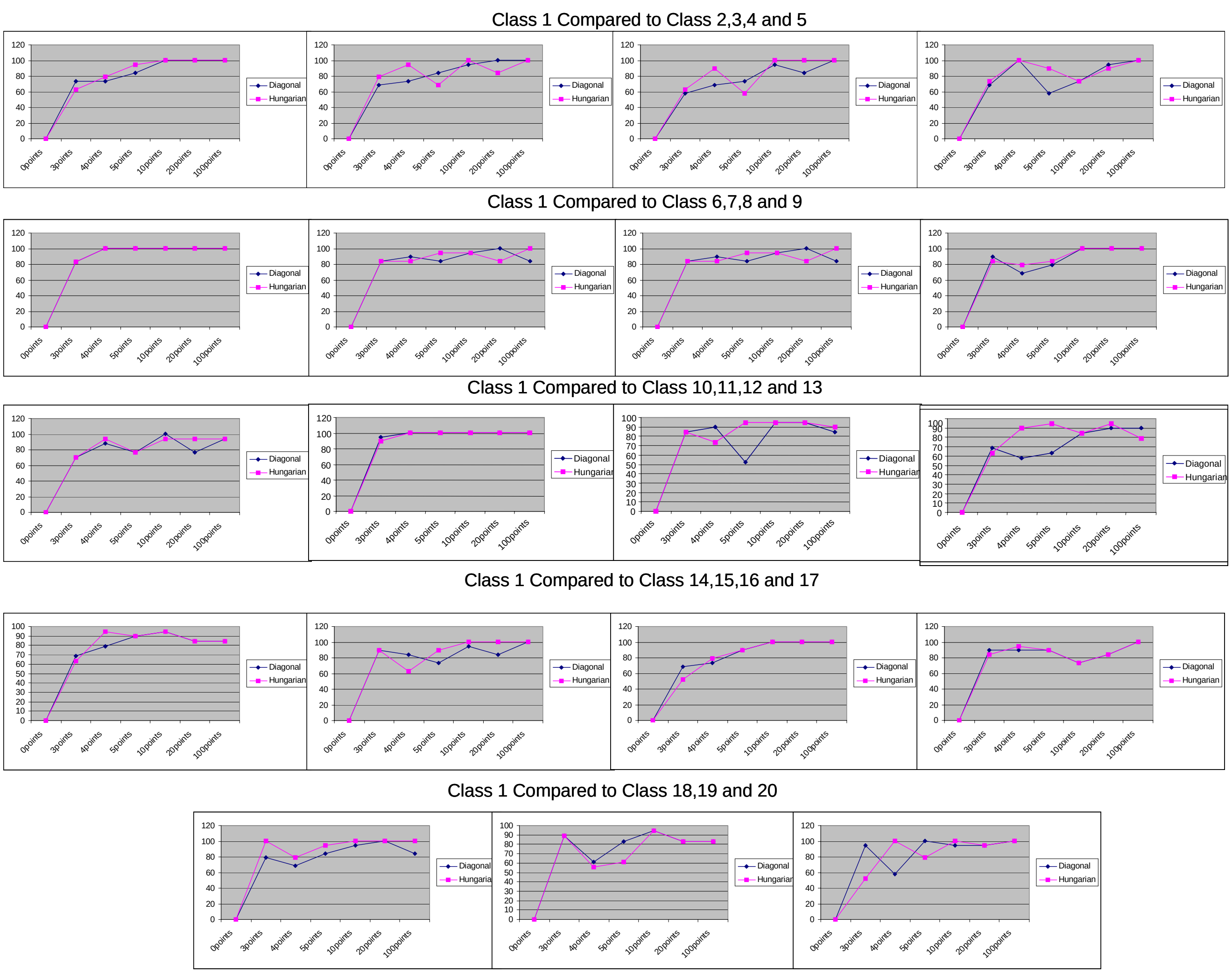


Figure 8. Results of Pottery Class Comparison Run

The timing comparison of the two methods is shown in (Figure 9) The x-axis tick marks represent 3,4,5,10,20 and 100 boundary points. The y-axis is the time in seconds to complete a shape comparison.

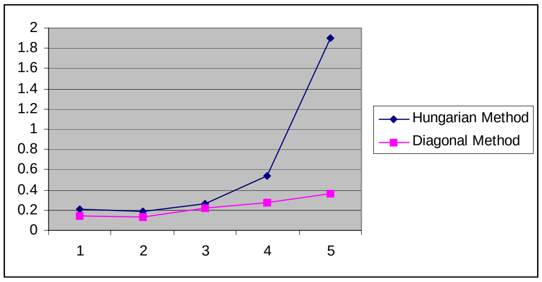


Figure 9. Timing Comparison of Diagonal Method and the Hungarian Method

Summary and Future Work

The results of these experiments show that in the case of ordered boundary points of an object, the least cost diagonal method using shape contexts is at least as good as shape contexts based on the Hungarian method but runs in $O(n^2)$. Future work will investigate the least cost diagonal method and rotational invariance as well as the relationship of the number of boundary points used and their impact on shape differentiation.[1]

References

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