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The Origins and Rise of Medieval Information Visualization

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Abstract — **This paper considers the origins of the earliest designs for medieval information visualization. Works by Macrobius, Boëthius, and Isidore of Seville are examined. These works are placed within the timeline of information visualization, and suggestions are made as to their antecedents.**

Keywords - information visualization, medieval art, medieval manuscripts, history.

I. Introduction

The Middle Ages (c. 450 - 1450) was a time of transition and upheaval. After the fall of the Roman Empire in the West, monastic communities arose as preservers and disseminators of ancient knowledge, and sources of proto-scientific research. Early scholars grappling with information both sacred and profane invented charts and diagrams to confer visual form onto abstract concepts from classical, religious, and secular texts. Begun as a few incidental charts created to help communicate concepts within their manuscripts, these graphical representations had evolved by the twelfth century into routine formats for rendering scientific, philosophical, and theological truths.

This paper explores some of the earliest examples of medieval information visualization. Preserved and replicated in manuscripts from the Carolingian Renaissance (c. 780 - c. 900) onward [1], these manuscripts disseminated the visualizations that were either devised to clarify the texts of early authors, or were created by early authors as parallel depictions of information contained within their texts. The works will be placed within the information visualization timeline, and suggestions will be made as to some of their precursors. The paper's reviews imagery that has been at least nominally addressed within the histories of art, religion, and science, reconsidering it from the perspective of the history of information visualization.

The paper begins with a brief review of the historical context underlying early medieval scholarship, focusing on the kinds of texts that were available for study during the Carolingian Renaissance. This is the earliest time period from which manuscripts exist containing the visualizations discussed herein. Sections follow describing contributions to information visualization by the early medieval authors Macrobius, Boëthius, and Isidore of Seville. A final discussion concerning the origins of these visualizations closes the paper.

II. Background

The decline of the Roman Empire throughout Western Europe and its ultimate disintegration in the fifth century gradually engendered a loss of classical knowledge linked to the long tradition of Hellenistic scholarship communicated through the ancient Greek language. Although Greece had been under Roman rule as early as 146 BCE, and formally annexed by Augustus in 27 BCE, Greek remained the recognized language of the Eastern Mediterranean, and the language of Roman erudition. A nearly complete disengagement with Greece meant that by about the year 450 CE the overwhelming majority of classical texts available to scholars were those written in the original Latin or translated from the Greek into Latin. In the latter case, few translations existed from classical Rome because, for the most part, the educated elite of that time were trained to read Greek and saw little reason to translate these works [2, Chap. 7].

The available Roman manuscripts that would eventually have the greatest impact on early medieval authors and scholars were written by Pliny the Elder (23 CE – 79 CE), a historian, naturalist, natural philosopher, and author of *Natural History*, a comprehensive encyclopedia of the natural world; Marcus Vitruvius Pollio (d. 15 BCE), architect and engineer who wrote *De Architectura* (On Architecture); Martianus Capella (c. 410 - 439 CE), writer of *On the Marriage of Philology and Mercury,* an encyclopedic work posed as an allegory, that encompassed the seven liberal arts (grammar, logic, rhetoric, arithmetic, geometry, astronomy, and music), and became a major influence on Medieval education; and finally Macrobius Ambrosius Theodosius (c. 395–423), commonly called Macrobius, writer of *Commentary on the Dream of Scipio* (*Commentarii in Somnium Scipionis*), which laid out a comprehensive philosophy of nature encompassing arithmetic, astronomy, and cosmology.

Works in Latin that were available included the Vulgate Bible, translated by St. Jerome in the late 4th century; Plato's *Timeus,* translated into Latin by Cicero (106 BCE – 43 BCE) and Calcidius (a late 4th century philosopher); Euclid's *Elements*; and Porphyry's *Introduction to Aristotle's Logic,* translated by Anicius Manlius Severinus Boëthius (480 – c. 524). In addition, Boëthius wrote handbooks based on Roman and Greek sources covering logic (*In Ciceronis Topica*), music (*De institutione musica*), and arithmetic (*De*

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institutione arithmetica). But by 524, the year of Boëthius's death, it remained to be seen whether any of these works, other than the Bible, would be extensively studied [3, Chap. 3].

The problems facing medieval scholarship were manifold. The gradual dissolution of the Roman Empire brought with it a degradation in education, and hence an increase in illiteracy. Early medieval monasteries were focused on studying the Bible, not pagan works such as those created by natural scientists and philosophers. Copying manuscripts was not a priority. Performed with little oversight, usually at the whim of a literate religious, the crude artisanal character of these works mitigated any formal effort at extensive dissemination. Ultimately, it would take over three centuries for these problems to be resolved.

One important thing that needed to occur was a reorientation of thought towards the kind of knowledge that was acceptable for study within a monastic community. This conceptual change would derive from the writings of Augustine of Hippo (354 – 430) (St. Augustine), North African Bishop of the Christian Church, who merged the Greek philosophical tradition with the Judeo-Christian religious and scriptural traditions [4]. St. Augustine was not only a major influence on medieval philosophy, but also influenced the writings of a long line of philosophers including Martin Luther, Emanuel Kant, Martin Heidegger, Friedrich Nietzsche, and Bertrand Russell. St. Augustine's message for early medieval scholars was that the Bible was not the only source of truth; truth could be discovered anywhere. Since the universe was God's creation, any truth would naturally stem from Him. The implications for medieval scholarship were two-fold. First, history, philosophy, dialectic, and rhetoric – the core of the Hellenic scholarly tradition – were neither secular nor human, they came from God. So these works could be employed as part of monastic practice. Second, since truth was "discovered," not created, St. Augustine's philosophy endowed the search for truth with an epistemic dimension that would eventually lead to the development of proto-scientific research practice by early scholars such as Venerable Bede (c. 672 – 735) [5].

Literacy was addressed during the reign of Charlemagne (c. 742 – 814). As the lands under his dominion continued to grow during the eighth century, there were insufficient literate individuals to help administer the expanding state. For the Church at that time, illiteracy meant that not all priests possessed the requisite skill to read the Vulgate Bible. Language and communication were issues as well. The decline of the Roman Empire had engendered a regionalization of Latin dialects, the future modern romance languages, which seriously impeded communication across Europe. During the latter quarter of the eighth century Charlemagne executed a program of reforms that would transform the state and become known as the Carolingian Renaissance. A major part of his program was to attract many of the leading scholars of his day to his court. With the aid of one of these scholars, the English monk Alcuin of York (c. 735 – 804), who arrived at his court in 782, a program of cultural revitalization and educational transformation was undertaken to restore old schools and found new ones throughout his empire under the guidance of a monastery, cathedral, or noble court. A standard curriculum was developed that established the trivium (grammar, logic, and rhetoric) and quadrivium (arithmetic, geometry, music, and astronomy) as the basis for education, and writing of textbooks was undertaken. A standardized version of Latin was also developed that became the common language of scholarship and supported pan-European administration of the empire. Writing was standardized too. The Carolingian minuscule was introduced to increase the uniformity, clarity, and legibility of handwriting. It was used between 800 and 1200 to write codices, pagan and Christian manuscripts, and educational texts.

III. Medieval Visualization

In general, early medieval scholars were more concerned with the study and preservation of classical texts than generating new scholarship. For example, in Eastwood's discussion of Roman astronomy's impact on Carolingian science education [6], he considers the four most influential and widely disseminated classical scientific texts that can be found in the manuscripts originating during the Carolingian Renaissance [2, p. 197]: Macrobius's *Commentary on the Dream of Scipio*, Pliny's *Natural History*, Martianus Capella's *On the Marriage of Philology and Mercury*, and Calcidius's *Commentarius* (Commentaries) on Plato's *Timaeus*. What is significant about these works from an information visualization perspective are the diagrams explicitly created to elucidate concepts within their respective texts. In some cases the diagrams that appeared in manuscripts needed to be invented by medieval readers, because none were specified in the original source (e.g. Pliny, Capella). In other cases, diagrams that were in an original manuscript (e.g. Calcidius) did not survive uncorrupted into the ninth century, necessitating either their reconstitution or invention anew. And finally there were manuscripts in which the original author (e.g. Macrobius) explicitly stated how these diagrams were to be constructed. But even precisely defined procedures were misinterpreted. As Eastwood has observed, Carolingian students and teachers alike were challenged by the process of transforming text into visualizations. Thus, given their insufficient knowledge of geometry, and their difficulty in assessing a diagram's correctness; the design, construction, validation, standardization, and integration of these diagrams into pedagogical practice evolved into a protracted process that formally did not abate until the infusion of scholarly Greco-Arabic texts into Western Europe by the twelfth century.

Most examples of information visualization are found in Carolingian manuscripts associated with the quadrivium, the segment of Carolingian education focused on arithmetic, geometry, music, and astronomy; with the great preponderance of charts and diagrams being astronomical or cosmological in nature (c.f. Eastwood [6] for a complete discussion). Works by three seminal authors will be considered: Macrobius's *Commentary on the Dream of Scipio* (*Commentarii in Somnium Scipionis*), Boëthius's *De institutione arithmetica* and *De institutione musica,* and Isidore of Seville's (c. 560 – 636) *Etymologiae* (*Etymologies*) and *De natura rerum*. These three scholars not only created manuscripts that significantly influenced medieval thought, but produced writings containing diagrams that delineate the origins of medieval information visualization.

IV. MACROBIUS

Macrobius' *Commentary on the Dream of Scipio* [7] is an explanation and amplification of text from the final section of Cicero's *De re publica*, a treatise on the state of the Roman republic. The *Dream of Scipio* appears in the sixth and last book of Cicero's work [8], in which he employs a Socratic dialog to communicate a cosmic vision experienced by Scipio Aemilianus, Roman military tribune and future consul, to elucidate Rome's relation to the cosmos. In effect, *Scipio's* dream is a cosmological treatise that has been transformed by Macrobius' *Commentary* into an early medieval astronomical primer.

Macrobius relied substantially on visualization as a means for conveying Cicero's ideas. He believed that the visual channel was a far faster means of communicating concepts than either text or speech [9]. Five diagrams are integrated into his *Commentary*, with instructions for drawing four of them offered as part of its narrative stream [9]. Figure 1 shows a drawing of a map of the Earth divided into frigid, temperate, and torrid zones taken from a manuscript created approximately 820 in northern France based on Macrobius' c.430 text (MS Lat. 6370, fol. 81r, Paris, Bibliothèque Nationale de France). The letters along the circle's periphery refer to their respective narrative text in *Commentary* and are intended to enhance the reader's understanding of Macrobius' explanation of zonal theory.

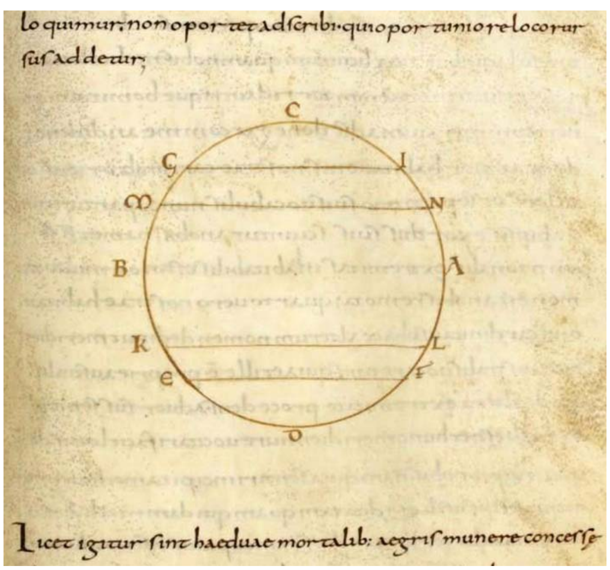

Figure 1. Zonal map of the Earth from Macrobius' *Commentary on the Dream of Scipio* (MS Lat. 6370, fol. 81r, Paris, Bibliothèque Nationale de France).

Macrobius' zonal map organized the spherical world into five climate zones: the northern and southern frigid zones, northern and southern temperate zones, and an equatorial tropical zone. Only two of these zones were believed to be inhabitable, with the known world populating the northern temperate zone's Eastern Hemisphere. Because most surviving zonal maps are traceable to illustrations in Macrobius' *Commentary*, this map is also known as a "Macrobian" map.

The significance of the visualizations in Macrobius' *Commentary* is not that they are early visual representations of information, which they are, but rather that they represent the results of a formalized process for visualizing information which Macrobius explicitly states as part of his narrative. The difference between "learning something" and "learning to do something" clearly was important to Macrobius, otherwise he could have easily omitted the procedural details for diagram creation. The ultimate result is that his *Commentary* is transformed, at least in part, from an informational treatise to a handbook along the lines of Vitruvius' *De Architectura*, in which information sets the context for a formal discussion of architectural and engineering practice.

V. BOËTHIUS

Boëthius, along with Augustine and Aristotle, is considered to be the fundamental philosophical author of late antiquity, his most famous work being *De consolatione philosophiae*, written during his imprisonment in Pavia before his execution there in 524 [10]. His treatises on arithmetic, geometry, and music were not only essential contributions to the quadrivium, but also contain a wealth of visualizations. The sources for his works on arithmetic, *De institutione arithmetica* [11], and music, *De institutione musica*, are believed to have been Pythagoras, Plato, Aristotle; and, in particular, his direct translations from the Greek of Nicomachus' (c. 60 – c. 120) works, *Introduction to Arithmetic* [12] and *Manual of Harmonics*.

Boëthius believed as Macrobius that visual representation of information was an important pedagogical tool for clarifying concepts. As a result, his works on arithmetic and music are filled with illustrations. Figure 2 shows a diagram from Book 1 of Boëthius's *De institutione arithmetica* taken from a mid-ninth century manuscript (MS Cod. Sang. 248, fol. 10a, St. Gallen, Stiftsbibliothek), which he uses as part of a discussion of "the nature of the odd times the even."

At the core of this diagram is a 4 x 4 square array consisting of even numbers that are related through multiplication. Boethius uses latitude and longitude to designate rows and columns, respectively, and connects these with arcs, each of which is labeled with a number that is the product of two numbers that anchor each arc. For example, the multiplication of far left (12) and right (96) numbers of the top row produces 1152 (īcLii), the numbers viewed along the apical arc.

Arcs are an important part of Boethius's graphical schema. He uses them in his discussions of arithmetic and geometric ratios, and they are integral to his treatise on music which has its foundation in ancient Greek musical theory. Figure 3 displays an arc diagram taken from a twelfth century version of Boëthius's *De institutione musica* (MS VadSlg 296, fol. 53r, St. Gallen, Kantonsbibliothek, Vadianische Sammlung). It shows the ratios: 4:2 (*dupla*) that yield the octave (*diapason*); 6:2 (*tripla*), that represents the twelfth (*diapason ac diapente*); and 6:4, a ratio that governs the fifth (*diapente*).

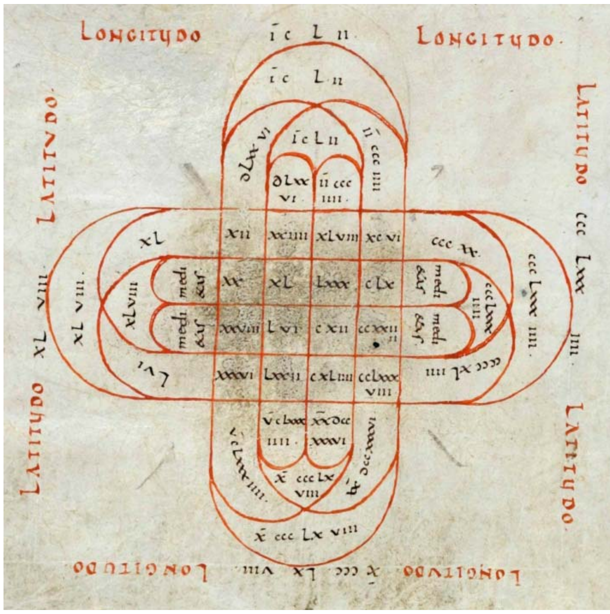

Figure 2. Aritmetic diagram from Boëthius’s *De Arithmetica* (MS Cod. Sang. 248, fol. 10a, St. Gallen, Stiftsbibliothek).

Figure 3. . Musical arc diagram from Boëthius’s *De institutione musica* (MS VadSlg. 296, fol. 53r, St. Gallen, Kantonsbibliothek, Vadianische Sammlung).

In Figure 4 Boëthius goes beyond this simple arc diagram by charting three tetrachord genera for the monochord. The monochord is a single stringed musical instrument invented by Pythagoras to study ratios that eventually became an integral part of medieval music education. A tetrachord is a four note series having a specific pattern of whole and half steps, that served as a basis for Greek and Medieval melodic construction. Here, the three monochord genera are plotted from top to bottom: the diatonic genus (e.g. whole, whole, half steps), chromatic genus, and enharmonic genus. What is immediately noticeable from this image is its overall complexity. It is probably the most complex visualization of early medieval times, and concomitantly the most difficult to craft and understand, even with direct reference to the associated text. Yet, this visualization and nearly all other illustrations in Boëthius' manuscript were copied and disseminated throughout the Middle Ages, making *De institutione musica* the standard musical textbook for over 400 years. Clearly, these illustrations were an important part of musical pedagogy that justified the investment of significant skill and effort required for their reproduction.

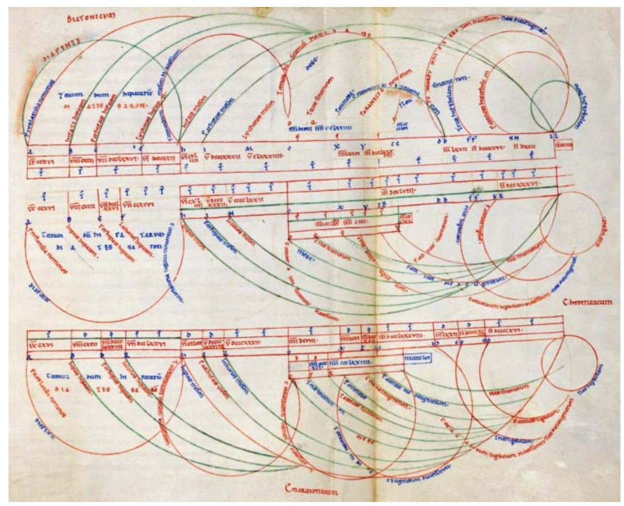

Figure 4. Musical arc diagrams for the monochord from Boëthius’s De institutione musica (MS VadSlg. 296, fol. 96r, St. Gallen, Kantonsbibliothek, Vadianische Sammlung).

VI. Isidore of Seville

Isidore of Seville is considered one of the greatest scholars of late antiquity who is best known as the first encyclopedist of the Middle Ages [13]. His *Etymologiae* (*Etymologies*) is an encyclopedic work in twenty books that quotes over 154 classical authors encompassing grammar, religion, law, agriculture, medicine, and more [14]. Through the Middle Ages *Etymologiae* was the textbook most in use, regarded so highly as a repository of classical learning that, in a great measure, it superseded the use of the individual works of the classics themselves, full texts that were no longer copied, and thus were lost. *Etymologiae* and *De natura rerum* [15] are two of his works that are of interest to the history of information visualization because they contain diagrams that he considered useful for communication of ideas. In particular, *De natura rerum* contains seven figures, six of which are circular diagrams called *rotae* that Isidore employs for conveying a variety of concepts including cartography, computus, the elements, and the relation of man to the cosmos. Two *rotae* will be considered here.

Figure 5 displays the first *rota* that appears in a copy of *De natura rerum*, written between the years 760 - 780 (MS Cod. Sang. 238, fol. 325, St. Gallen, Stiftsbibliothek). It is designed to communicate the concordance between the Roman (Julian) and Egyptian calendars [16]. Egyptian calendars were composed of twelve 30 day months, with five days remaining at the end of the year. This *rota* serves to convey the number of days that the start of each Egyptian month precedes the onset of each Roman month. Beginning in this figure at a position just above 9 o'clock, the labels of the outer ring are read as the months January through December, for the twelve radial divisions of the circle. The second ring inward contains Roman numerals followed by the letters *KL*, which specify the number of days that the beginning of the Egyptian month precedes the kalends (KL), the first day of the Roman month. The three inner rings may be read from the central roundel outward in the following way to give the number of days in each month in the Julian calendar: *Roman numerals – diebus (days) – month name*.

Figure 5. Calendar, *De natura rerum*, Isidore of Seville, c. 760-780 **(**MS Cod. Sang. 238, fol. 325, St. Gallen, Stiftsbibliothek).

The centers of many of these kinds of *rota* typically remain blank, but this example contains an illustration of an individual flanked by two birds. It has been suggested that this image represents Christ as a young man, but it may just as easily represent the author of the document himself. Lending credence to the former proposition is that *De natura rerum* embodies Isidore's understanding of the physical universe arranged into a narrative format that follows the Bible's order of creation. It should be remembered that Isidore was a Church Bishop whose charge was to spread Christian teaching and address paganism, so an image of Christ at the center of "time" makes conceptual sense.

It is also interesting to note that Isidore's *rota* carries with it a less explicitly communicated layer of meaning related to the Roman-Egyptian calendars. Roman calendars represented a computistical approach to fixing time, based on a government's authority at preordaining important dates for its population to observe; while Egyptian calendars represented time as a natural phenomenon, experienced as a progression of the sun's movements through the seasons. In the end, the Church would take a computistical approach [17][18] to accommodate both methods for measuring the passage of time in its calculation of important feast days, and thus affirm its magisterial authority over its followers.

The second *rota* from Isidore's *De natura rerum* is shown in Figure 6. It is an *Annus-Mundus-Homo* diagram from a manuscript produced on or about the year 800 in the women's cloister of Chelles Abby on the river Marne near Paris (MS Cod. Sang. 240, fol. 137, St. Gallen, Stiftsbibliothek). This diagram expresses a combination of Greek thought on the nature of the universe and the state of mankind.

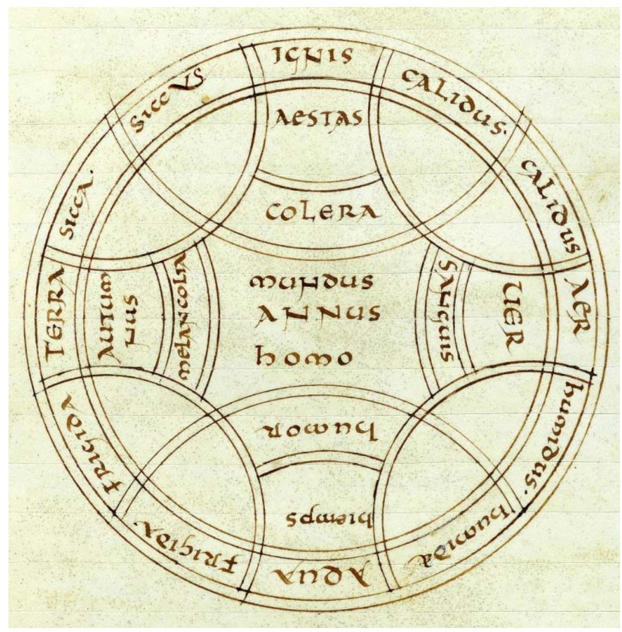

Figure 6. *Annus* - *Mundus* - *Homo* diagram, Isidore of Seville, c. 800 **(**MS Cod. Sang. 240, fol. 137, St. Gallen, Stiftsbibliothek).

The ancient Greek cosmology of Empendocles, Pythagoras, and Plato begins with two properties of matter and their oppositions – hotness and coldness, moistness and dryness. When taken in combination, these properties form the four basic elements (earth (terra), air (aer), fire (ignis), and water (aqua)) displayed in logical opposition along the cardinal directions of Figure 6. Fire and water are opposites, as are earth and air. These elements are linked by shared properties. For example, fire and air share the property hotness (calidus); water and air share the property moistness (humidus), and so on.

Two additional sets of relationships are layered upon the diagram. The first set contains the four seasons: aestas (summer), autumnus (autumn), hiems (winter), and ver (spring) displayed as rational opposites around the *rota* in counterclockwise order. The second set is related to the concept of the four humors of Hippocratic medicine used to describe the human temperament. These are exhibited clockwise within the innermost ring of the *rota* as colera (bile), sanguis (sanguine), phlegmatic (here incorrectly given as *humor* instead of *pituita*), and melancholia (melancholy).

This *rota* aligns the cosmic with the human, promulgating both the ancient Greek notion and Isidore's thesis that man is a small scale (microcosm) parallel of the universe (macrocosm) [19]. It is also a visual schema for guiding multilevel thought. The *rota's* center points to three levels of engagement: the observer (homo), the world (mundus), and the year (annus). The concentric rings of information underscore the dynamic relationships therein, such as the mobility of elements from ignis to aer to aqua to terra; the cycles of the seasons; or the transitions in qualities from calidus (hot) to humida (humid) to frigida (cold) to sicca (dry). The schema's generality lends it to be used for displaying the direction of the winds, temperatures, and more. In all, *rotae* became the standard means for visualizing a multiplicity of concepts through the thirteenth century.

VII. ORIGINS

Pinpointing either the true author or inventor of these visualizations is an open issue. Unlike today's manuscripts, in which rigorous attribution to external sources is essential, the works of medieval manuscripts may be a blend of adaptations from earlier authors, original content, and personal commentary that, at best, offer only minimal credit to works of others. Medieval manuscripts may not be exact transcriptions of lost originals. Copyists may have modified or adapted each manuscript to suit their particular needs.

One approach to assessing whether a diagram was at least intended to be part of a manuscript is to look at where it resides. If it is found in a marginal gloss, it is likely to have been added later. If the image is found within the flow of the narrative text, then space was made for its insertion, possibly at the author's direction. This is the case in Macrobius' *Commentary* where a space was left for a figure (c.f. Figure 1). It holds true for Boëthius and Isidore as well, since their texts explicitly refer readers to explanatory diagrams. But fixing invention is another matter.

The visualizations of each author discussed here have at least conceptual antecedents. Macrobius' zonal map of the earth in Figure 1 dates back to the Pythagoreans with references to the cosmography of Eratosthenes (c. 275-194 BCE), Posidonius (c. 151-35 BCE) and Crates of Mallos (c. 168 BCE) [20]. Yet, until Macrobius, there seems to be no extant physical record of a diagram of this kind.

Boëthius' arc diagrams, in their simplest form, have been used for logic. Their application is traceable back to Plato, in works such as *Timaeus* and *The Republic,* and may have been employed as far back as Pythagoras [21]. However, none are found in any existing copies of the original versions of the manuscripts [22].

Isidiore's *rota* shown in Figure 6 may be seen to mirror Greco-Roman designs. For example, the 2nd century mosaic of Neptune shown in Figure 7 contains a central medallion surrounded by women representing the four seasons. Winter is the most heavily clothed of the four, surrounded by branches of winter berry. Moving counter-clockwise from winter in the lower left corner brings spring with blossoms, summer with ripened wheat, and autumn surrounded by grapes. A variation on this schema may be found in a 4th C mosaic from Dorset, England where Neptune is replaced by Christ and the four winds replace the four seasons [23].

Figure 7. Neptune Mosaic, 2nd century CE. Bardo Museum.

VIII. FINAL THOUGHTS

This paper has explored the origins of some of the earliest designs for medieval information visualizations by focusing on works by Macrobius, Boëthius, and Isidore of Seville, three authors of late antiquity whose manuscripts were revived during the Carolingian Renaissance as part of cultural and educational reform. It has placed these visualizations within a timeline of information visualization, and suggested that their conceptual antecedents may be found in antiquity. Yet, it may not be the act of invention that makes these visualizations important, but rather their innovative application and extension that marks these works as special. Disseminated by the Carolingian network of schools, they influenced the way information was communicated for the next four hundred years.

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