Contention Resolution in Multiple-access Channels: k-Selection in Radio Networks

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Shared Resource Contention

- Unique resource to be shared among many users
- All contenders must have access eventually
- Only one user at a time may have access

$\Longrightarrow$ Contention.

- Unknown number of contenders

$\Longrightarrow$ up to n.

- E.g.: k-Selection in Radio Networks:

  "unknown size-k subset of network nodes
  must access a unique shared channel of communication,
  each of them at least once."

- Q: k-Selection with unknown k in $O(k)$?

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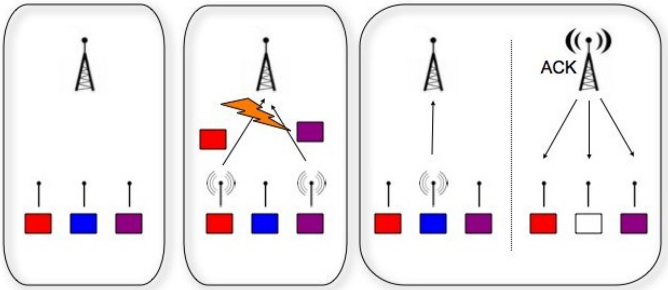
Radio Network Model and Notation

- labeled stations called *nodes*.
- no information except own ID (unique, arbitrary) and n.
- time slotted in communication steps.
- nodes are potentially reachable in one comm step $\rightarrow$ *single-hop*.
- time complexity = communication steps (negligible computation cost).
- piece of information to deliver called *message*.
- node is *active* if holds a message to deliver.
- message assignment is external (called message *arrival*).
- *batched* message arrivals (*static* k-Selection).
- number of messages left to deliver called *density*.

Radio Network Model

- communication through radio broadcast on a shared channel.
- in one step:
  - no message transmitted → all nodes receive background noise.
  - more than one node transmits → all other nodes receive interference noise. (collision, messages garbled, etc.)
  - exactly one node transmits → all other nodes receive message and the sender receives an ack.
    (successful transmission, message delivered, single transmission, etc.)
- nodes can not distinguish between interference and background noise.



Related Work

Randomized

- With collision detection:
  - Willard'86:
    - Expected $\log\log k + o(\log\log k)$ for *Selection* with unknown n.
  - Martel'94:
    - k-Selection expected $O(k + \log n)$ with known n.
      Kowalski'05: can be improved to expected $O(k + \log\log n)$ using Willard's.
- Without collision detection:
  - Kushilevitz, Mansour'98:
    - For any given protocol, $\exists k$ s.t.
      expected $\Omega(\log n)$ to get even the first message delivered.

Related Work

Beyond Radio Networks

- Greenberg, Leiserson'89: *randomized routing of bounded number of messages in fat-trees* $\Rightarrow$ k-Selection if constant edge-capacities. $\Rightarrow$ logarithmic congestion parameter $\Rightarrow$ $O(k \text{ polylog } n)$.
- Gerèb-Graus, Tsantilas'92: *arbitrary h-relations realization* in $\Theta(h + \log n \log\log n)$ w.h.p. $\Rightarrow$ k-Selection if $h = k$. Needs h known.
- Bender, Farach-Colton, Kuszmaul, Leiserson'05: *Back-off strategies for contention resolution of batched arrivals of k packets on simple multiple access channels*
  log log-iterated back-off $\rightarrow \Theta(k \log\log k / \log\log\log k)$
  w.p. $\geq 1 - 1/k^{\Theta(1)}$, without knowledge of a bound on k.

Related Work

Deterministic

- *Tree* algorithms: $O(k \log(n/k))$ [H'78, MT'78, C'79]. Adaptive, with collision detection.
- Greenberg, Winograd'85: tree algorithms take $\Omega(k \log_k n)$.
- Greenberg, Komlòs'85: $\exists$ *oblivious* protocols $O(k \log(n/k))$ even without collision detection if k and n are known.
- Clementi, Monti, Silvestri'01: matching lower bound. Also holds for *adaptive* protocols if no collision detection.
- Kowalski'05: oblivious deterministic protocol $O(k \text{ polylog } n)$ without collision detection, using Indyk'02 explicit selectors.

Result

- Back-on/back-off Randomized k-Selection in one-hop Radio Network in $(e+1+\xi)k + O(\log^2(1/\varepsilon))$ with probability $\geq 1-\varepsilon$ unknown k ($\xi > 0$ arb. small constant).
  - Optimal (modulo a small constant factor < 4) if $\varepsilon \in \Omega(2^{-\sqrt{k}})$.
  - Given $\Omega(k \log\log k / \log\log\log k)$ [Bender et al.'05] for monotonic back-off, shows separation using back-on strategies.
  - Improves over loglog-iterated back-off $O(k \log\log k / \log\log\log k)$ [Bender et al.'05] by exploiting back-on and knowledge of n.
  - Error probability is parametric $\Rightarrow$ suitable to be used in multi-hop Radio Networks.

Protocol for node x

without constants

- Algorithm AT: (if $\delta > \log(1/\varepsilon)$ messages left)

  Concurrent Task 1:

  $t = \log(1/\varepsilon)$. (set up a step counter)

  $\hat{\delta} = \log(1/\varepsilon)$. (set up a density estimate)

  for each communication step

  transmit $\langle x, message \rangle$ with probability $1/\hat{\delta}$.

  $t = t - 1$.

  if $t \leq 0$

  $t = \log(1/\varepsilon)$. (new round)

  $\hat{\delta} = \hat{\delta} + \log(1/\varepsilon)$. (update estimate)

  Concurrent Task 2:

  upon receiving a message from other node

  $\hat{\delta} = \max\{\hat{\delta} - 1, \log(1/\varepsilon)\}$. (update estimate)

  $t = t + 1$. (stretch round)

  Concurrent Task 3:

  upon delivering message, stop.

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Protocol for node x

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- Algorithm BT: (if $\delta \leq \log(1/\varepsilon)$ messages left)

  for each communication step

  transmit $\langle x, message \rangle$ with probability $1/\log(1/\varepsilon)$.

Protocol for node x

without constants

But we do not know δ $\rightarrow$ interleave both:

Concurrent Task 1:
  $t = \log(1/\varepsilon)$, $\hat{\delta} = \log(1/\varepsilon)$.
  for each communication step
    if step is even (Algorithm BT)
      transmit $\langle x, message\rangle$ with probability $1/\log(1/\varepsilon)$.
    if step is odd (Algorithm AT)
      transmit $\langle x, message\rangle$ with probability $1/\hat{\delta}$.
      $t = t - 1$.
      if $t \leq 0$
        $t = \log(1/\varepsilon)$, $\hat{\delta} = \hat{\delta} + \log(1/\varepsilon)$.
Concurrent Task 2:
  upon receiving a message from other node
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Analysis

Time Efficiency

- Failure steps:
  - Algorithm AT:
    - At most $k/\log(1/\varepsilon)$ rounds until the estimate reaches k.
    - Each round includes $\log(1/\varepsilon)$ failure steps.
  - Algorithm BT:
    - After $\leq \log(1/\varepsilon)$ messages are left, in $O(\log^2(1/\varepsilon))$ steps w.p. $1-\varepsilon$ BT delivers all.
- Success steps:
  - One step per message delivered $\Rightarrow k$ overall.
- Overall:
  - $(e+1+\xi)k + O(\log^2(1/\varepsilon))$
    (there are some constants...)

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Correctness sketch

Lemma

Until the estimate $\hat{\delta}$ is some "constant close" to the number of messages left δ, within a round, the probability of passing a message is not positively correlated with time.

Lemma

If $\hat{\delta}$ is "logarithmically close" to δ at the begining of a round, until the number of messages delivered in this round is logarithmic, the probability of delivering a message is at least constant.

$\Longrightarrow$ we can bound from below logarithmically the number of messages delivered in each round where the estimate is "close" to the number of messages left using Chernoff.

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Lemma

If δ is more than some logarithmic number M, after running the AT-algorithm for $(e+1+\xi)k$ steps, where $\xi > 0$ is any constant arbitrarily close to 0, $\delta \leq M$ with probability at least $1-\varepsilon$.

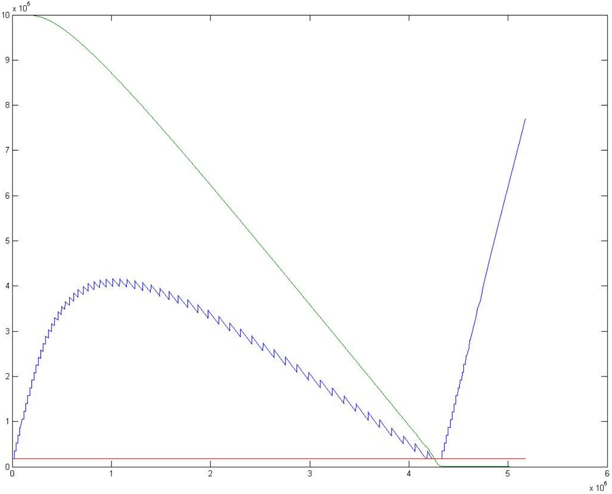
Proof.

- Using Chernoff, the probability of reducing δ logarithmically in each round after $\hat{\delta}$ is "logarithmically close" is at least $1-\varepsilon/(k+\varepsilon)$.
- Using an inductive argument over rounds and conditional probabilities, $\delta \leq M$ in one pass with probability at least $1-\varepsilon$.

□

Analysis

Illustration of estimate progress



Analysis

Correctness sketch

Together with Algorithm BT...

Theorem

For any one-hop Radio Network, under the model detailed, the protocol described solves the k-selection problem within $(e+1+\xi)k + O(\log^2(1/\varepsilon))$ communication steps, where $\xi > 0$ is any constant arbitrarily close to 0, with probability at least $1-\varepsilon$.

Application

Farach-Colton, Fernández Anta, Mosteiro
Optimal Memory-Aware Sensor Network Gossiping

Phases of the algorithm:

1. Partition nodes in *masters* and *slaves*
   $\Rightarrow$ MIS $\rightarrow O(\log^2 n)$
2. Every master reserves blocks of time steps for local use
   $\Rightarrow$ Coloring $\rightarrow O(\log n)$
3. Every master maintains set of messages received
   $\Rightarrow$ back-on/back-off $\rightarrow O(\Delta + \log^2 n)$
4. Every master disseminates local set
   $\Rightarrow$ flooding among masters $\rightarrow O(D)$

Overall:

$$O(\log^2 n + \log n + \Delta + \log^2 n \log \Delta + D) \in O(\Delta + D) \text{ w.h.p.}$$

Future Work

1. Remove the knowledge of n.
2. Generalize the system model:
   - Continuous message arrival.
   - Arbitrary wake-up.
3. Evaluation of the algorithm:
   - Study its practicality by simulation.
   - Compare it with other algorithms (e.g., Bender et al).
   - Improve its constants.

Thank you