Beeping Deterministic CONGEST Algorithms in Graphs

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September 16, 2025

Beeping Model

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- Nodes can *listen* or *beep*
- Beeps are heard by **all** neighbors
- At receiver: silence means 0, one or more beeps mean 1

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transmit 010 000 011

Beeping Model

transmit	010	000	011

receive	0_0	011	0__

Motivation

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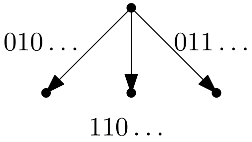
- suitable for cheap devices with low energy consumption (e.g., Sensor Networks)
- non-adaptive communication schedules in the Beeping Model are equivalent to superimposed codes
- studying natural communication in biological networks

Main results – simulation of CONGEST round

Problem: Each node has a (different) message of $O(\log n)$ bits for each of its neighbors. How do we transmit all these messages?



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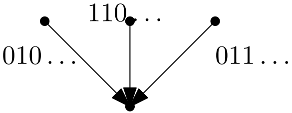

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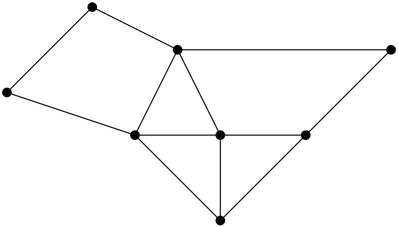

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- Matches the randomized result by Davies (PODC 2023) up to *polylog n* factor.
- Matches the lower bound $\Omega(\Delta^2 \log n)$ up to *polylog n* factor.

Main results – simulation of CONGEST round

- The algorithm utilizes multiple different families of *avoiding selectors*.
- The algorithm regularly delivers messages in such a way that the degree of the graph is effectively reduced and we can use selectors for smaller graph degree.

Main results – MIS

Simulation of CONGEST round applied to MIS algorithm by Ghaffari et al. (SODA 2021) gives the following result.

Corollary

There exists deterministic MIS algorithm in the Beeping Model that works in $O(\Delta^2 \text{ polylog } n)$.

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Simulation of CONGEST round applied to MIS algorithm by Ghaffari et al. (SODA 2021) gives the following result.

Corollary

There exists deterministic MIS algorithm in the Beeping Model that works in $O(\Delta^2 \text{ polylog } n)$.

- Beats the previous best result $O(\Delta^3 + \Delta^2 \log n)$ by Beauquier et al. (INFOCOM 2018) by a factor of Δ.

Our results – additional tools

Theorem

B-bit h-hop CONGEST simulation can be done in $O(h \cdot B\Delta^{h+2} \text{ polylog } n)$ rounds.

Theorem

B-bit h-hop CONGEST simulation must use at least $\Omega(B\Delta^{h+1})$ rounds.

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Note that repeated application of 1-hop CONGEST simulation would yield Δ^{2h} factor.

Our results – additional tools

Problem: Each node knows its ID. The task is to learn the IDs of all neighbors.

Theorem

Learning Neighborhood can be done in the Beeping Model in $O(\Delta^2 \log^2 n)$ *rounds.*

Our results – additional tools

Network decomposition is a central tool in distributed graph algorithms.

Theorem

$(\log n,\ \log^2 n)$*-Network Decomposition can be done in the Beeping Model in* $O(\Delta^2 \log^8 n)$ *rounds.*

Theorem

Cluster Gathering can be done in the Beeping Model in $O(\Delta^2 \log^4 n)$ *rounds.*

Summary of results and related work

problem	protocol type	beeping rounds	ref
simulation of one CONGEST round	randomized	$O(\Delta \min(n, \Delta^2) \log n)$ whp	PODC2020
		$O(\Delta^2 \log n)$ whp	PODC2023
	deterministic	$O(\Delta^4 \log n)$	INFOCOM2018
		$O(\Delta^2 \textit{ polylog } n)$	our result
	rand./det.	$\Omega(\Delta^2 \log n)$	PODC2023
B-bit h-hop simulation	deterministic	$O(h \cdot B\Delta^{h+2} \textit{ polylog } n)$	our result
	rand./det.	$\Omega(B\Delta^{h+1})$	our result
Learning Neighborhood	deterministic	$O(\Delta^2 \log^2 n)$	our result
Cluster Gathering		$O(\Delta^2 \log^4 n)$	our result
$(\log n, \log^2 n)$-Network Decomposition		$O(\Delta^2 \log^8 n)$	our result
MIS		$O(\Delta^3 + \Delta^2 \log n)$	INFOCOM2018
		$O(\Delta^2 \textit{ polylog } n)$	our result

Future Work

- Are there some graph problems that do not require Δ^2 factor in round complexity?
- Round complexity of Local Broadcast?

Thank you!