Probabilistic Lower Bounds on the Length of a Longest Edge in Delaunay Graphs of Random Points in a d-Ball

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The Problem

- Longest Delaunay edge in multidimensional Euclidean spaces.
- Multidimensional body of volume 1.
- Set of points distributed uniformly at random in it.

- Motivation:
  - Length of longest edge: bound communication cost in wireless nets.
  - Points deployment: RGG's.

- Length of longest Delaunay edge strongly influenced by boundaries:
  - with boundary (e.g. disk).
  - without boundary (e.g. sphere (ball surface)).

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Previous Work

- Longest Delaunay edge
  - inside a hypercube of size n (infinite Poisson point set)
    - Bern, Eppstein, Yao (JCGA 1991): observe $\Theta\left(\sqrt[d]{\log n}\right)$ in expectation.
  - RGG ($d = 2$):
    - Kozma, Lotker, Sharir, Stupp (PODC 2004):
      $O\left(\sqrt[3]{(\log n)/n}\right)$ w.h.p. for points "close" to boundary.
      $O\left(\sqrt{(\log n)/n}\right)$ w.h.p. for points "away" from boundary.
  - d-sphere and d-ball:
    - Arkin, Fernández-Anta, Mitchell, Mosteiro (CCCG 2011):
      Upper and lower bounds with ε error probability down to constants.
      Tight for $e^{-cn} \leq \varepsilon \leq n^{-c}$ and $d \in O(1)$.
      LBs for a d-ball only for $d \in \{2, 3\}$.
- Multidimensional Delaunay tessellations: (construction algorithms)
  - Devijver, Dekesel (PRL 1983)
  - Lemaire, Moreau (CG 2000)

Our results

We complete the study in [1] showing

lower bounds for a d-ball, for any $d > 1$

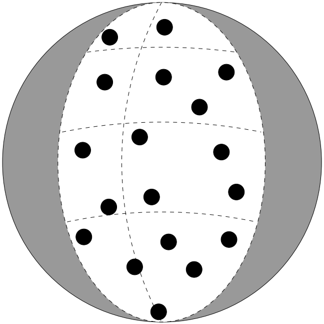
- down to constants
- parametric error probability ε
- asymp tight for $e^{-cn} \leq \varepsilon \leq n^{-c}$ and $d \in O(1)$

[1] E. Arkin, A. Fernández Anta, J. S. B. Mitchell, and M. A. Mosteiro, CCCG 2011.

Preliminaries

Definition

Let P be a set of points in a d-ball, two points $a, b \in P$ form an edge of $D(P)$, if and only if there is a d-ball B such that, a and b are located in the surface area of B, and the interior of B does not contain any other point of P.



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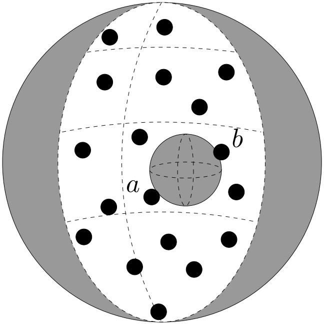

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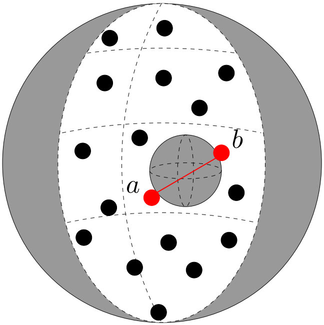


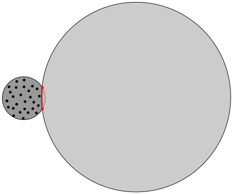
Results

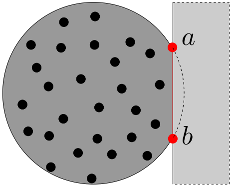
Proof techniques

- Lower bounds: show configuration such that

  “long” Delaunay edge is “not very unlikely”.

For enclosing bodies with boundaries...





... witness d-ball may be huge!

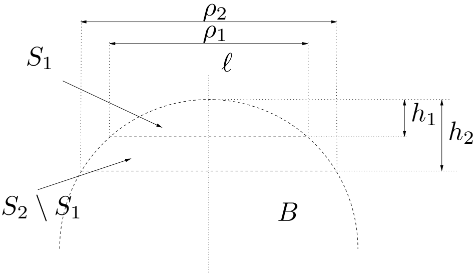
Results

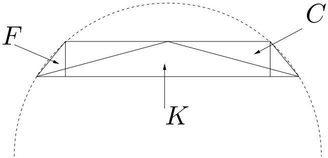
For points on a d-ball

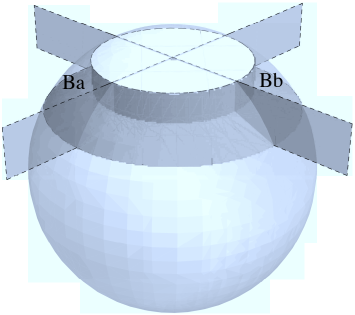
	[1] Upper bounds: w.p. $\geq 1-\varepsilon$, $\nexists\ \widehat{ab} \in D(P)$	[1] Lower bounds: w.p. $\geq \varepsilon$, $\exists\ \widehat{ab} \in D(P)$	Our results: w.p. $\geq \varepsilon$, $\exists\ \widehat{ab} \in D(P)$
d	$V_d(d(a,b)) \geq \dfrac{\ln \frac{\binom{n}{2}\binom{n-2}{d-1}}{\varepsilon}}{n-d-1}$	—	$V_d(x) = \dfrac{\ln \frac{\alpha(d)}{\varepsilon}}{\kappa_2(d)\left(n-2+\ln \frac{\alpha(d)}{\varepsilon}\right)}$ $d(a,b) \geq \frac{x}{\sqrt{d-1}}$
2	$\sqrt[3]{\frac{16}{\sqrt{\pi}} \ln \frac{\binom{n}{2}(n-2)}{\varepsilon} \over n-3}$	$\sqrt[3]{\dfrac{\ln \frac{\alpha_2}{\varepsilon}}{2\sqrt{\pi}\left(n-2+\ln \frac{\alpha_2}{\varepsilon}\right)}}$	$\sqrt[3]{\dfrac{4 \ln \frac{\alpha(d)}{\varepsilon}}{7\sqrt{\pi}\left(n-2+\ln \frac{\alpha(d)}{\varepsilon}\right)}}$
3	$\sqrt[4]{\frac{96}{\pi^{3/2}} \ln \frac{\binom{n}{2}\binom{n-2}{2}}{\varepsilon} \over n-4}$	$\sqrt[4]{\dfrac{\sqrt[3]{\frac{48}{\pi^4}} \ln \frac{\alpha_3}{\varepsilon}}{\left(n-2+\ln \frac{\alpha_3}{\varepsilon}\right)}}$	$\sqrt[4]{\dfrac{4\sqrt[3]{\frac{48}{\pi^4}} \ln \frac{\alpha(d)}{\varepsilon}}{\kappa_2(3)\left(n-2+\ln \frac{\alpha(d)}{\varepsilon}\right)}}$

$V_d(x)$: volume of a spherical cap of base diameter x.
[1] E. Arkin, A. Fernández Anta, J. S. B. Mitchell, and M. A. Mosteiro, CCCG 2011.

Proof sketch







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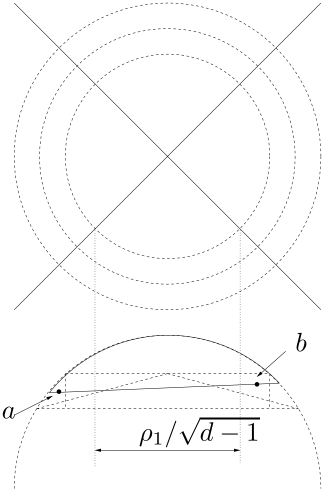

$Pr(\exists a) \in \Omega(1)$
$Pr(\exists b) \in \Omega(1)$
$Pr(S_2 \text{ is empty}) \geq \varepsilon$

Conclusions

- We have completed the study in AFMM-CCCG11 for arbitrary d.
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Thank you!