Polynomial Counting in Anonymous Dynamic Networks

with Applications to
Anonymous Dynamic Algebraic Computations

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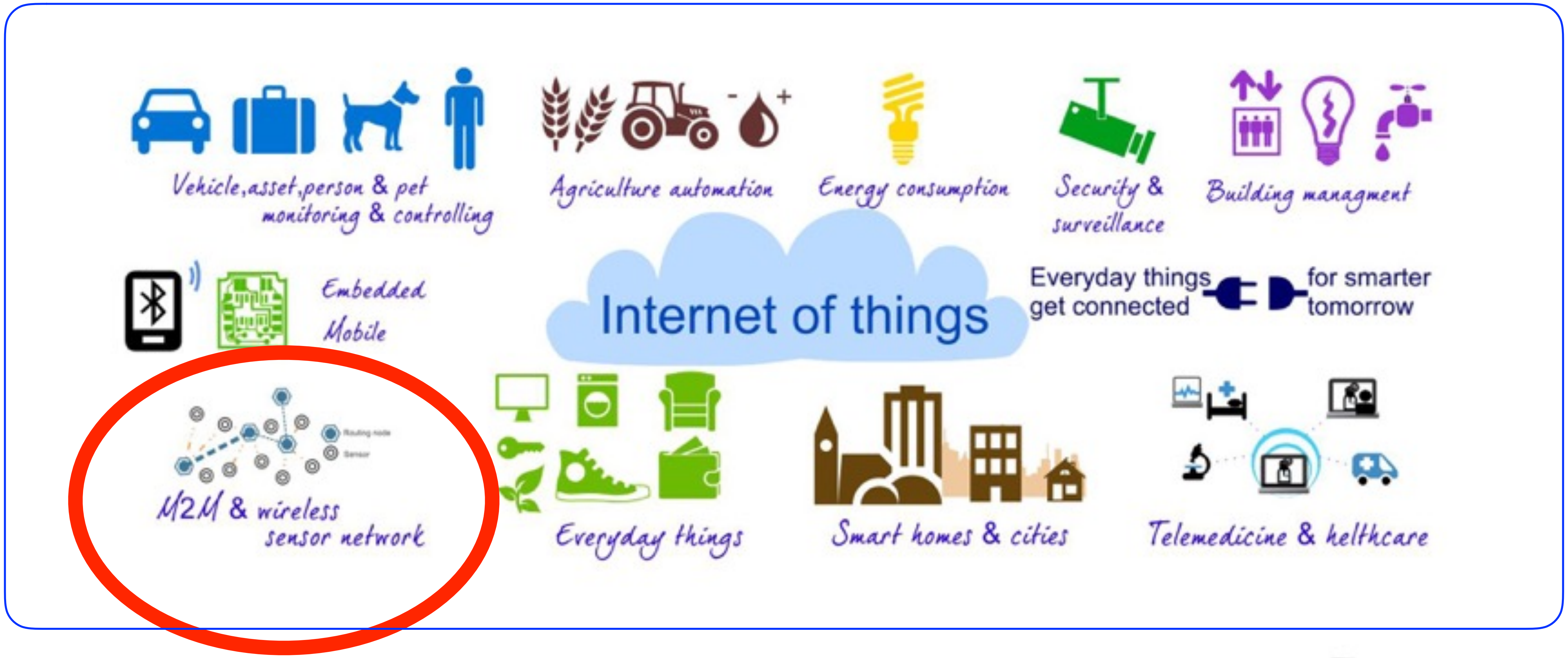
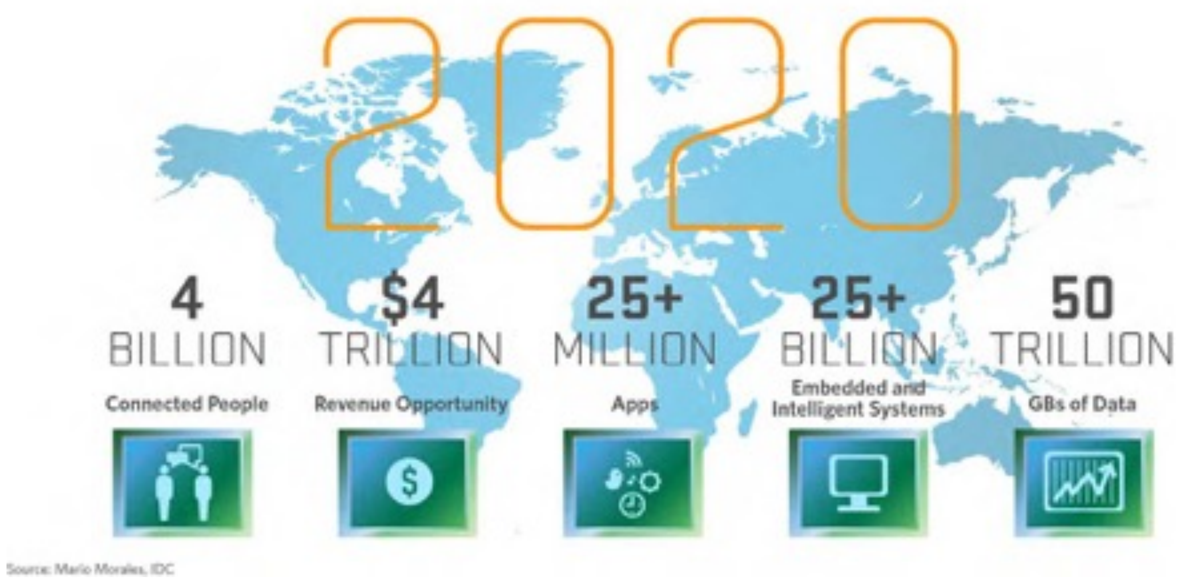
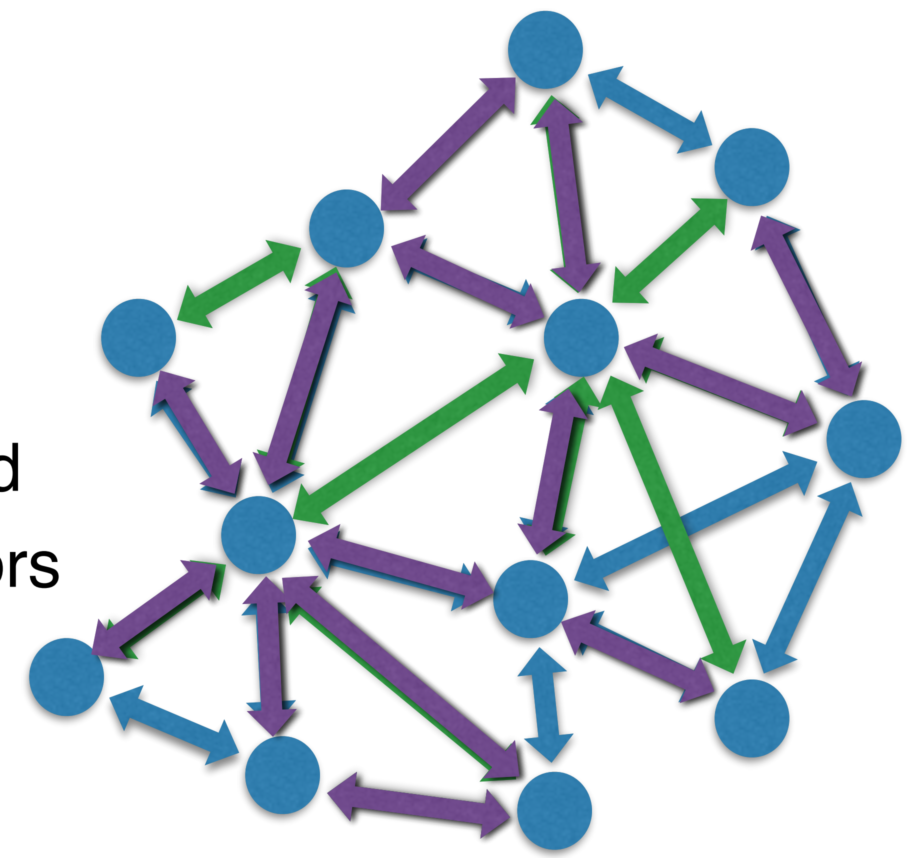
ICALP 2018

The Internet of Things





Anonymous Dynamic Networks



- **Fixed set of n nodes**
  - No identifiers or labels
  - A special node, called the leader [1]
- **Synchronous communication :** At each round
  - a node broadcasts a message to its neighbors
  - receives the messages of its neighbors
  - executes some local computation
- **1-interval connectivity** [2]
  - communication links may change from round to round, but
  - at each round the network is connected

[1] O. Michail, I. Chatzigiannakis, and P. G. Spirakis. Naming and counting in anonymous unknown dynamic networks. SSS 2013.
[2] F. Kuhn, N. A. Lynch, R. Oshman. Distributed computation in dynamic networks. STOC 2010.

The Counting Problem

How do you count the size of your group,

if the members are all identical and move?

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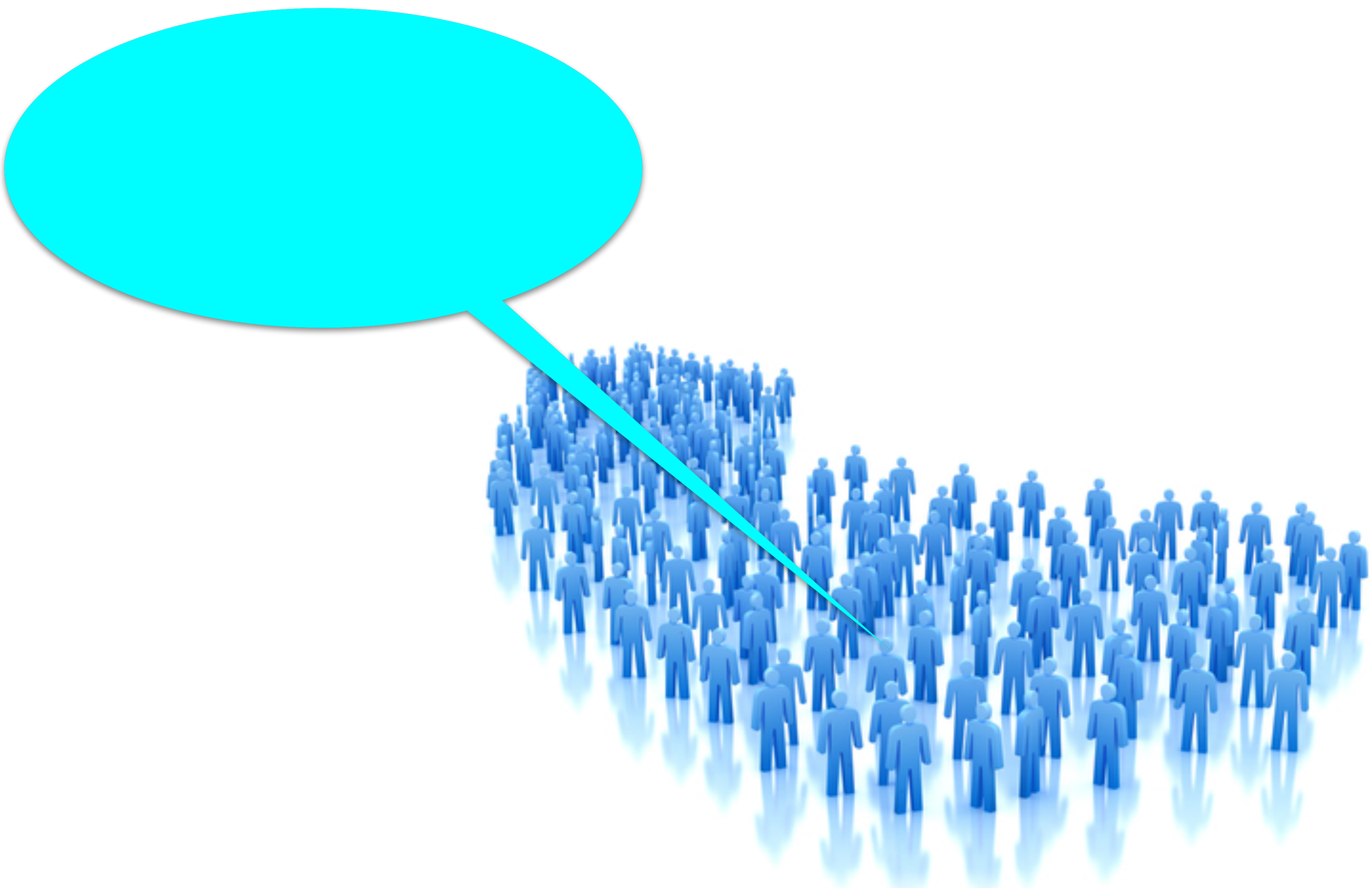

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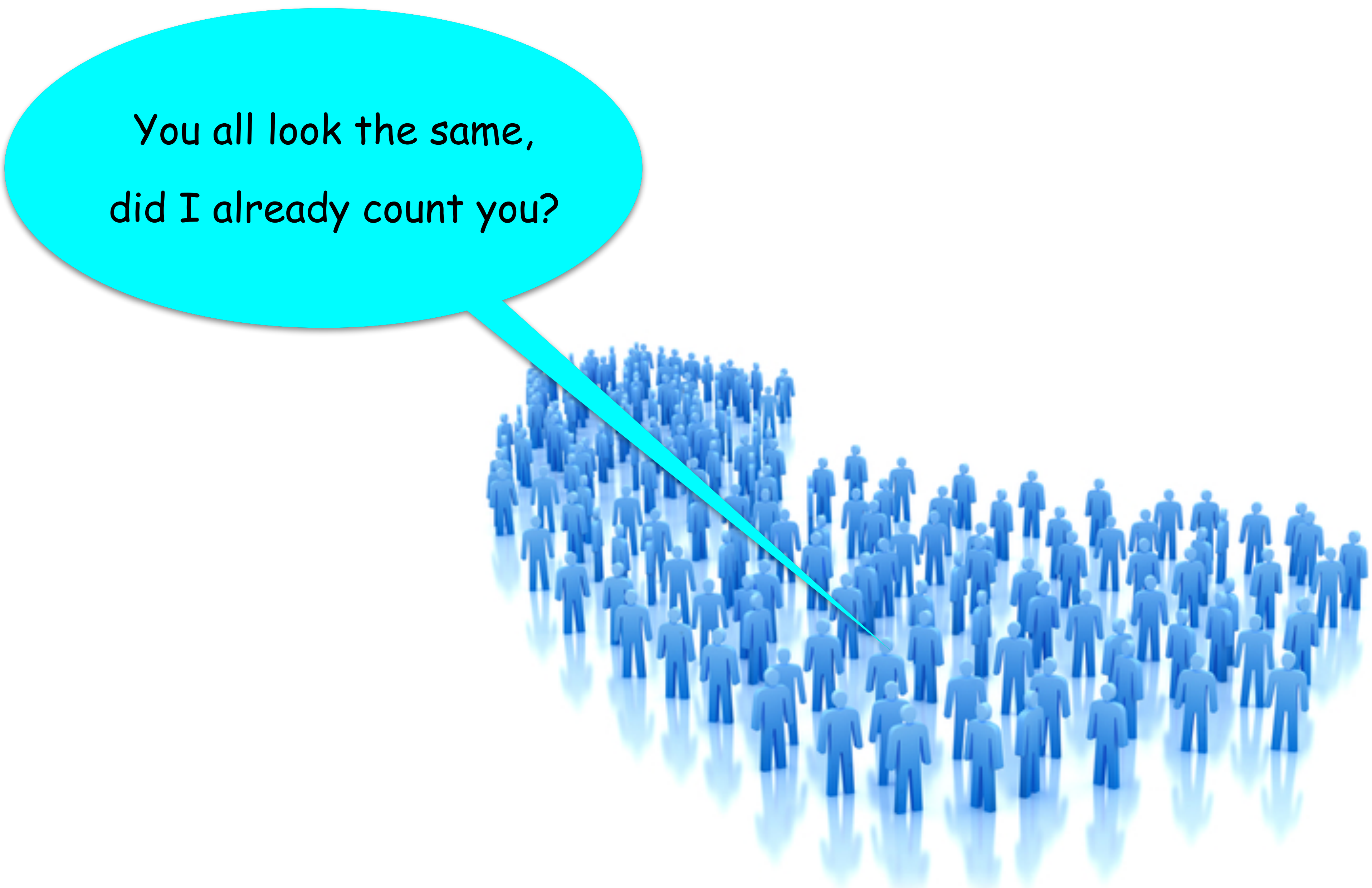

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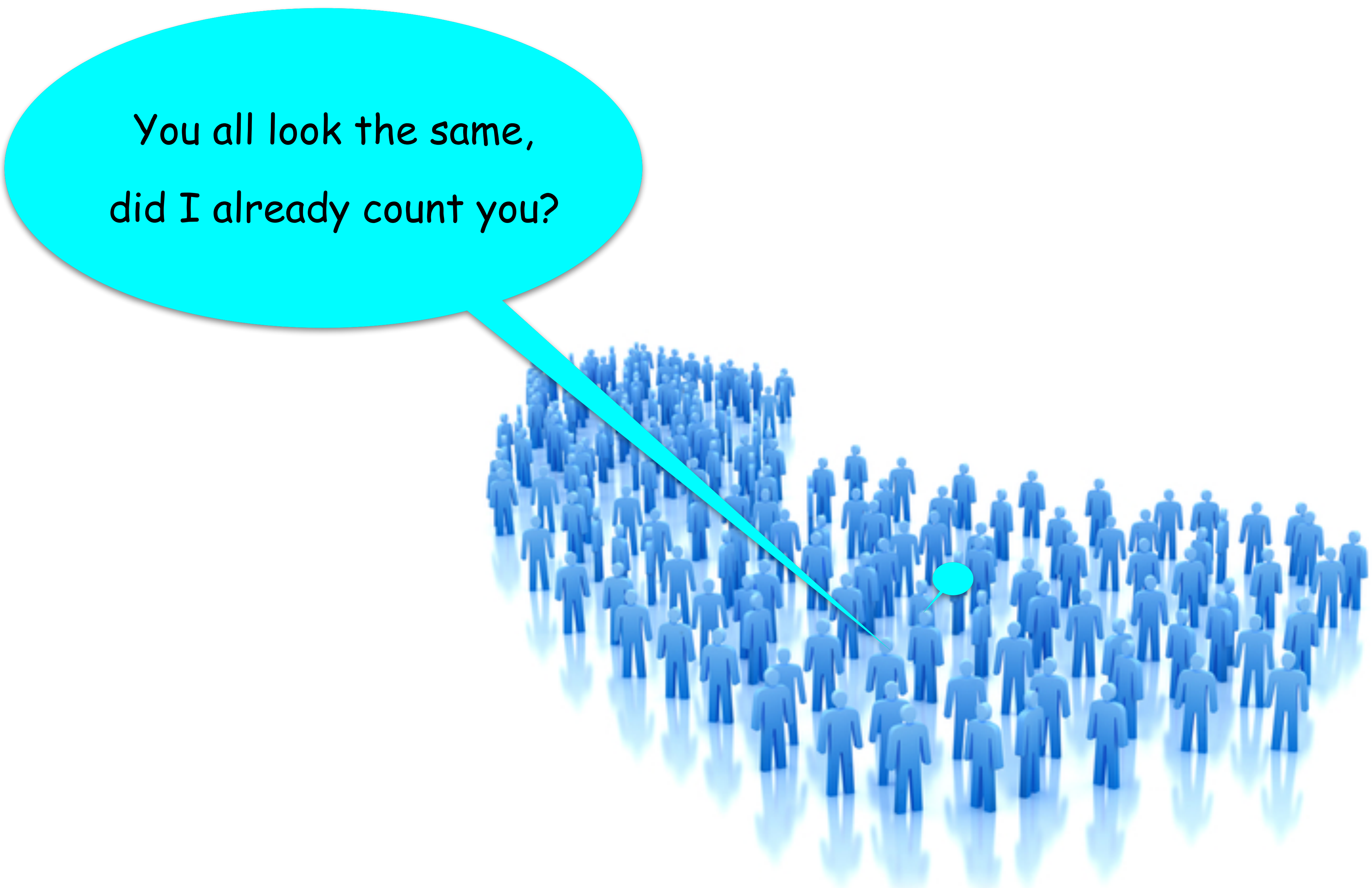

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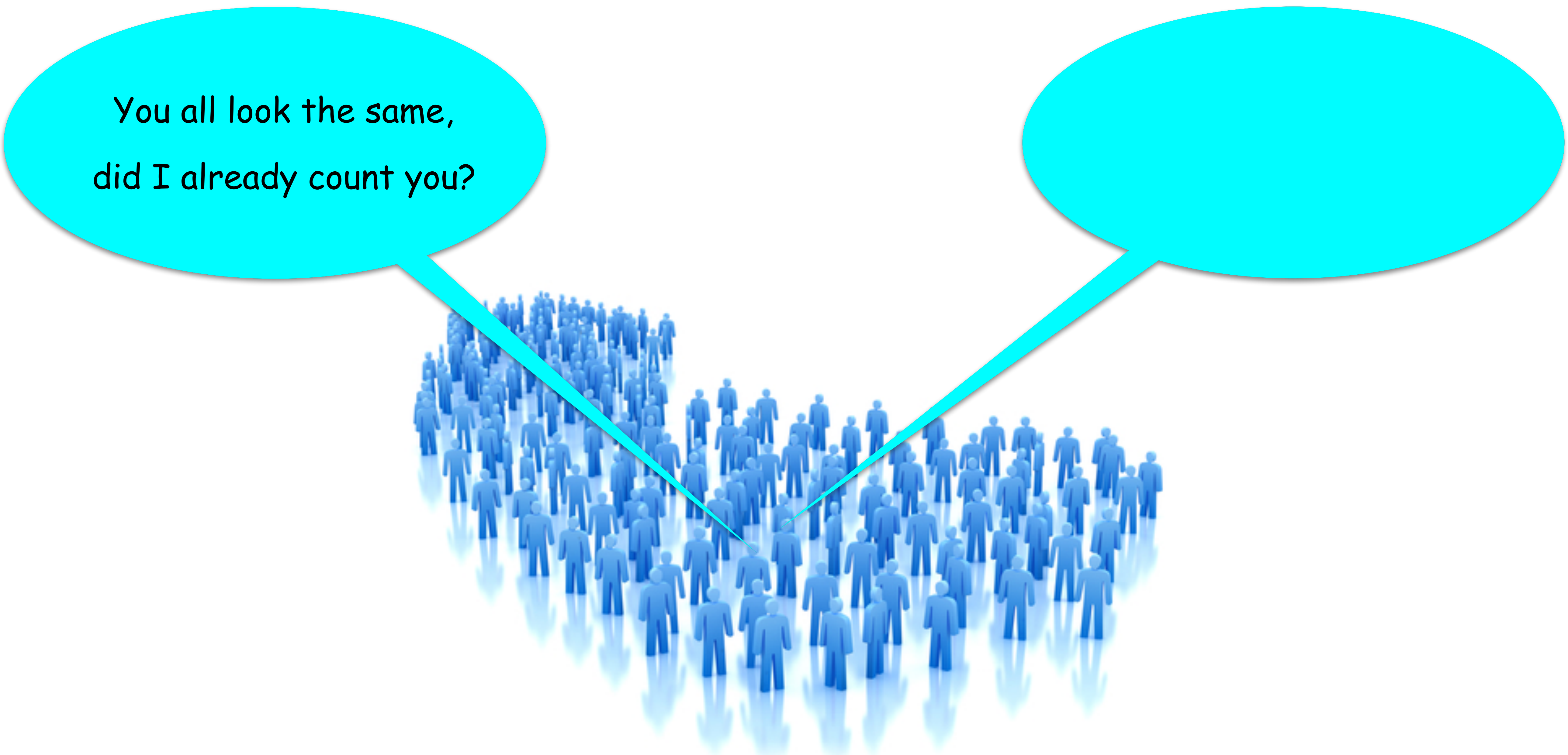

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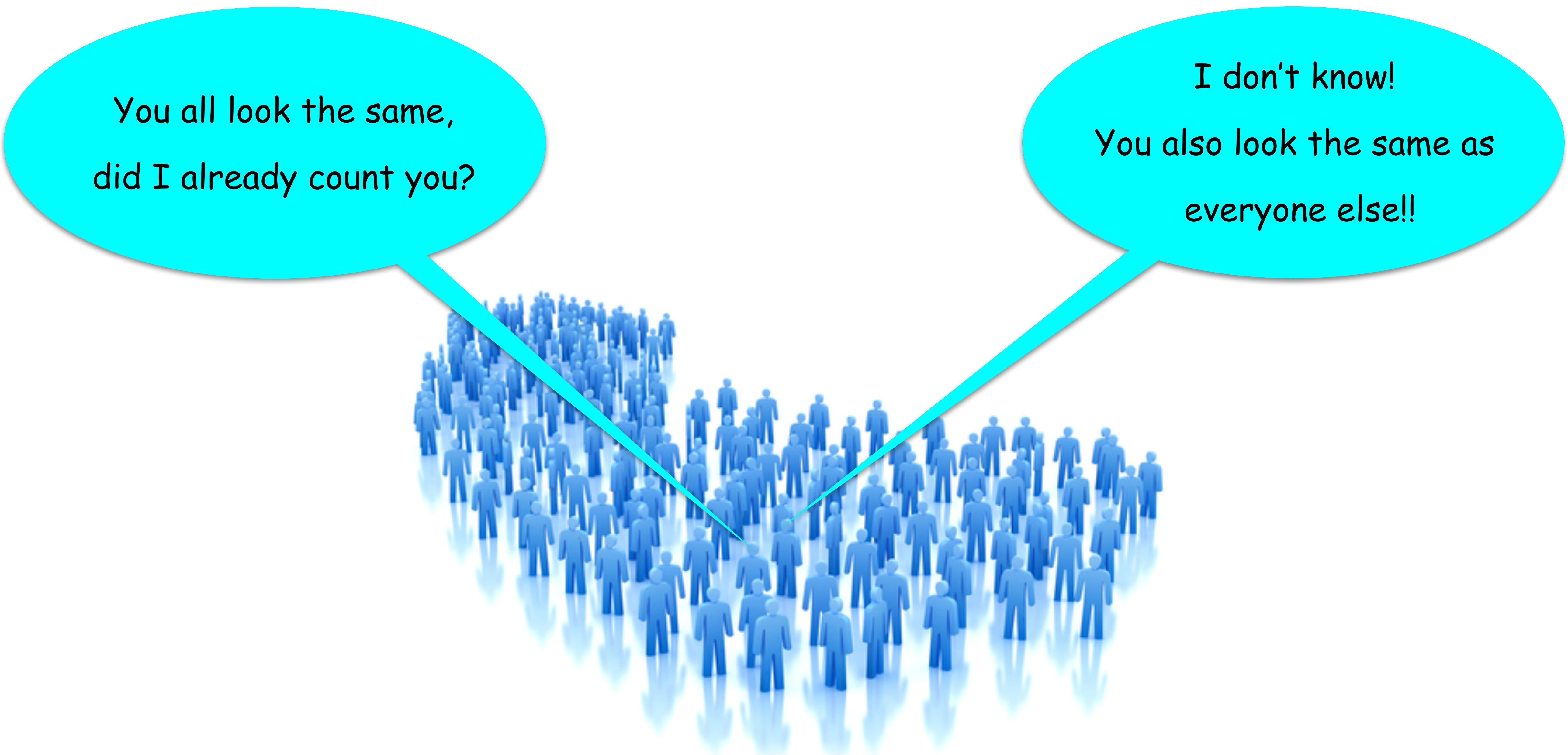


Why do we care?

The problem is clean, but why do we care?

Distributed algorithms

need the number of processors to decide termination.

We need a protocol: « Given a system of *n* nodes,

all nodes eventually terminate knowing *n* »

Previous work

- **Previous Counting Protocols**
  - Guarantee only an exponential upper bound on the network size [1] or
  - They guarantee the exact size but
    - Take double-exponential number of rounds [2] or
    - Take exponential number of rounds, but do not terminate [2] or
    - Terminate but no running-time guarantees [3].
  - Recently, exact-size exponential time Counting with termination:
    - [5] Incremental Counting (IC): needs dyn. max degree d_{max}, poly space.
    - [6] EXT Counting: no d_{max}, but exponential space.

Exponential speedup, but still not practical

- **Lower bound on the time complexity**
  - Ω(D) where D is the dyn. diameter.
  - Ω(log n) even if D is constant [4].

Huge gap

- **Experimental work**
  - IC on trees, paths, stars, G(n,p) [7].

Polynomial in practice

[1] O. Michail, I. Chatzigiannakis, and P. G. Spirakis. Naming and counting in anonymous unknown dynamic networks. SSS 2013.

[2] G. A. Di Luna, R. Baldoni, S. Bonomi, and I. Chatzigiannakis. Conscious and unconscious counting on anonymous dynamic networks. ICDCN 2014.

[3] G. A. Di Luna, R. Baldoni, S. Bonomi, and I. Chatzigiannakis. Counting in anonymous dynamic networks under worst-case adversary. ICDCS 2014.

[4] G. A. Di Luna and R. Baldoni. Investigating the cost of anonymity on dynamic networks. 2015.

[5] A. Milani and M. A. Mosteiro. A faster counting protocol for anonymous dynamic networks. OPODIS 2015.

[6] R. Baldoni and G. A. Di Luna. Non trivial computations in anonymous dynamic networks. OPODIS 2015.

[7] M. Chakraborty, A. Milani and M. A. Mosteiro. Counting in practical anonymous dynamic networks is polynomial. NETYS 2016.

Previous work

restrictions/ shortcomings

algorithm	needs		computes	stops?	complexity	
	size upper bound N	dynamic maximum degree u.b. $d_{\max}$			time	space
Degree Counting [20]		✓	$O(d_{\max}^n)$	✓	$O(n)$	
Conscious [10]	✓	✓	n	✓	$O(e^{N^2}N^3) \Rightarrow O(e^{d_{\max}^{2n}}d_{\max}^{3n})$ using [20]	
Unconscious [10]			n	No	No theoretical bounds	
$\mathcal{A}_{\mathcal{O}^P}$ [11]		Oracle for each node	n	Eventually	Unknown	
EXT [9]			n	✓	$O(n^{n+4})$	EXPSPACE
Incremental Counting [21]		✓	n	✓	$O\left(n\,(2d_{\max})^{n+1}\,\frac{\ln n}{\ln d_{\max}}\right)$	
Methodical Counting [This work]			n	✓	$O(n^5 \ln^2 n)$	PSPACE

[20] O. Michail, I. Chatzigiannakis, and P. G. Spirakis. Naming and counting in anonymous unknown dynamic networks. SSS 2013.

[10] G. A. Di Luna, R. Baldoni, S. Bonomi, and I. Chatzigiannakis. Conscious and unconscious counting on anonymous dynamic networks. ICDCN 2014.

[11] G. A. Di Luna, R. Baldoni, S. Bonomi, and I. Chatzigiannakis. Counting in anonymous dynamic networks under worst-case adversary. ICDCS 2014.

[21] A. Milani and M. A. Mosteiro. A faster counting protocol for anonymous dynamic networks. OPODIS 2015.

[9] R. Baldoni and G. A. Di Luna. Non trivial computations in anonymous dynamic networks. OPODIS 2015.

Contributions

- Methodical Counting (MC) algorithm:
  - no knowledge of network characteristics
  - computes the exact size of the network
  - all nodes obtain n and terminate
  - **first polynomial time** guarantees
  - **exponentially faster** than best previous work
- Design of control mechanisms:
  - for mass-distribution-based computations,
    to detect wrong convergence-time estimation
- Novel approach opens path:
  - to study more complex computations using same techniques
- Extensions to algebraic and other computations:
  - sum, average, max, min, multiple Boolean functions, others

MC Key Ingredients

Key idea:

- distribute a potential value iteratively (resembling previous works),
- but let the leader participate in the process as any other node,
- **leader removes potential but only after it has accumulated enough!**

MC Key Ingredients

Our approach allows to leverage previous work on

lazy random walks in evolving graphs [1].

But, **not a simple de-randomization,**

so, we use lazy random walks bounds,

but only when parameters are temporarily fixed,

and the number of received messages is not invalid.

- In ADNs, **neighbors cannot be distinguished**.
- Even **number of neighbors unknown** at transmission time,

  only after receiving but may change for next round.

- **Unknown network parameters =>**

  potential received could be bigger than 1.

- **Mixing and cover time** of lazy random walks depend on n =>

  cannot be used for termination.

[1] C. Avin, M. Koucky, and Z. Lotker. How to explore a fast-changing world (cover time of a simple random walk on evolving graphs). ICALP 2008.

MC Structure

epochs:

- one for each estimate $k=2,3,\ldots,n$
- initially, "potential" value: $\Phi_{non\text{-}leader}=1$, $\Phi_{leader}=0$

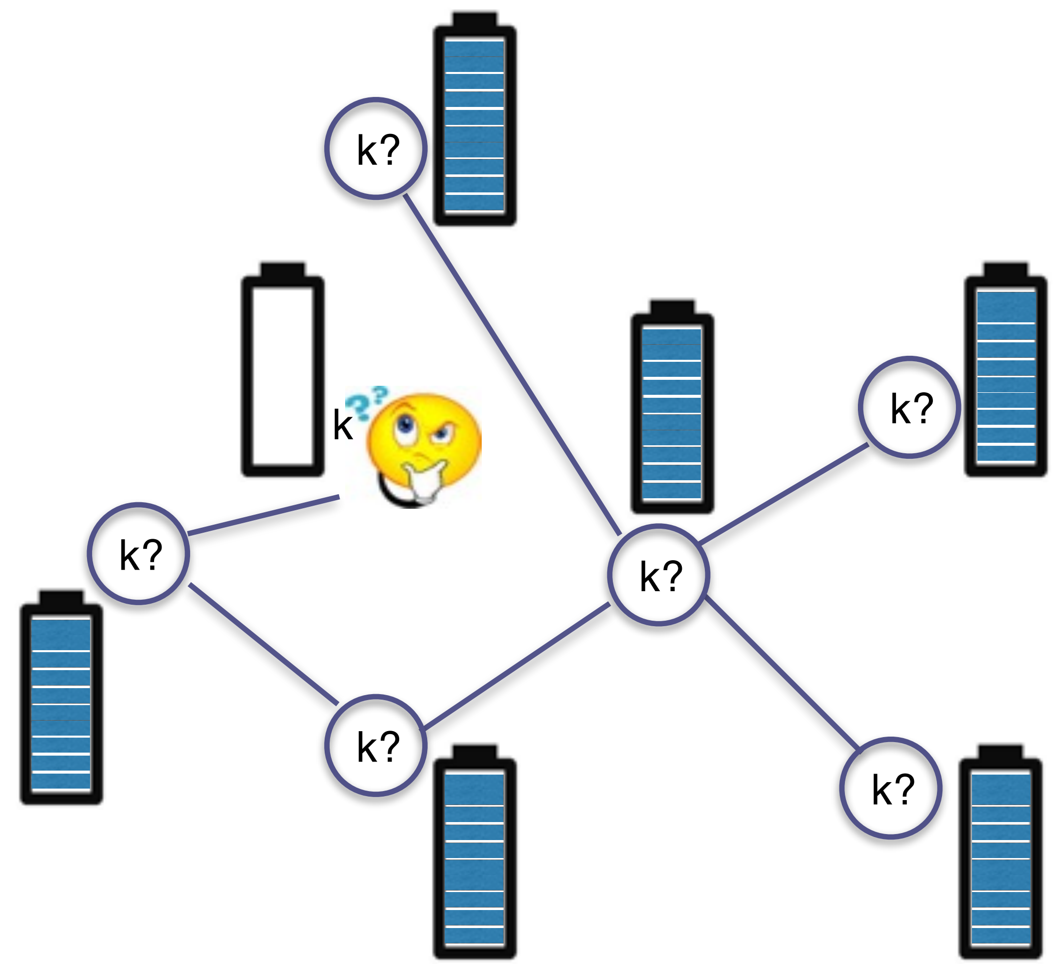
p(k) phases:

(to let the leader remove "enough" potential ρ)

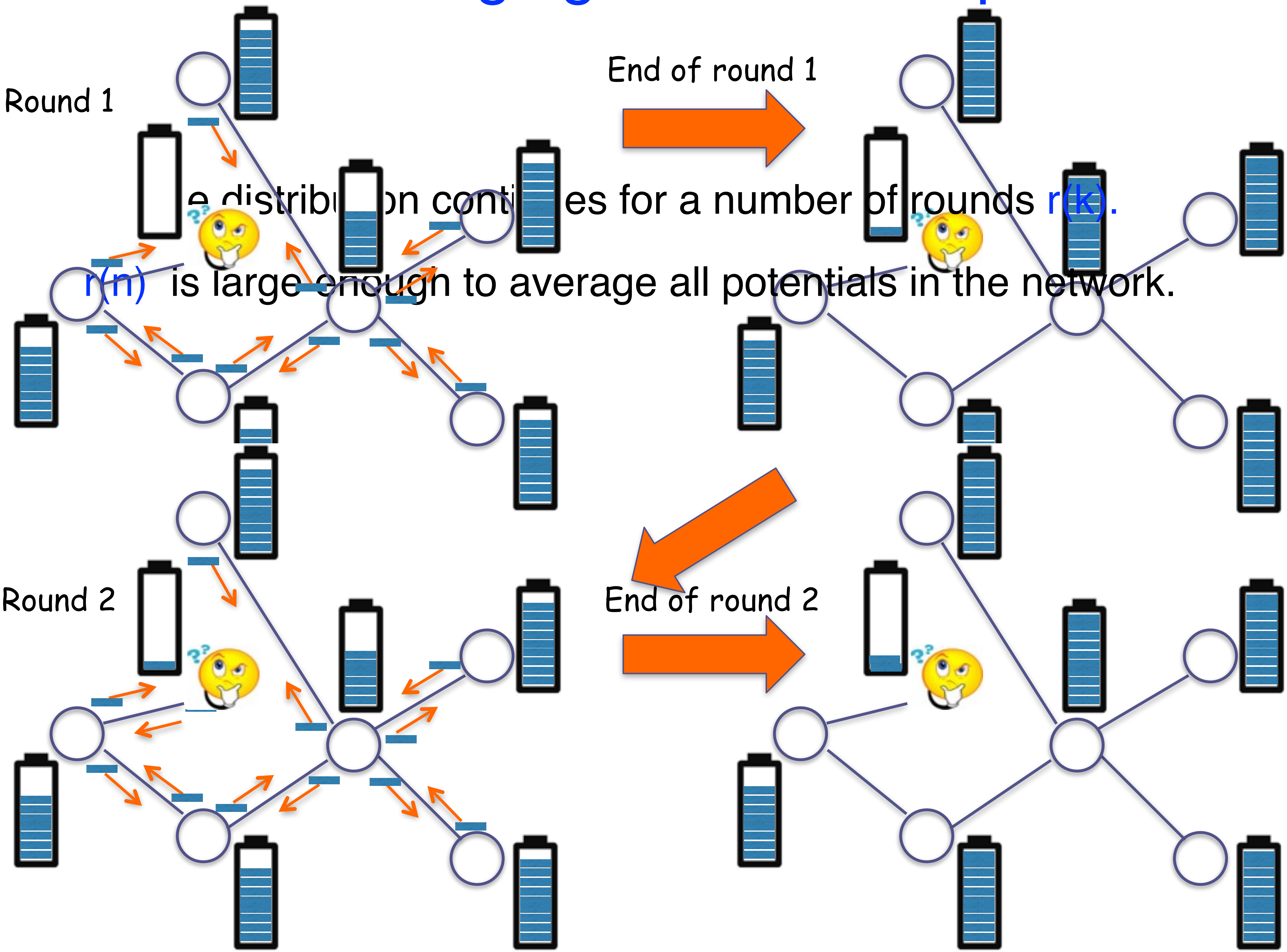
r(k) rounds:

(to "average" the current potentials Φ)

let's see how…



MC Averaging Phase Example



e distrib on conti es for a number of rounds r(k).

r(n) is large enough to average all potentials in the network.

MC Epoch Example

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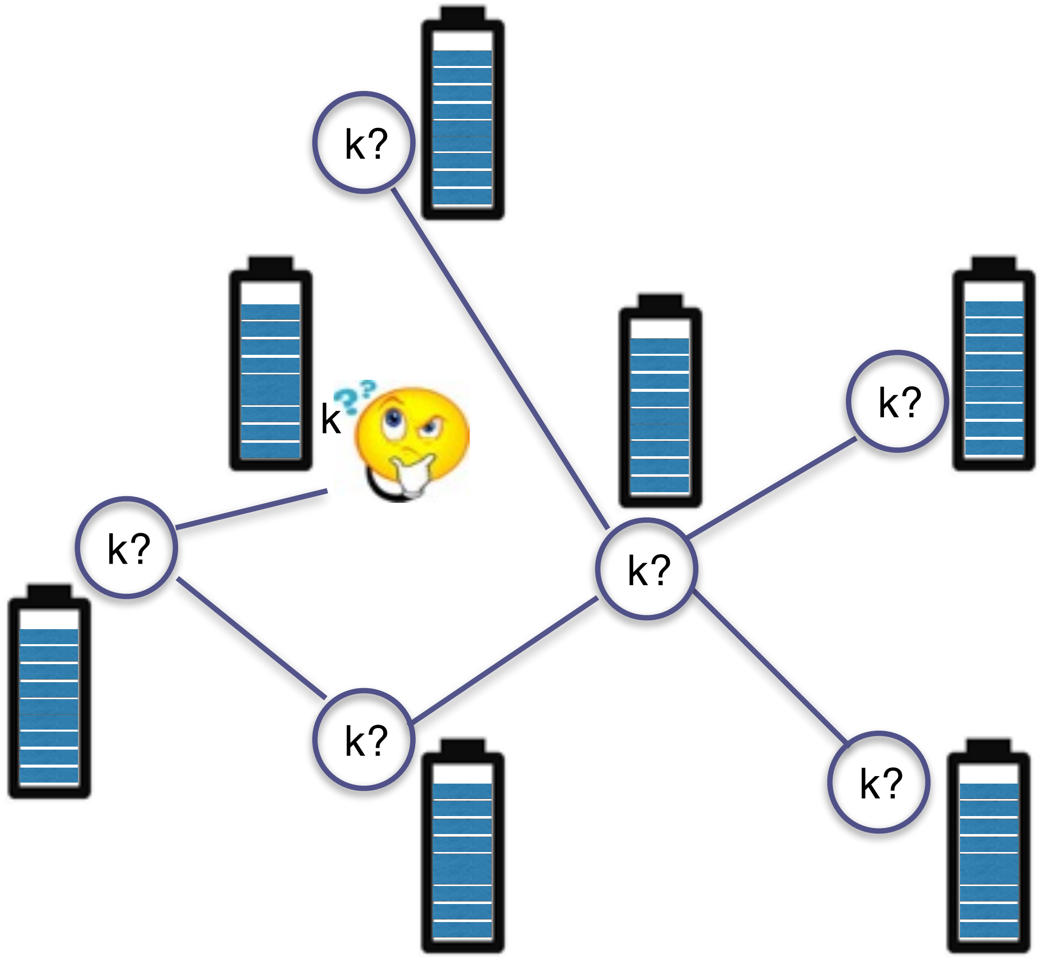
mass distribution:

- broadcast Φ and receive neighbors' Φ_i
- $\Phi = \Phi + \Sigma_{i \in N} \Phi_i/d(k) - |N|\Phi/d(k)$

- leader "removes" its potential: $\rho=\rho+\Phi$, $\Phi=0$



$\rho=$

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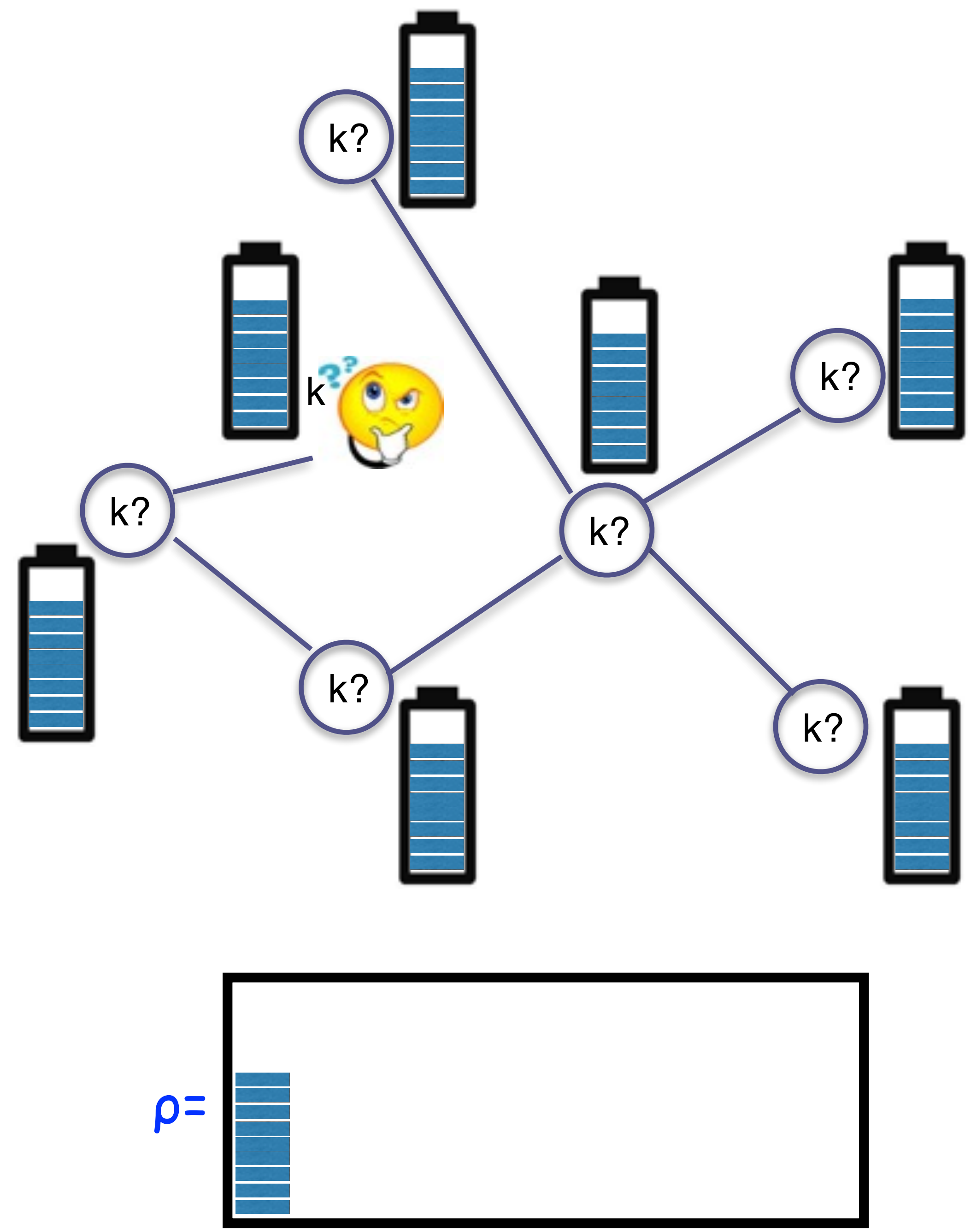

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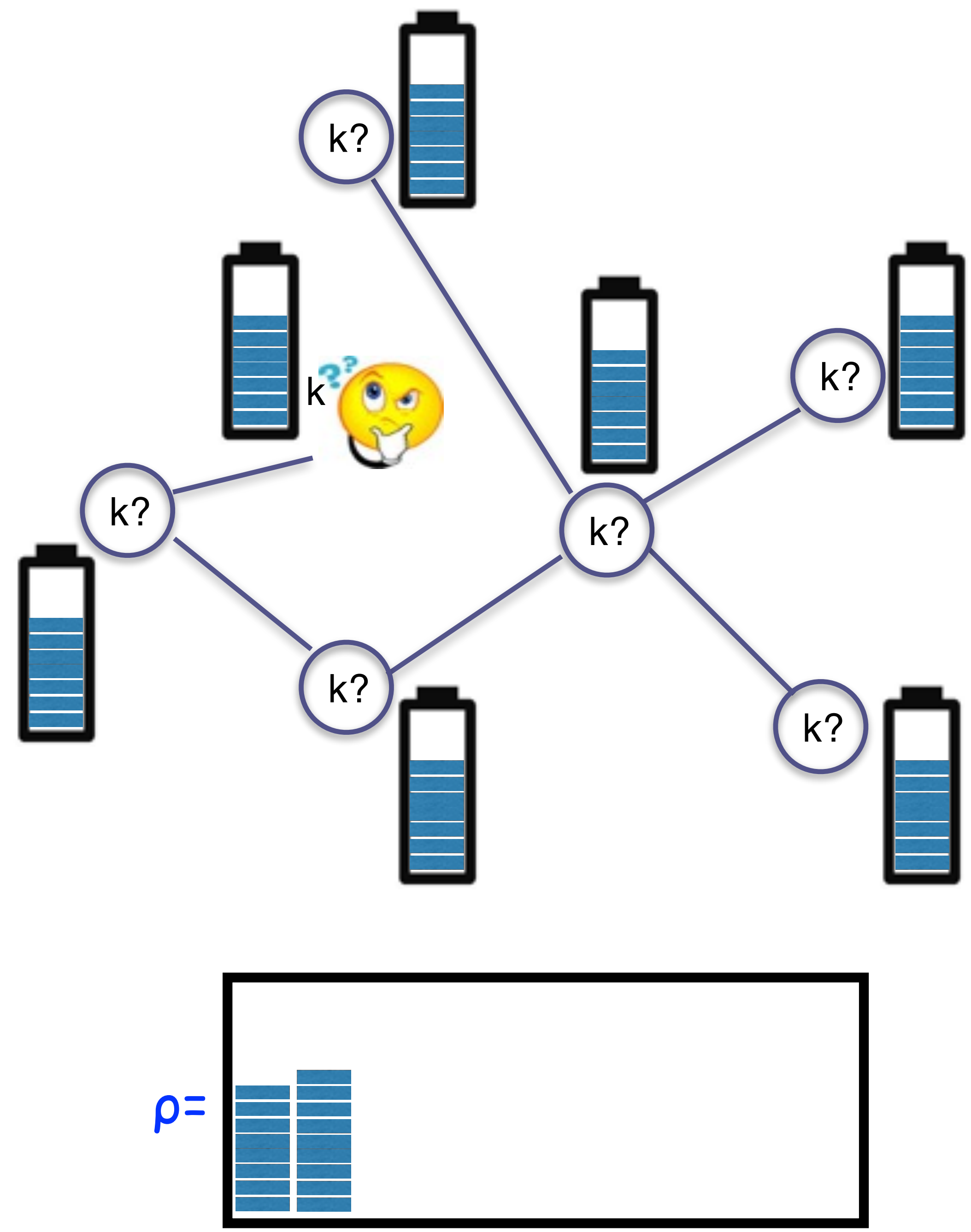

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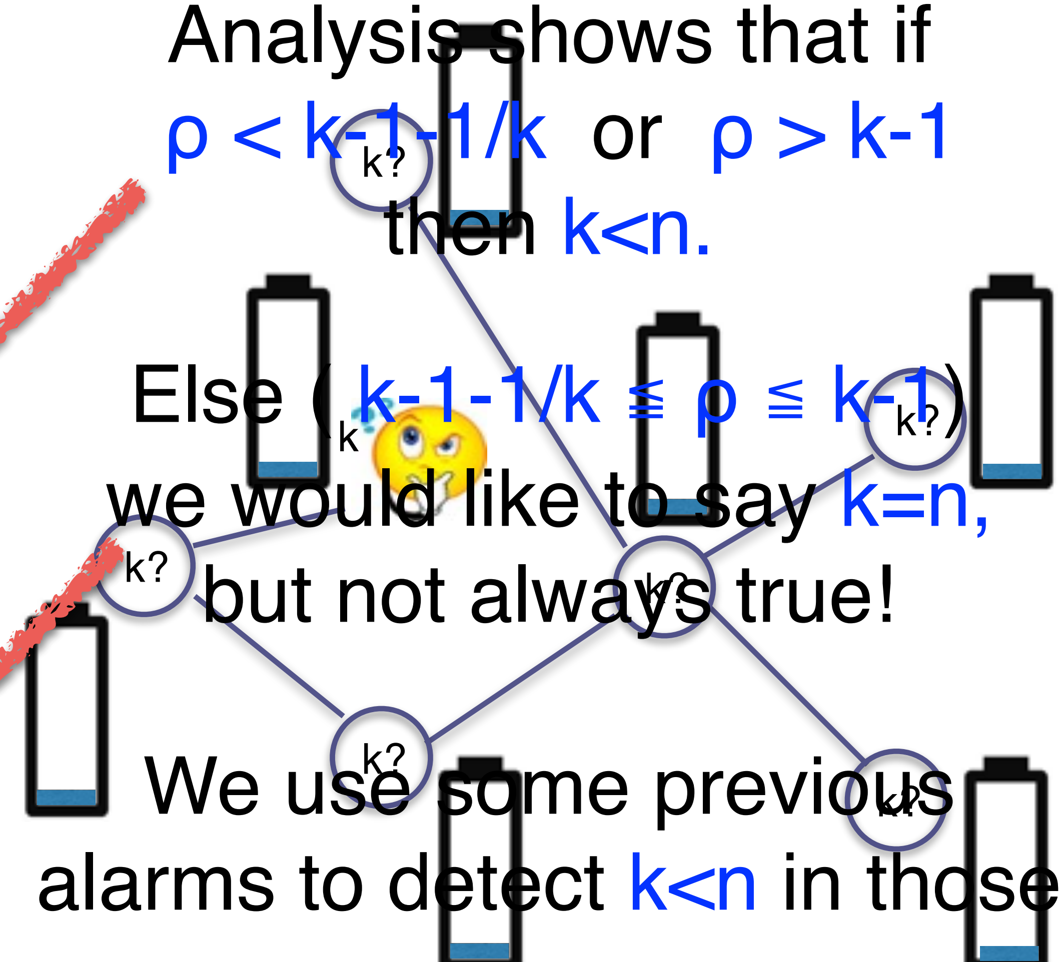
- if $\rho < k\text{-}1\text{-}1/k$ or $\rho > k\text{-}1$
  - try next k
- else notify all nodes that $k=n$

After $p(k)$ phases…



Analysis shows that if

$\rho < k\text{-}1\text{-}1/k$ or $\rho > k\text{-}1$

then $k<n$.

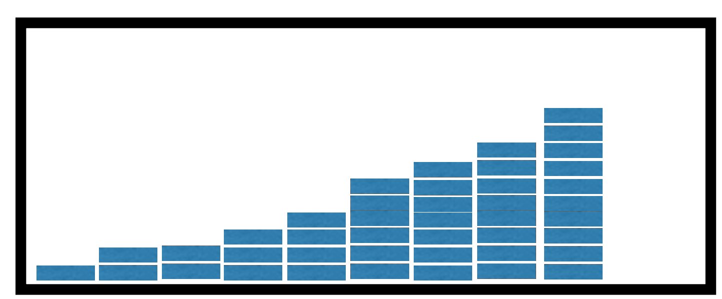
Else ($k\text{-}1\text{-}1/k \leqq \rho \leqq k\text{-}1$)

we would like to say $k=n$,

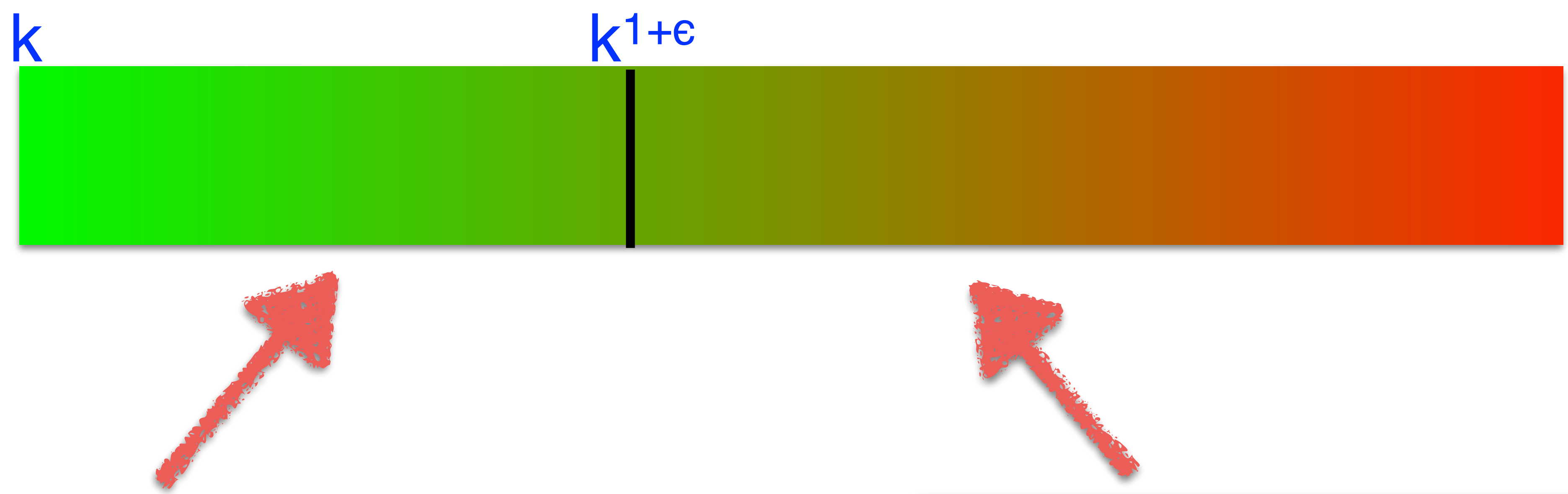
but not always true!

We use some previous alarms to detect $k<n$ in those cases…

$\rho=$



MC Alarms (for k<n)



If n is “close” to k then
leader removes
“too much” potential.

If n is “far” from k then
not “many” nodes
have “low” potential,

so, leader receives alarm from
nodes with “high” potential
“soon” after first phase.

MC Epoch Example

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- leader "removes" its potential: $\rho=\rho+\Phi$, $\Phi=0$

- if $k-1-1/k <= \rho <= k-1$ and status = normal
  - notify all nodes that $k=n$
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We use some previous alarms to detect $k<n$ in those cases.

And now the leader can notify $k=n$ when ρ is in that range.

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Main Theorem

THEOREM 6.2. *Given an Anonymous Dynamic Network with n nodes, after running* METHODICAL COUNTING *for each estimate $k = 2, 3, \ldots, n$ with parameters*

$$d = k^{1+\epsilon},$$
$$p = \left\lceil \frac{(2+\epsilon)k^{1+\epsilon}}{1 - 1/k} \ln k \right\rceil,$$
$$r = \left\lceil \left(4 + 2\epsilon + \max\left\{0, -\frac{2\ln(k^{\epsilon} - 1)}{\ln k}\right\}\right) dk^{2+2\epsilon} \ln k \right\rceil,$$
$$\tau = 1 - 1/k^{1+\epsilon},$$

where $\epsilon > 0$, all nodes stop after $\sum_{k=2}^{n}(pr + k)$ rounds of communication and output n.

COROLLARY 6.1. *The time complexity of* METHODICAL COUNTING *is $O(n^5 \log^2 n)$.*

MC Extensions

SUM: assume each node i stores a value v_i, and we need to compute the exact sum.

Compute n and SUM simultaneously:

For each node i

- Append to potential Φ_i the bit representation of value v_i as a sequence of values (initially in $\{0,1\}$ but later averaged iteratively and independently).

$$\langle \phi_i, v_{i0}, v_{i1}, v_{i2}, \ldots \rangle$$

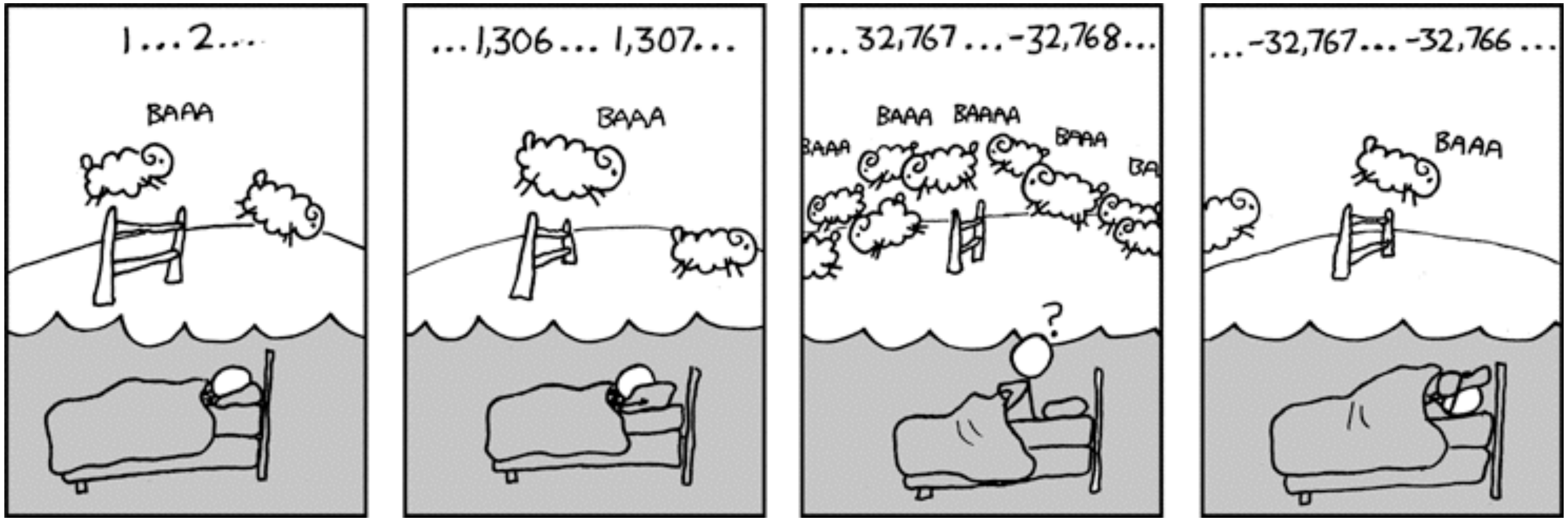
- Apply same algorithm to each v_{ij} independently, as well as to the potential.
- Store the v_{ij}'s at the end of each first phase, call them v'_{ij}.
- At the end of each epoch while $k<n$, reset to the original v_{ij}'s.
- At the end of last epoch ($k=n$), compute $\sum_j \lceil n v'_{ij} \rceil 2^j$.

Others: AVG, Boolean (AND, OR, XOR, etc.), some database queries.

Future and Ongoing Work

- Many leaders.
- Improve upper and lower bounds.
- Other computations in ADNs.
- Asynchronous protocol.

Thank you!



1...2...
BAAA
...1,306... 1,307...
BAAA
...32,767...-32,768...
BAAA
BAAA
BAAAA
BAAA
?
...-32,767...-32,766...
BAAA