Polynomial Anonymous Dynamic Distributed Computing without a Unique Leader

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Anonymous Dynamic Networks

- **Fixed set of n nodes**
  - No identifiers or labels
- **Synchronous communication :** At each round
  - a node broadcasts a message to its neighbors
  - receives the messages of its neighbors
  - executes some local computation
- **1-interval connectivity** [1]
  - communication links may change from round to round, but
  - at each round the network is connected

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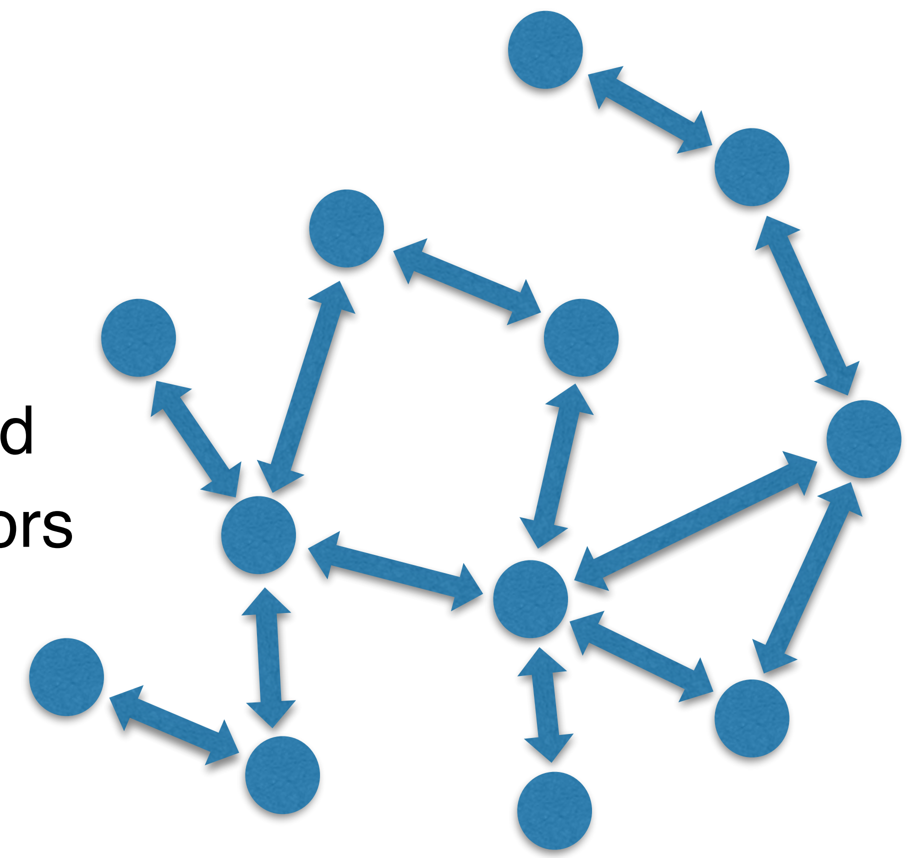
Anonymous Dynamic Networks



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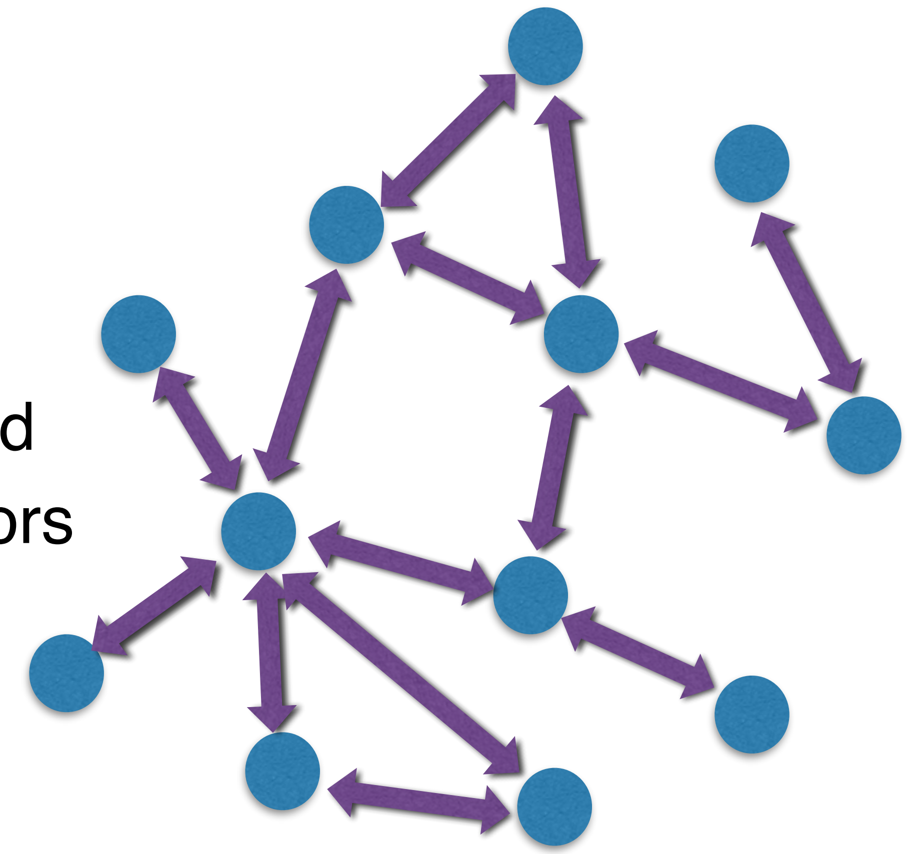

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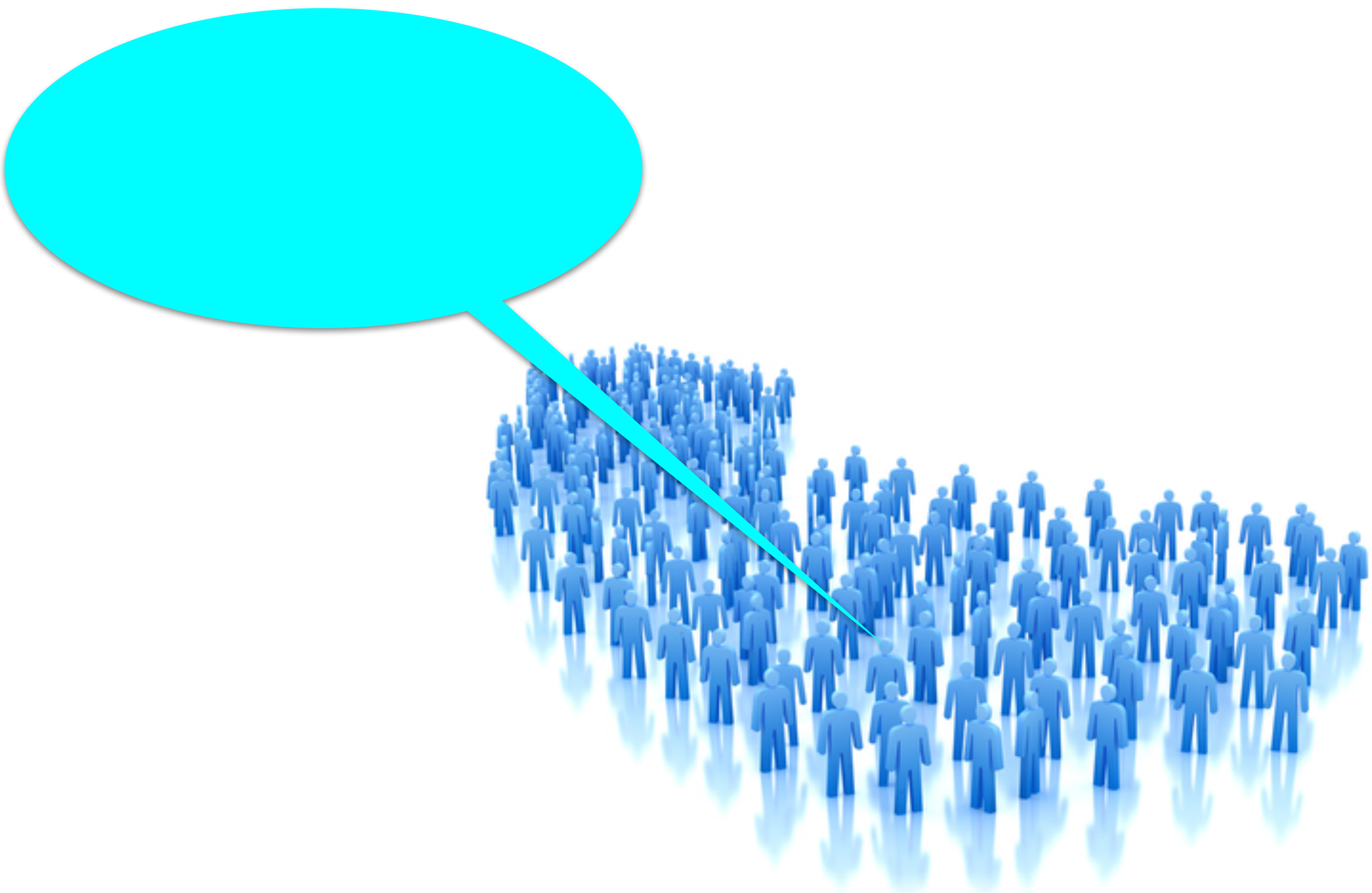

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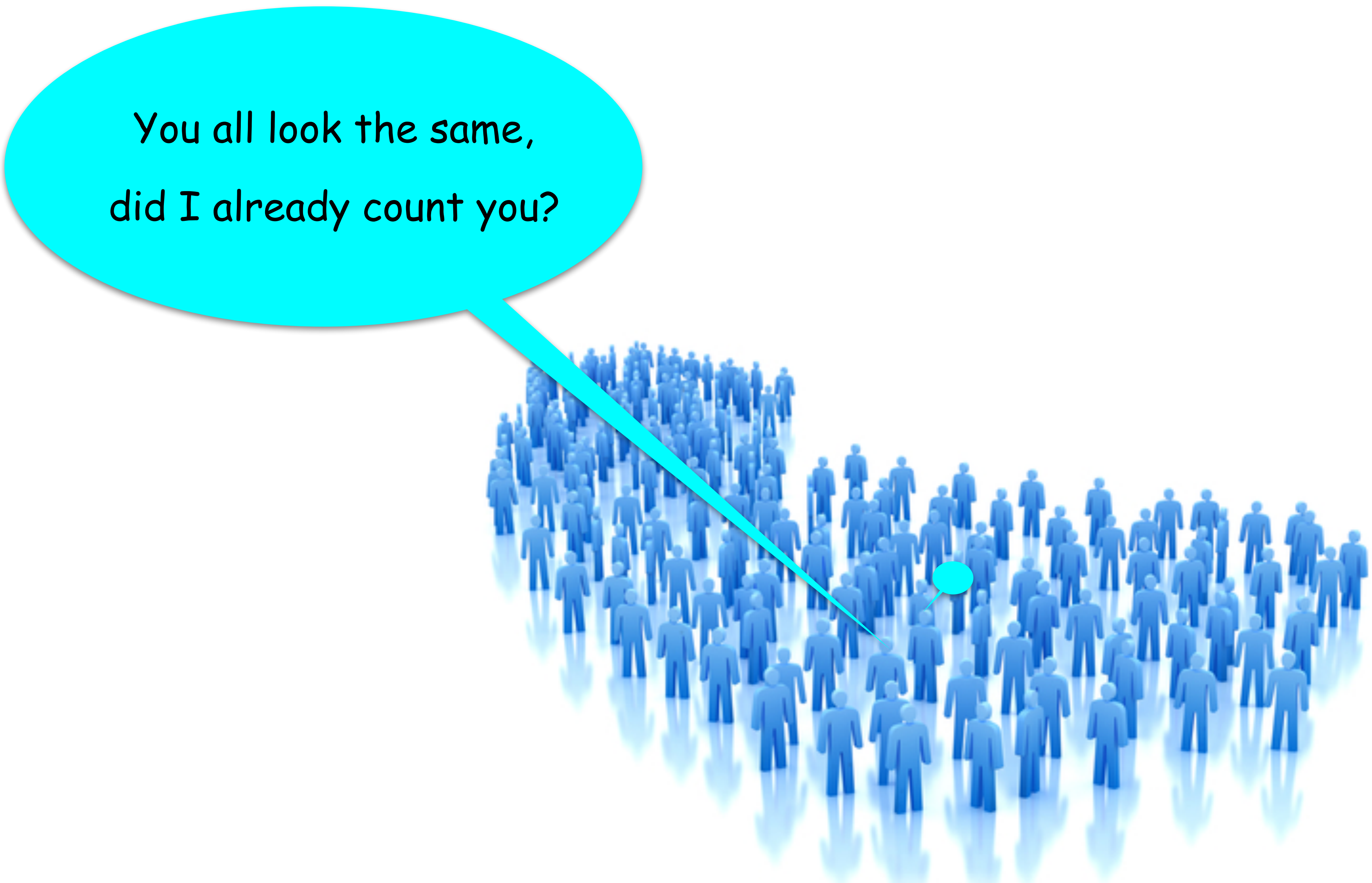

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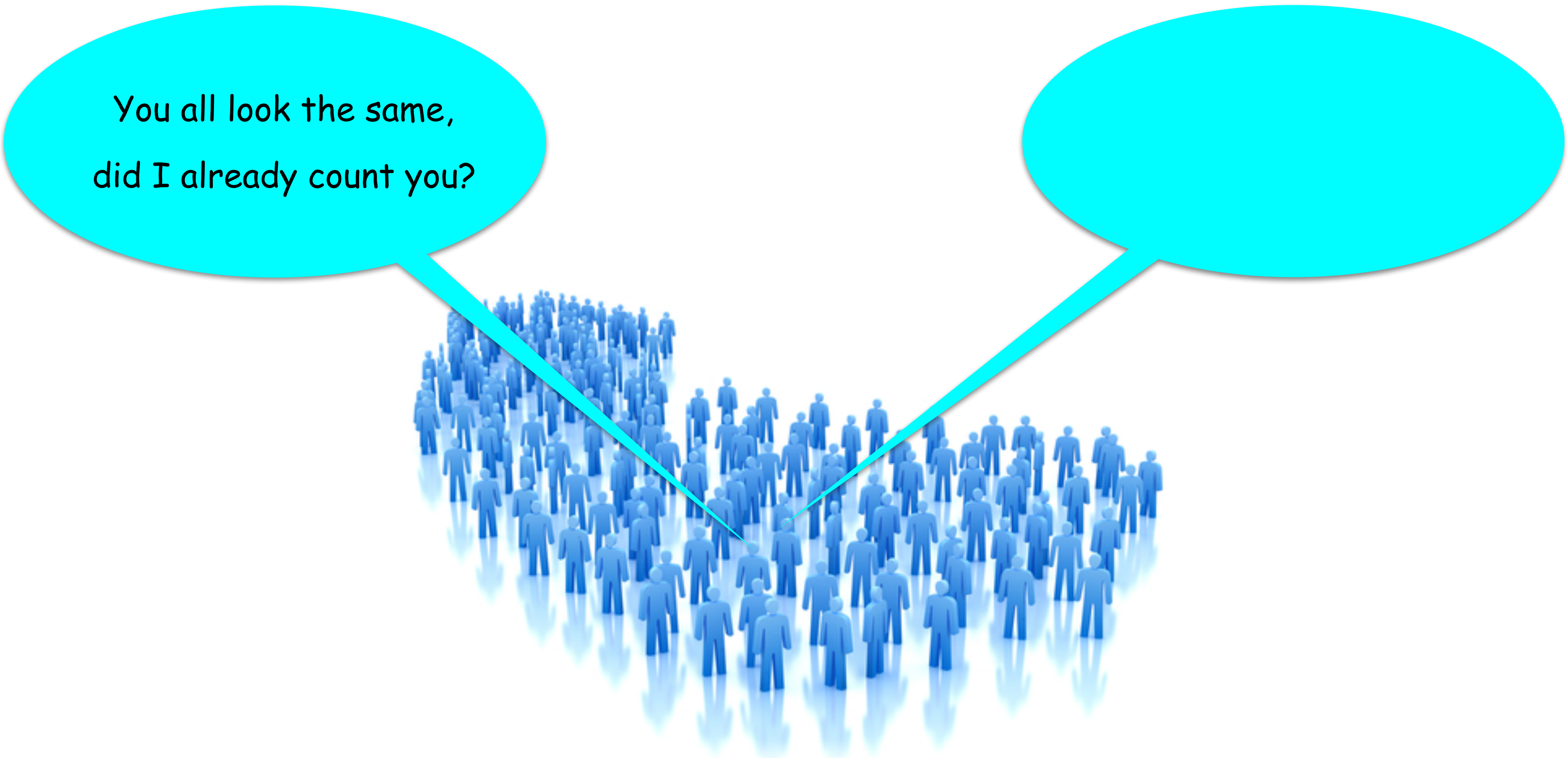

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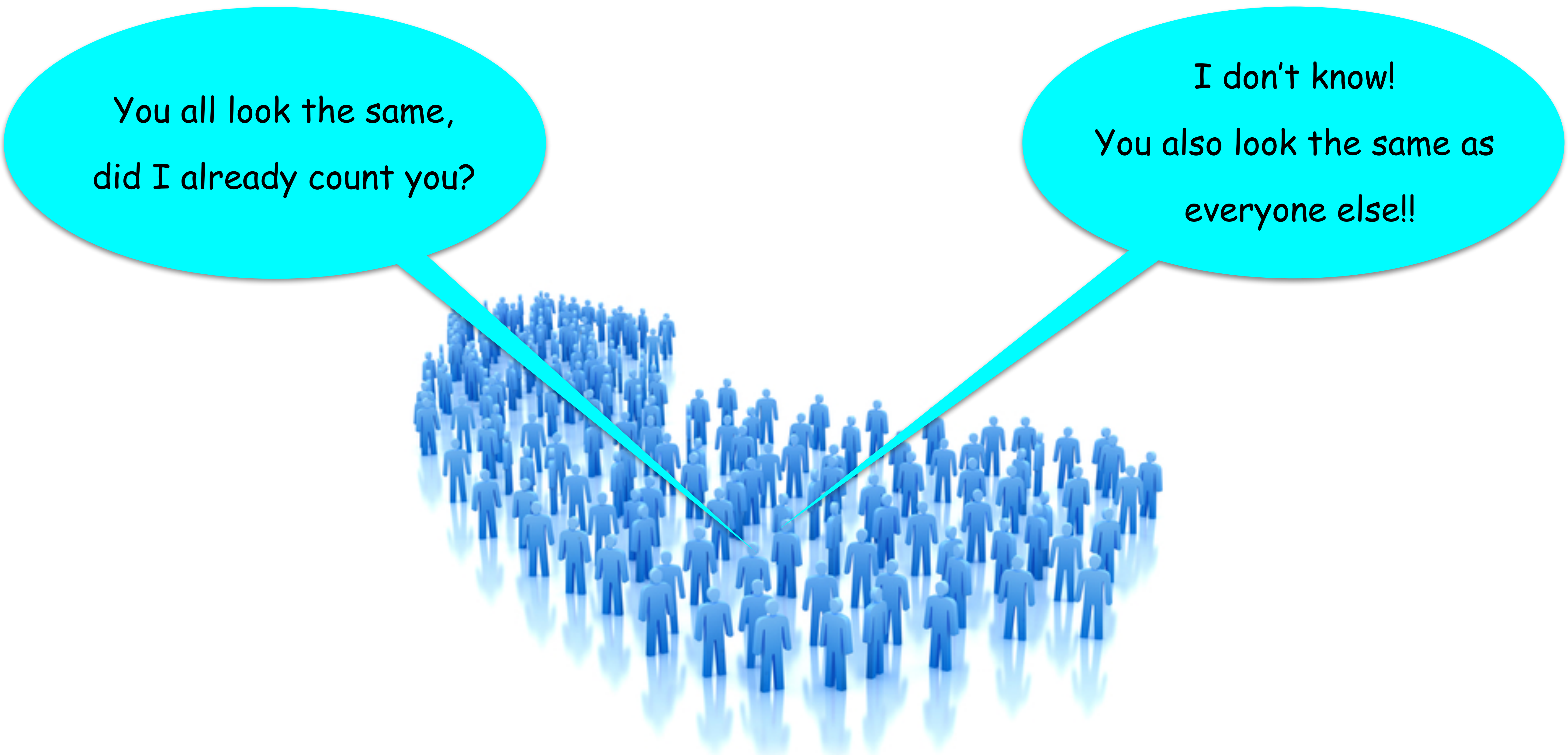

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Distributed algorithms

need the number of processors to decide termination.

We need a protocol: « Given a system of n nodes,

all nodes eventually terminate knowing n ».

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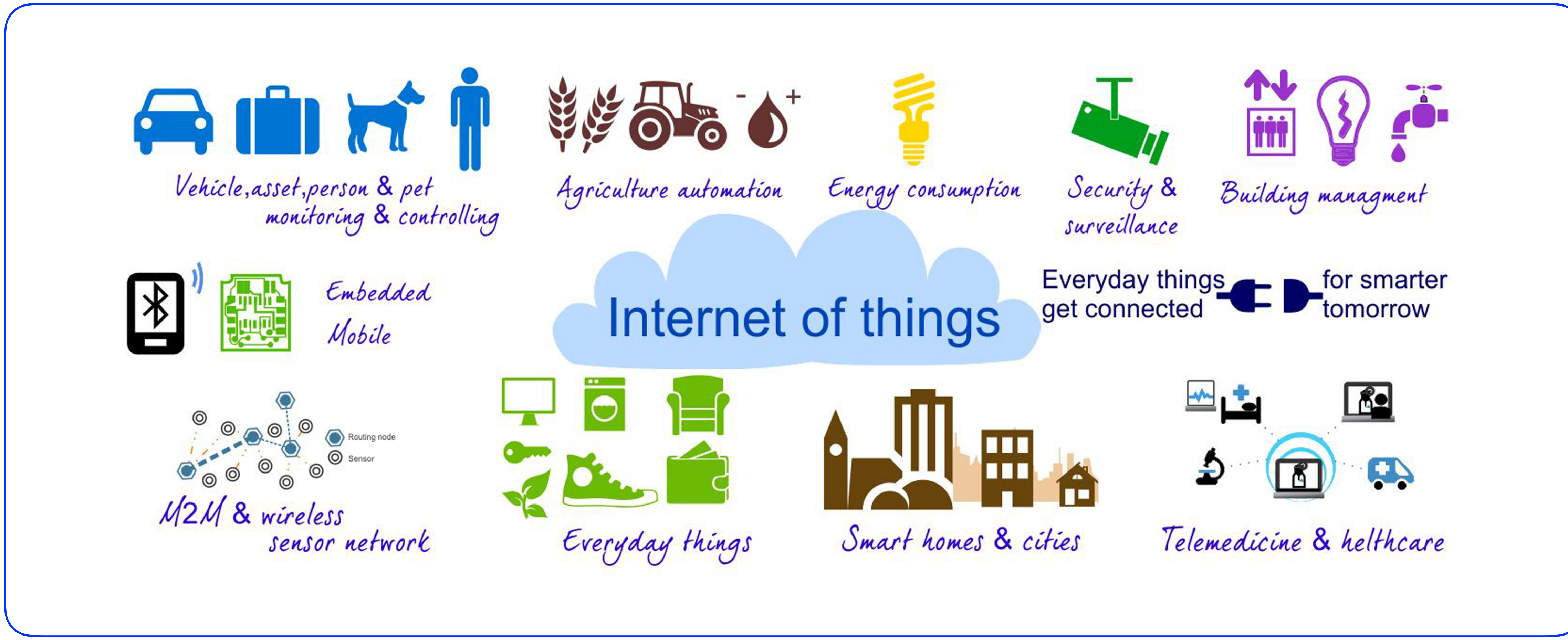
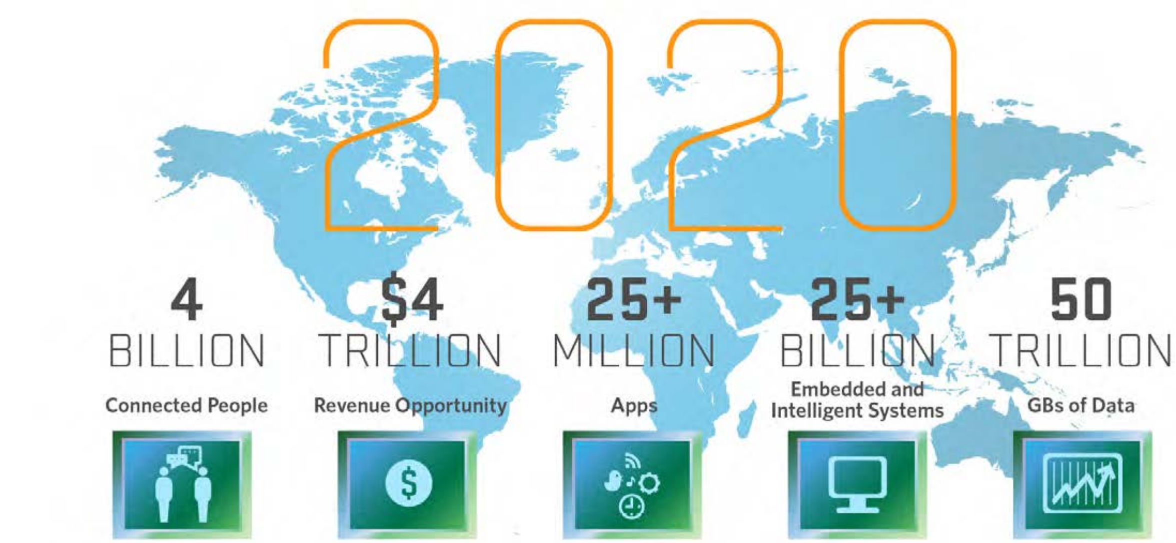
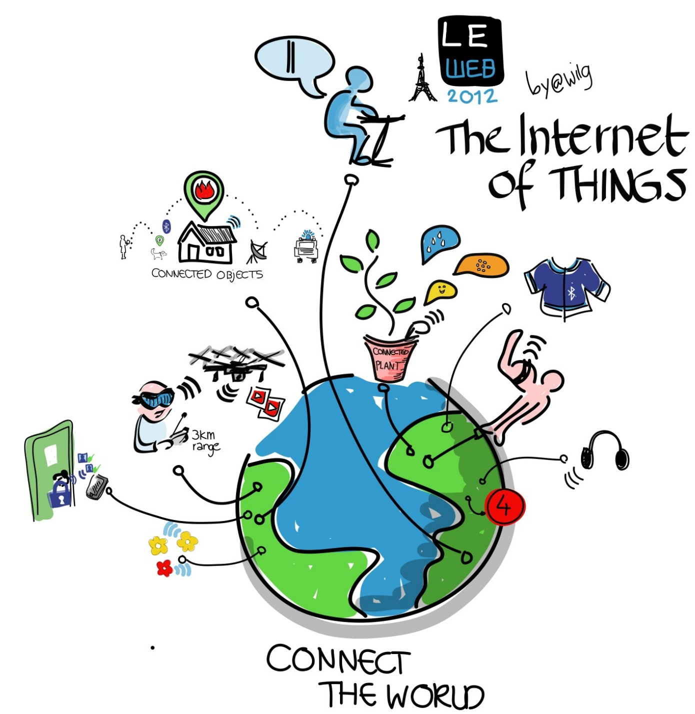




Source: Mario Morales, IDC



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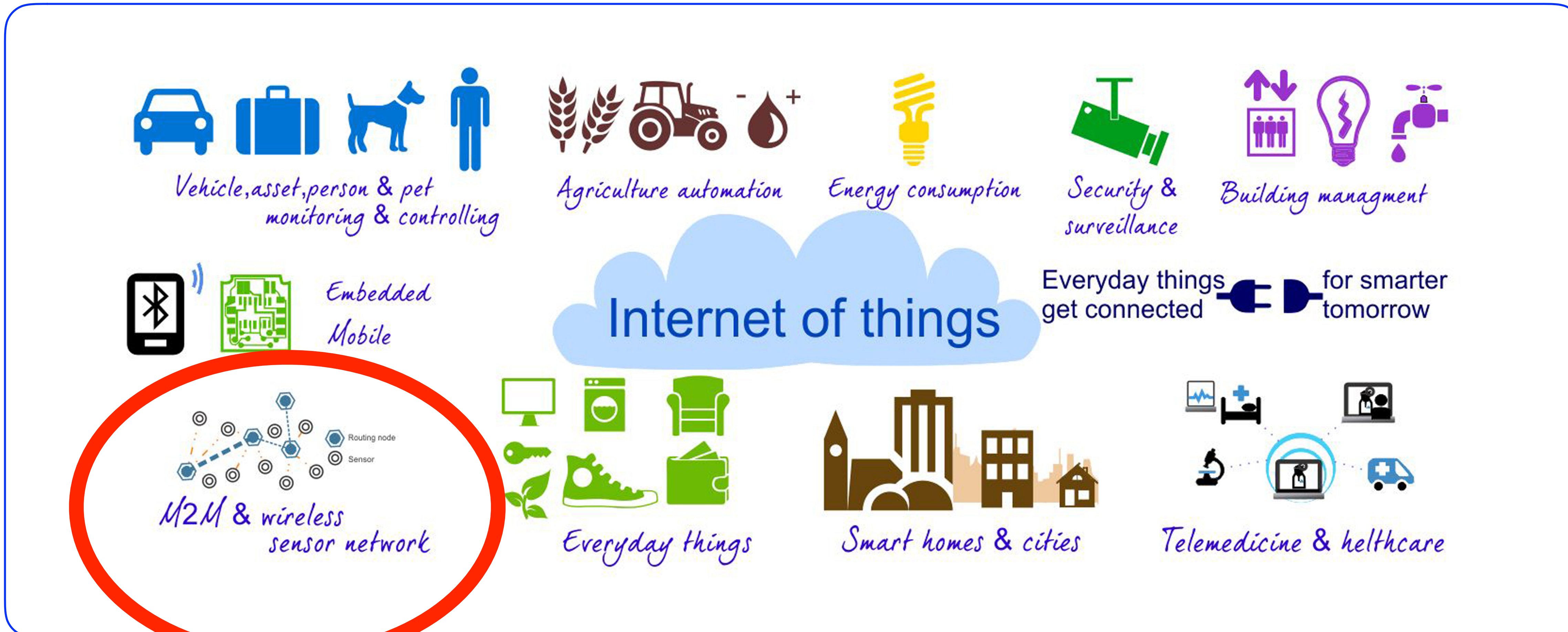

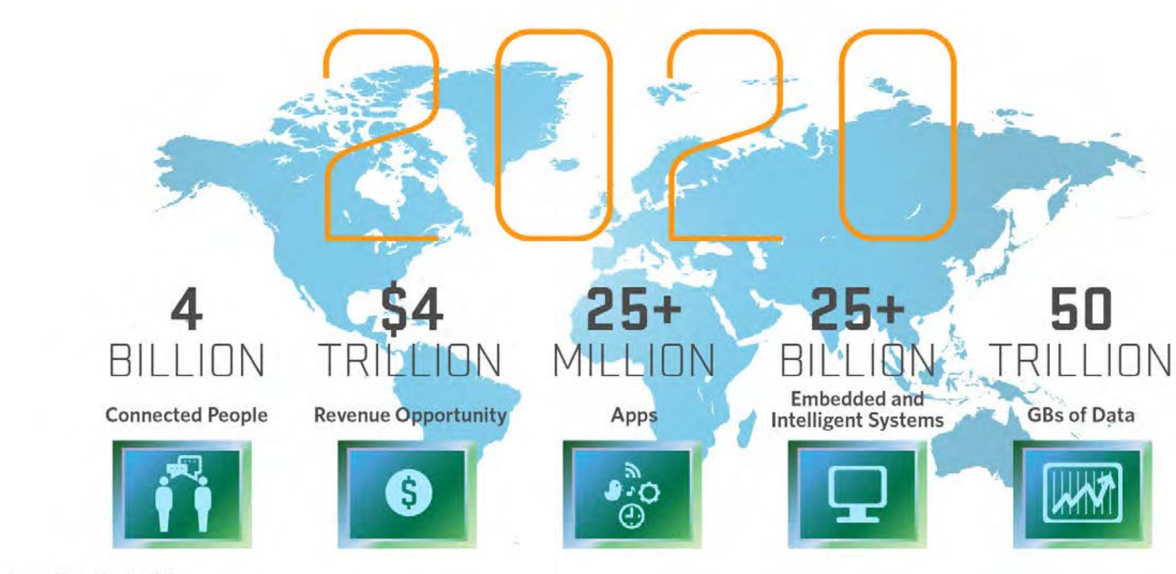

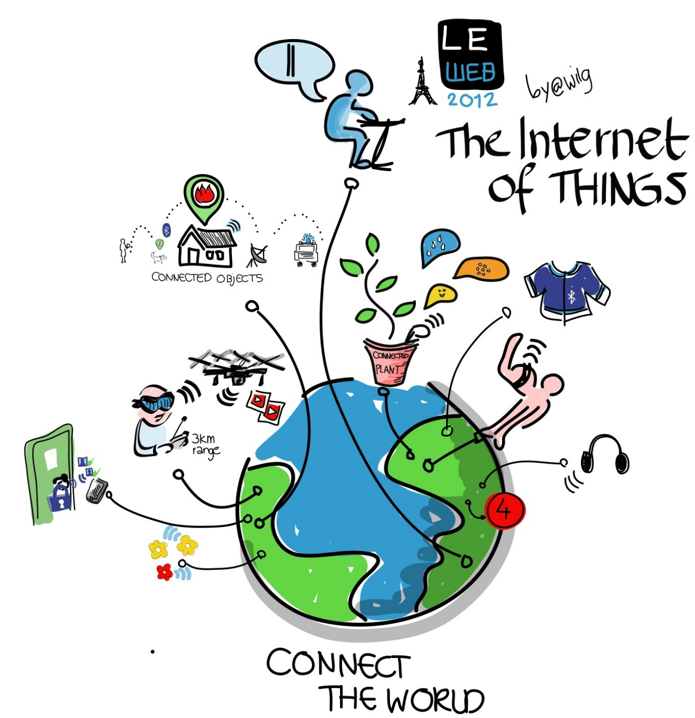

Previous work

algorithm	needs			computes	stops?	complexity	
	distinguished nodes	size upper bound N	dynamic maximum degree u.b. $d_{\max}$			time	space
Degree Counting [5]	1		✓	$O(d_{\max}^n)$	✓	$O(n)$	
Conscious [2]	1	✓	✓	n	✓	$O(e^{N^2}N^3) \Rightarrow O(e^{d_{\max}^{2n}} d_{\max}^{3n})$ using [5]	
Unconscious [2]	1			n	No	No theoretical bounds	
$\mathcal{A}_{\mathcal{O}^P}$ [3]	1		Oracle for each node	n	Eventually	Unknown	
EXT [1]	1			n	✓	$O(n^{n+4})$	EXPSPACE
Incremental Counting [6]	1		✓	n	✓	$O\left(n\,(2d_{\max})^{n+1}\,\frac{\ln n}{\ln d_{\max}}\right)$	
Methodical Counting [4]	1			n	✓	$O(n^5 \ln^2 n)$	PSPACE

restrictions/ shortcomings

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Questions

- Can we count deterministically with more than one special node?
- What information about the special nodes is needed?
- How "special"? Can the nodes be identical and just have two different programs?
- Can we let the nodes choose program at *random* and make them all identical?
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Results

ℓ black nodes and n-ℓ white nodes:

- Impossibility:
  - Deterministic counting: not possible if ℓ is unknown
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- Impossibility:
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- Methodical Multi-Counting (MMC) algorithm:
    - *all* nodes obtain n and terminate
    - no network info needed except ℓ
- Leader-less Methodical Counting (LLMC) algorithm:
    - first algorithm applicable to ADNs with all identical nodes

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Leader-less Methodical-Counting [This work]	0			n prob. $\geq 1-\zeta$ for any $\zeta > 0$	✓	$O(\eta^{4+\epsilon}\log^3 \eta)$ for any $\epsilon > 0$ and $\eta = \max\{n, \lceil\lceil 12/\zeta\rceil\rceil\}$	PSPACE

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first with $\ell \neq 1$

$\sim n\ell$ / log n speedup (for $\zeta \in \Omega(1/n)$),

faster than MC even for $\ell = 1$

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- How the black nodes communicate?
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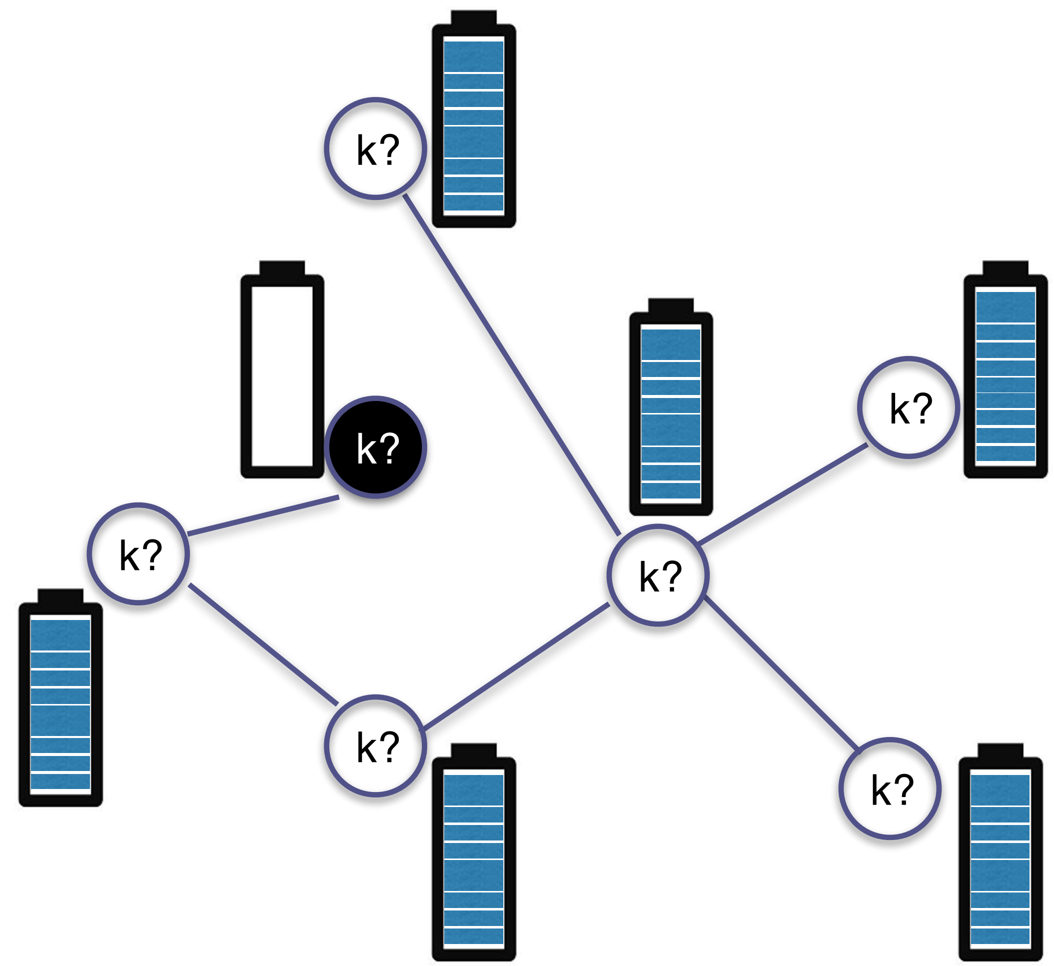
MMC Key Ingredients:

- try network size estimates $k = \ell+1, 2(\ell+1), 4(\ell+1), \dots$ binary search after estimate $k > n$
- share some potential values iteratively
- all nodes (black and white) share potential
- black nodes remove potential from the system every now and then
- carefully designed alarms allow to detect correct or wrong estimate

MMC Structure

epochs:

- one for each estimate $k=l+1, 2(l+1), 4(l+1), \ldots$
- initially, "potential" value: $\Phi_{white}=l$, $\Phi_{black}=0$



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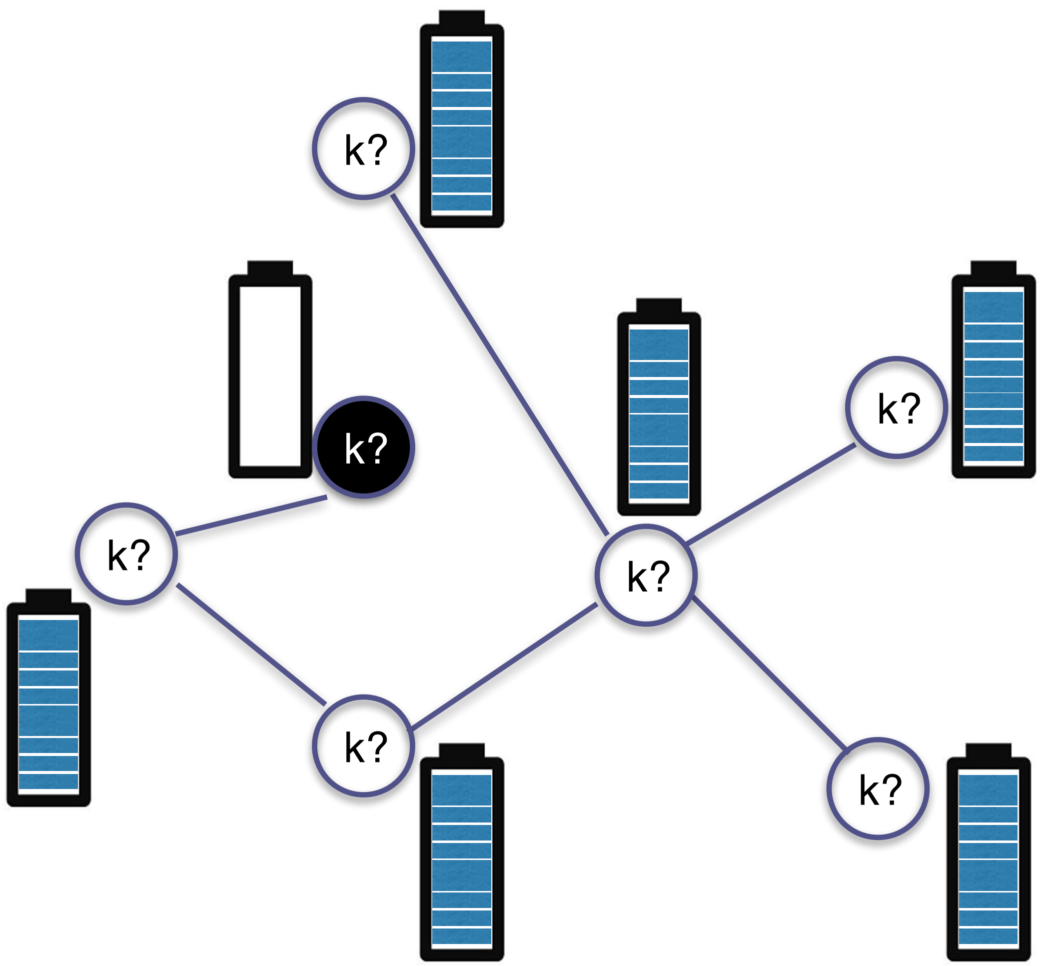

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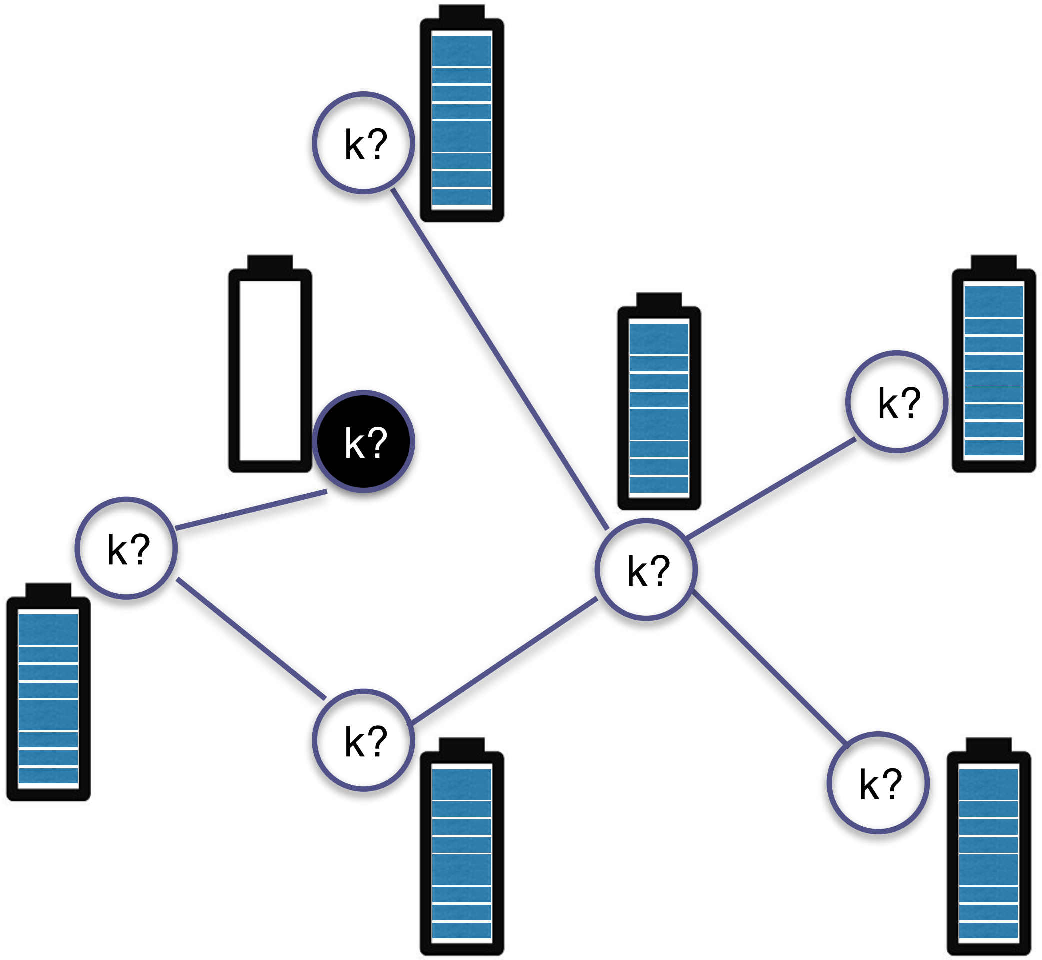

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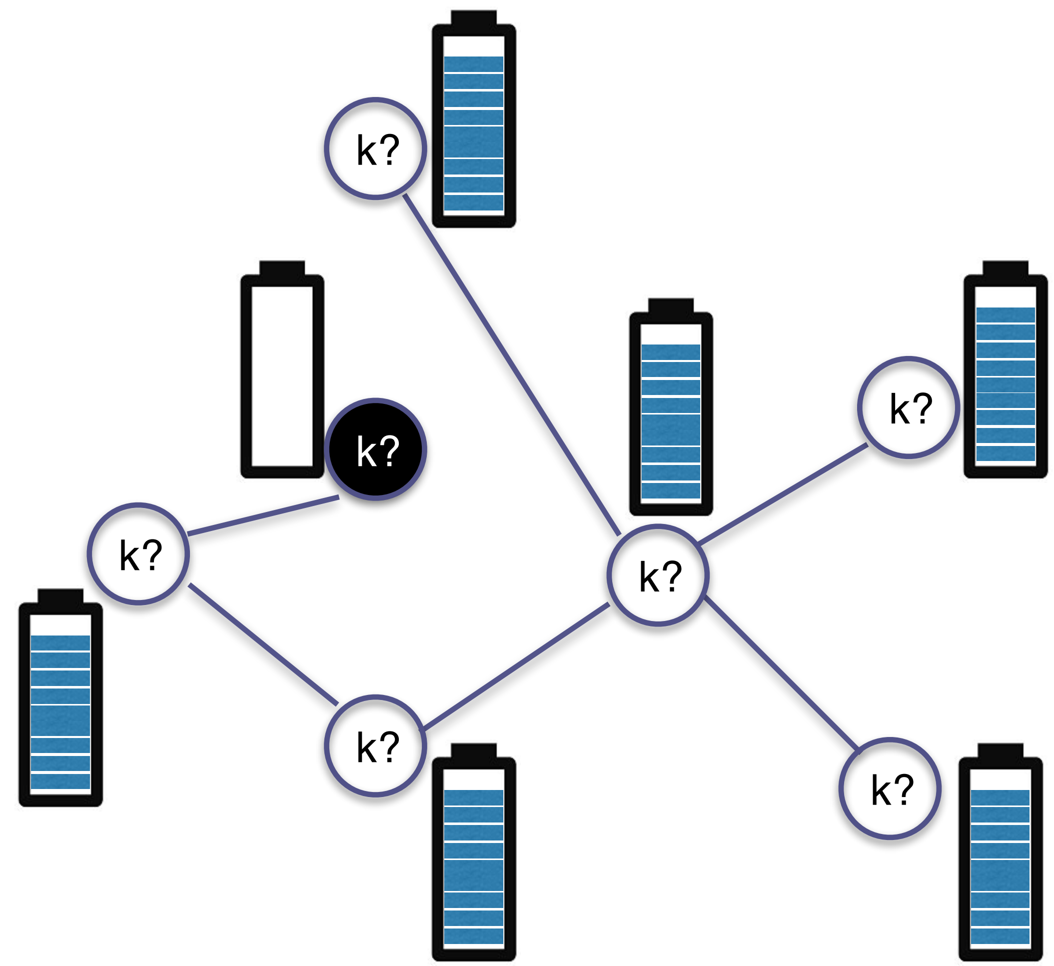

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- broadcast Φ and receive neighbors' Φ_i
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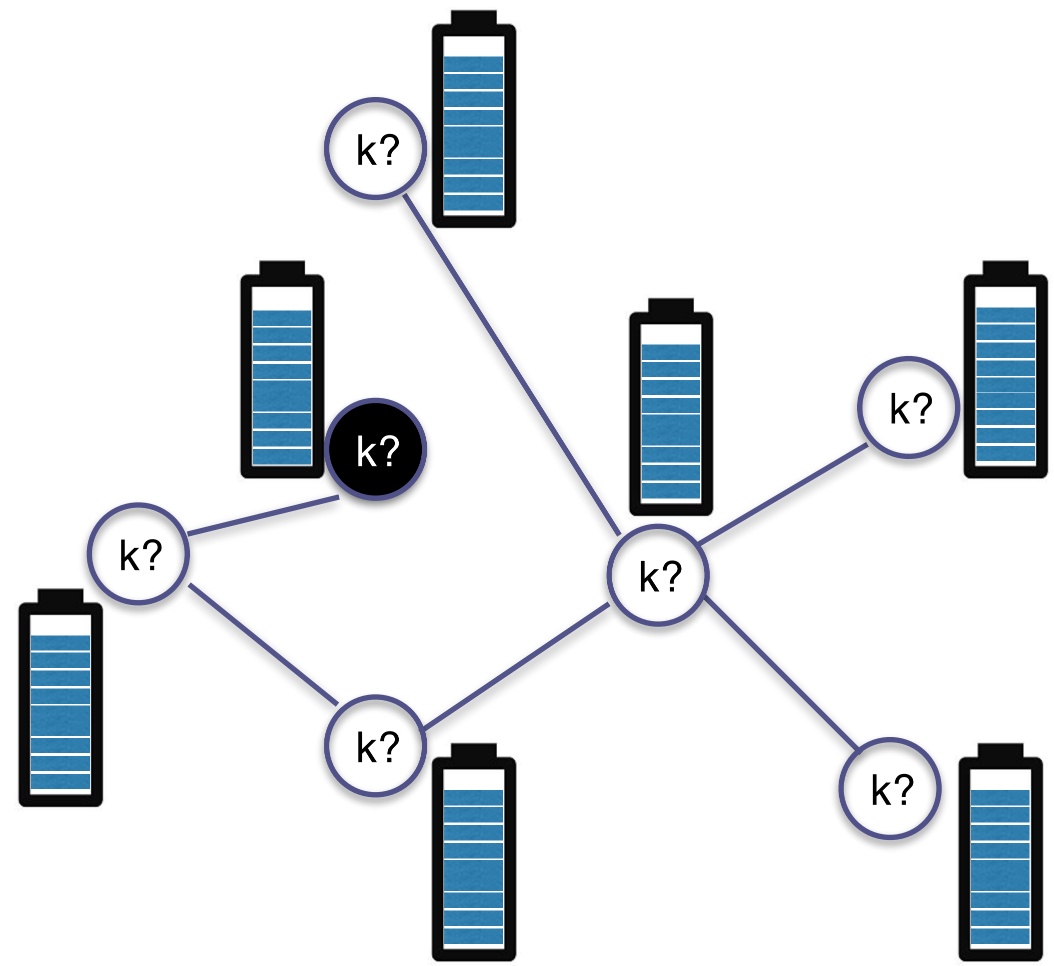

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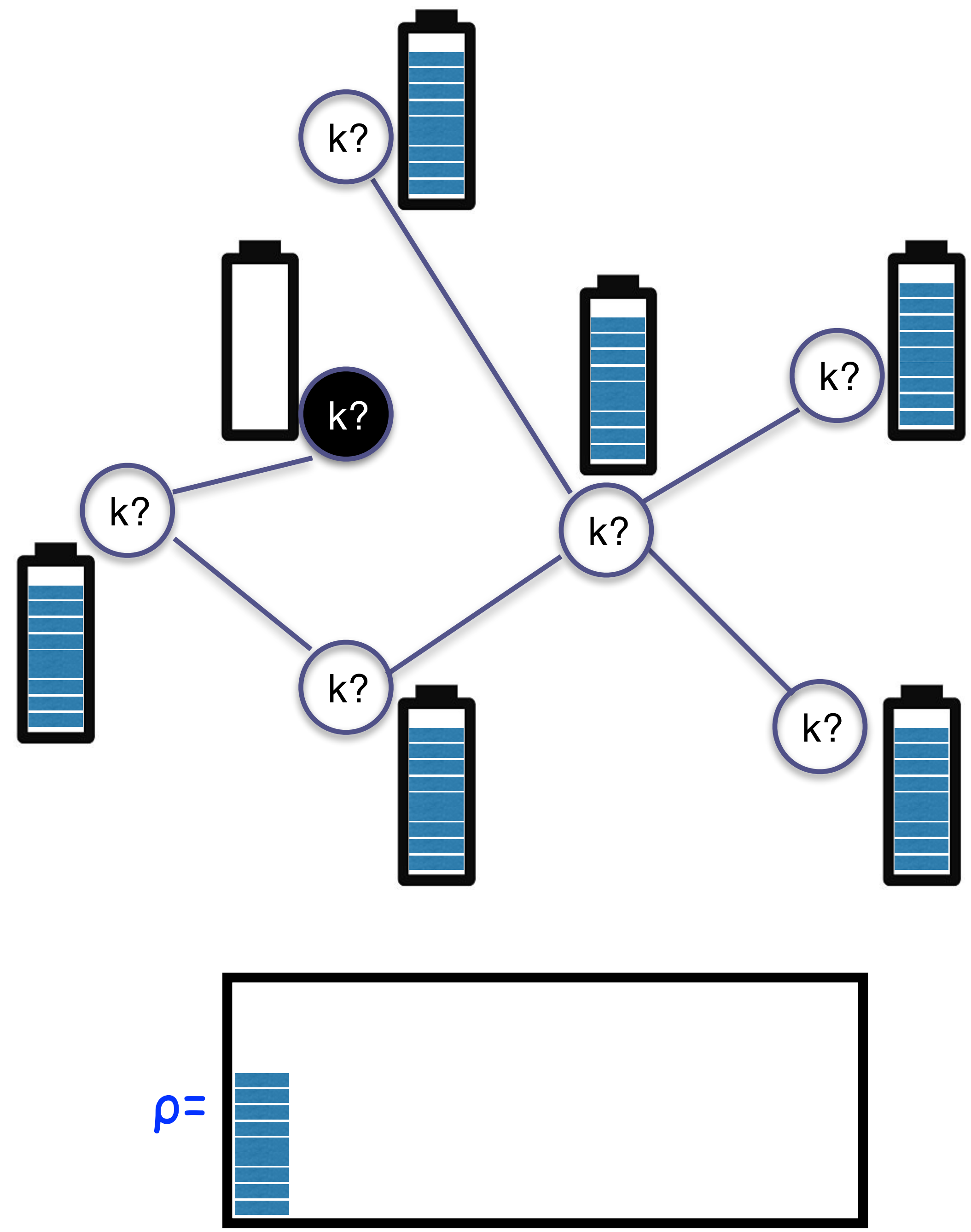

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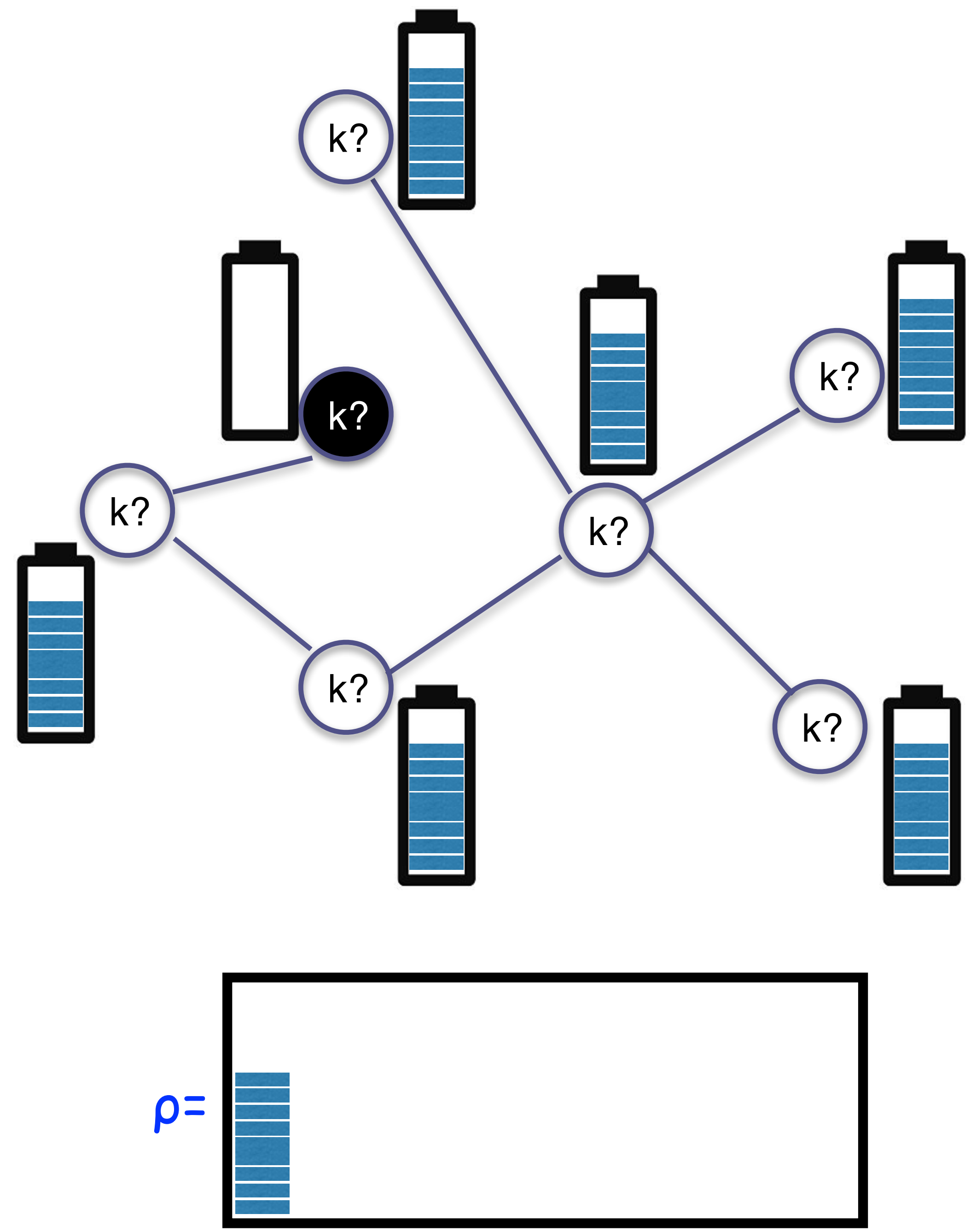

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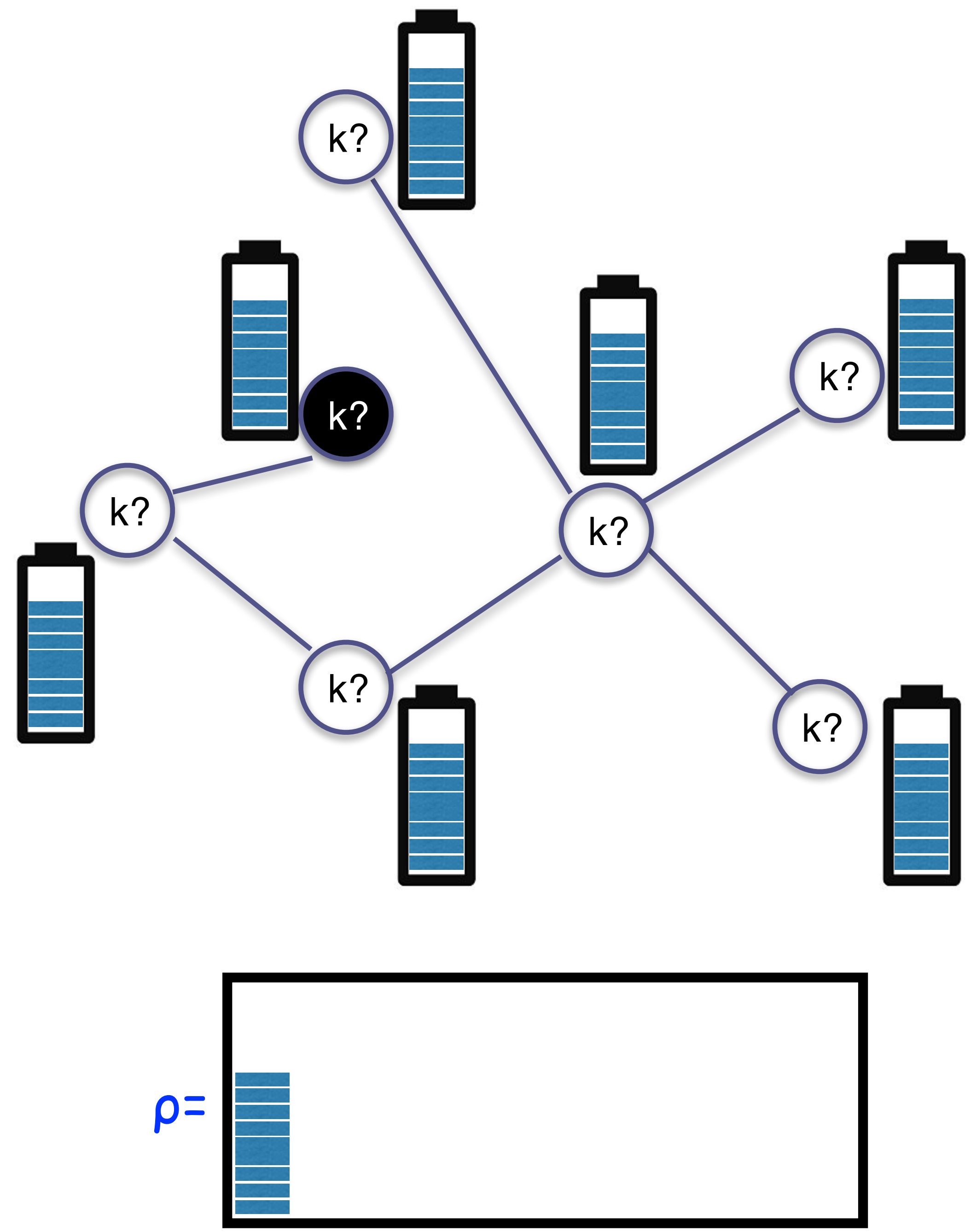

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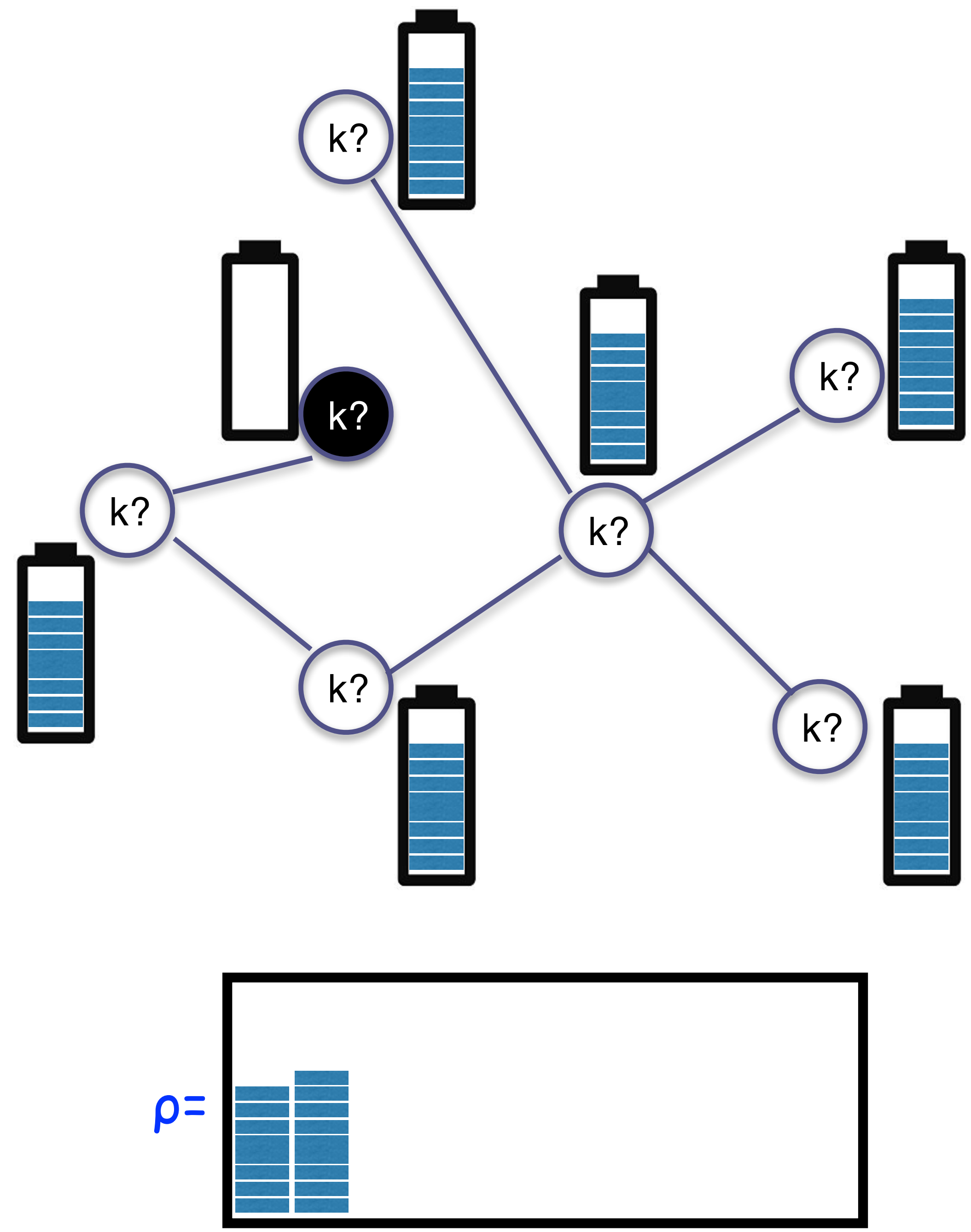

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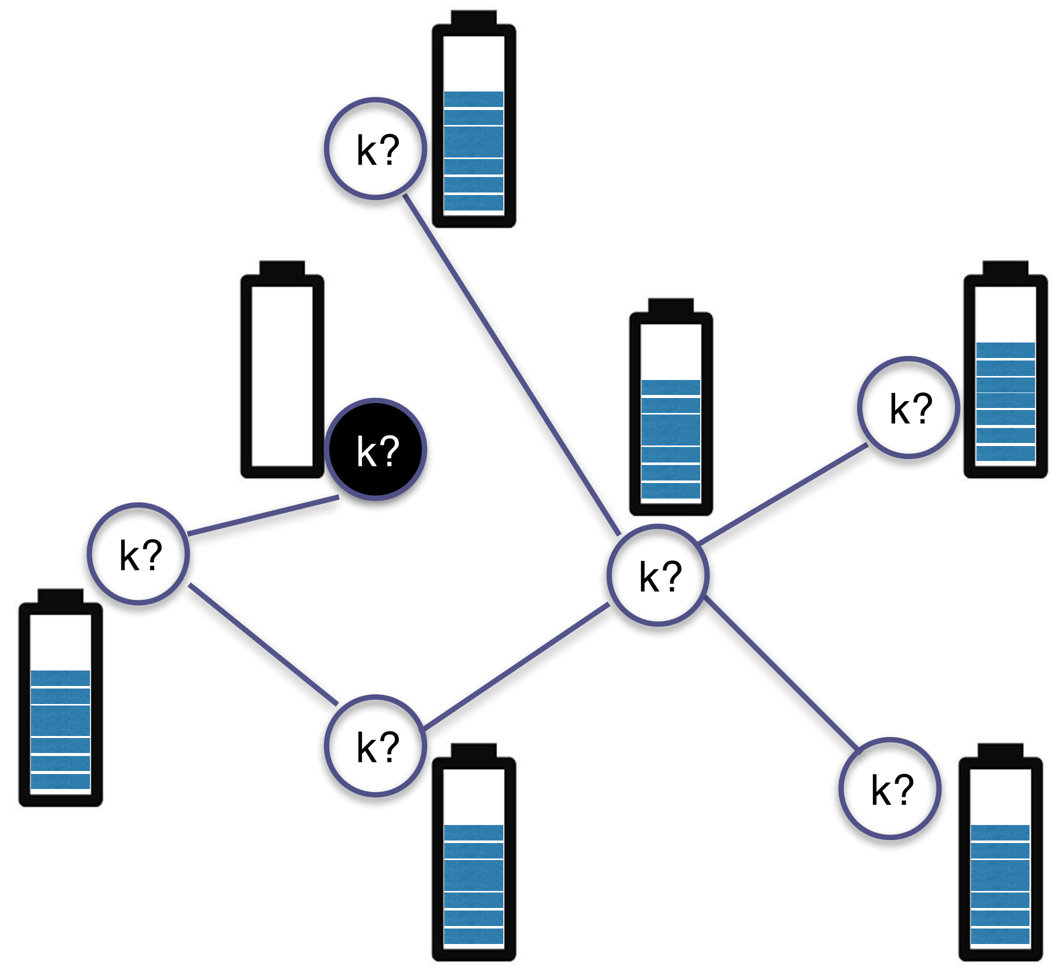

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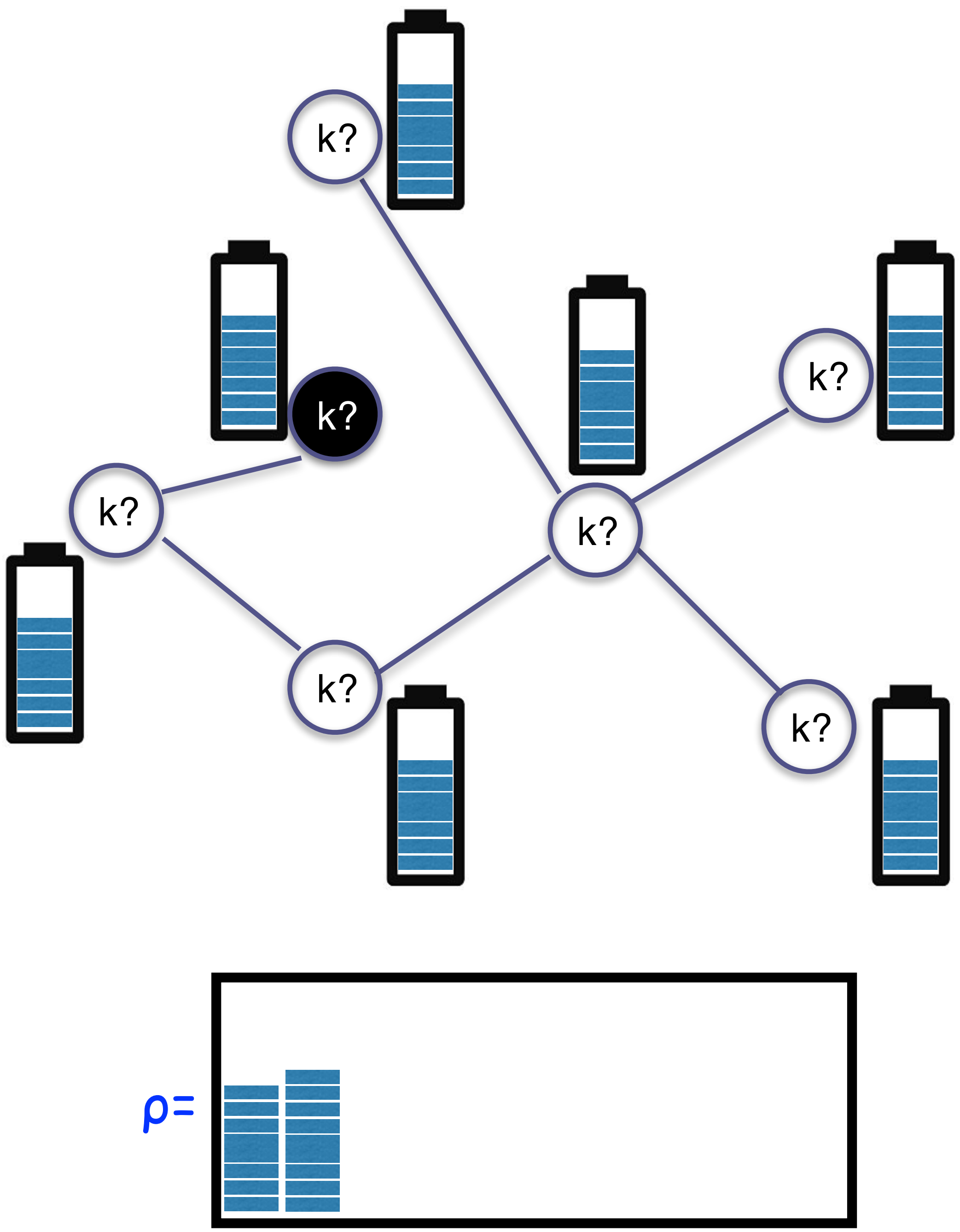

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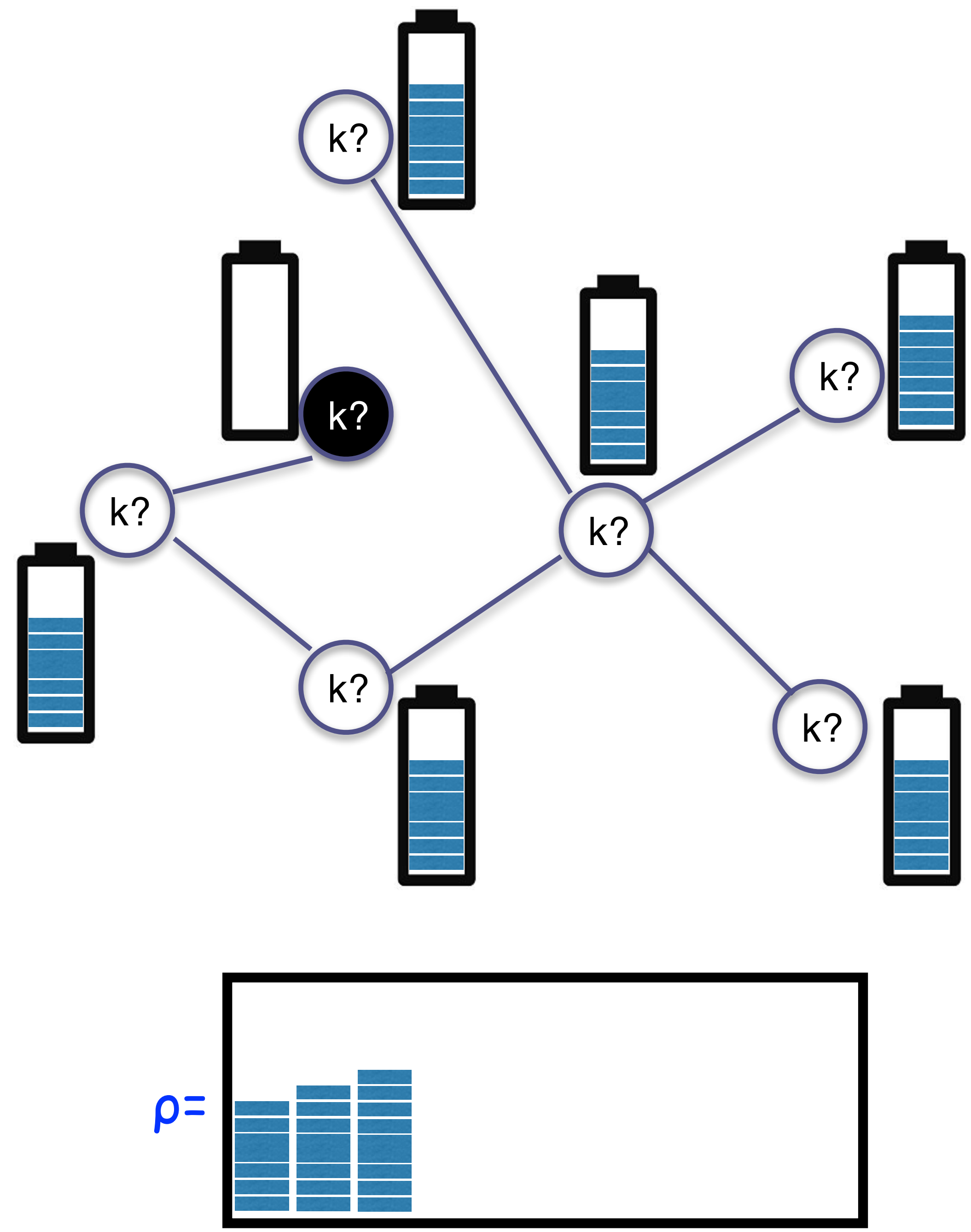

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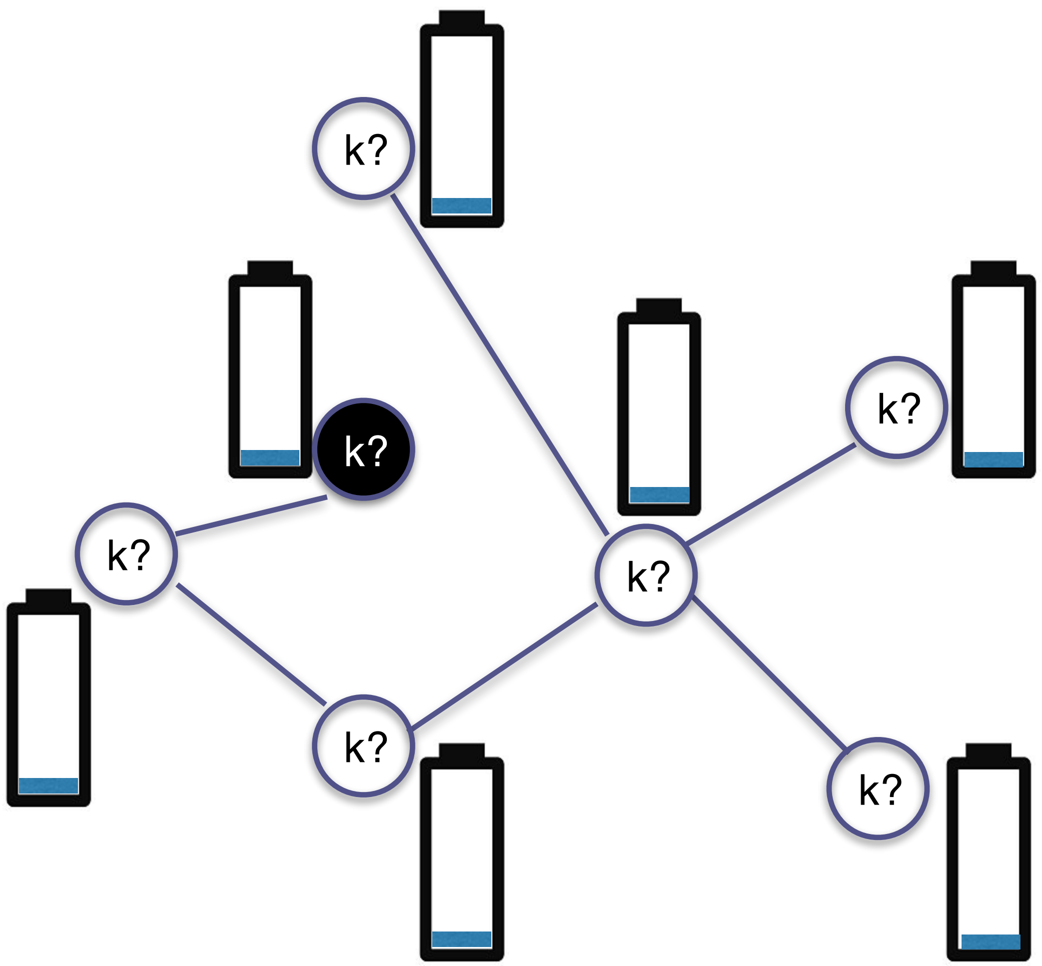
(to let blacks remove "enough" potential ρ)

$r(k)$ rounds:

(to "average" the current potentials Φ)



$\rho=$

MMC Epoch Example

epochs:

- one for each estimate $k=l+1, 2(l+1), 4(l+1), \ldots$
- initially, "potential" value: $\Phi_{white}=l$, $\Phi_{black}=0$

p(k) phases:

(to let blacks remove "enough" potential ρ)

r(k) rounds:

(to "average" the current potentials Φ)

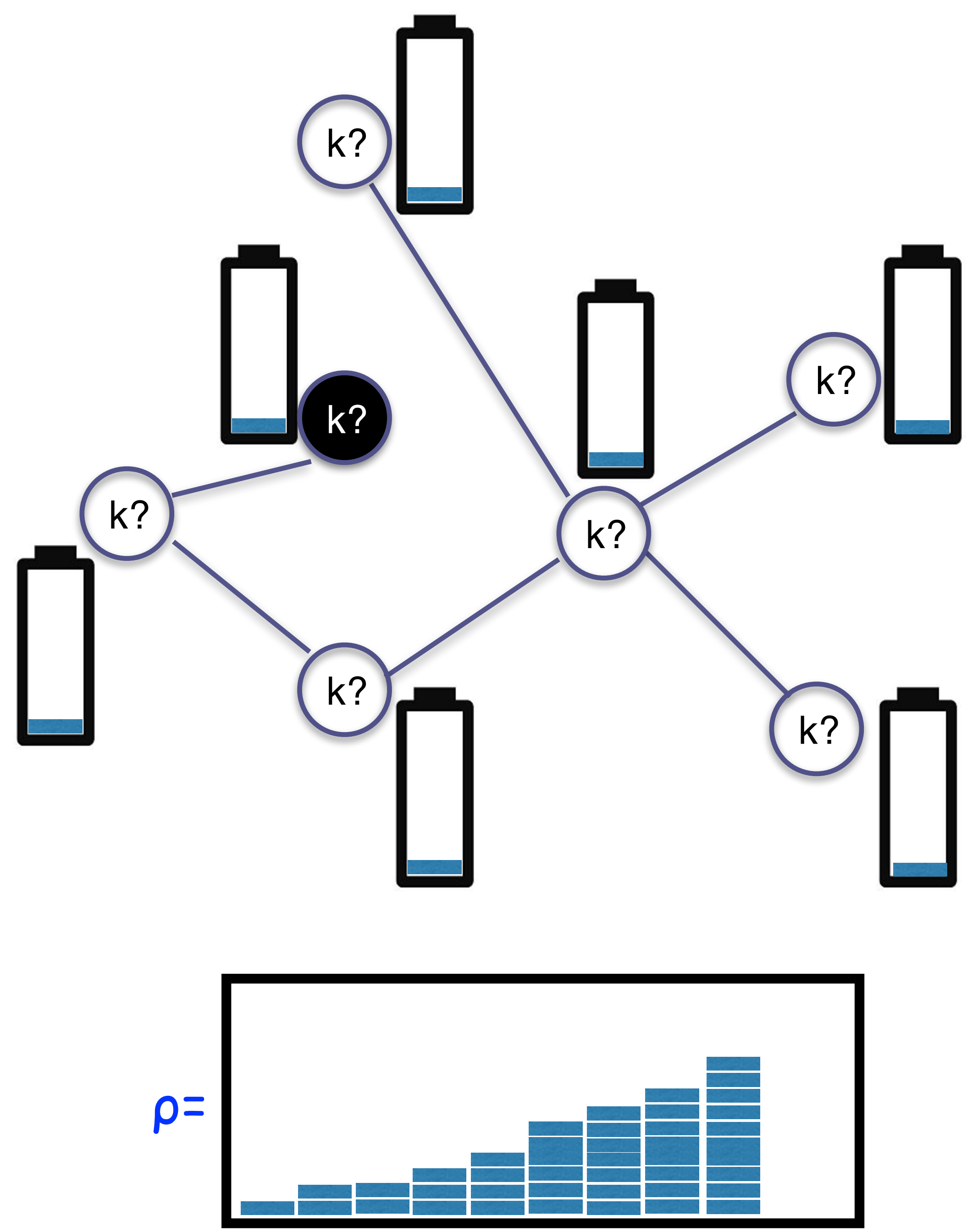
mass distribution:

- broadcast Φ and receive neighbors' Φ_i
- $\Phi = \Phi + \Sigma_{i \in N} \Phi_i/d(k) - |N|\Phi/d(k)$

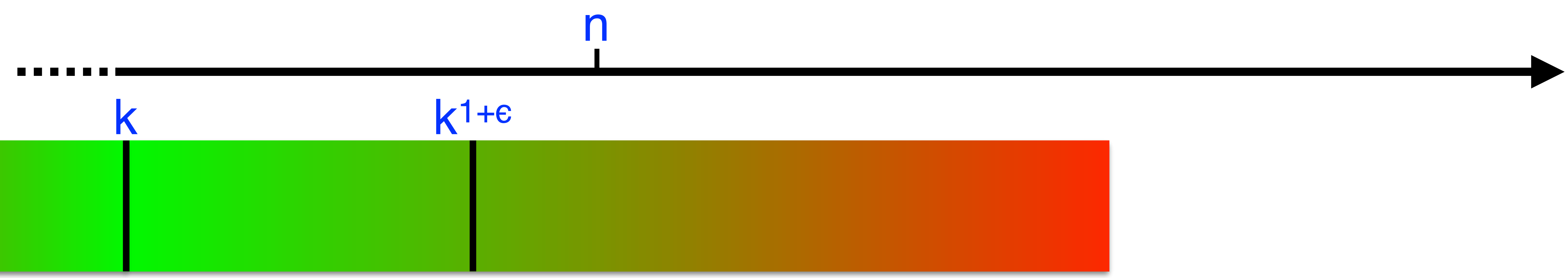
- blacks "remove" their potential: $\rho=\rho+\Phi$, $\Phi=0$

- blacks decide according to ρ
- blacks notify if $k >= n$
- try next k if needed

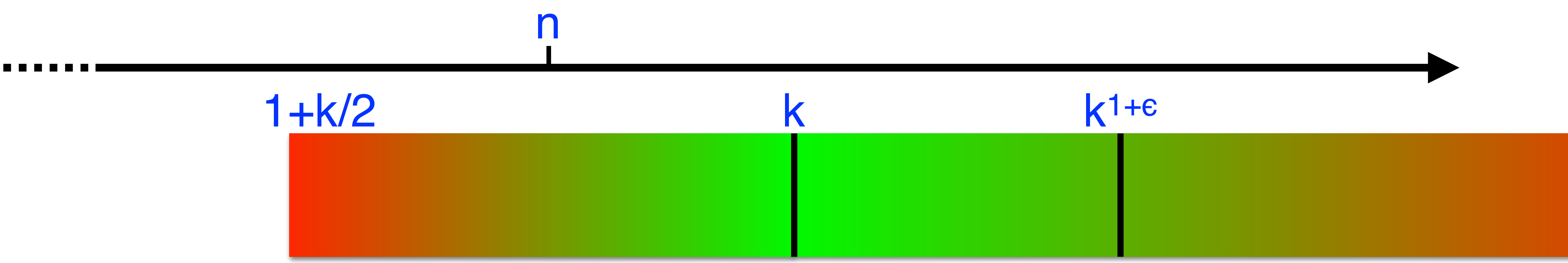
After p(k) phases…



MMC Alarms



MMC Alarms



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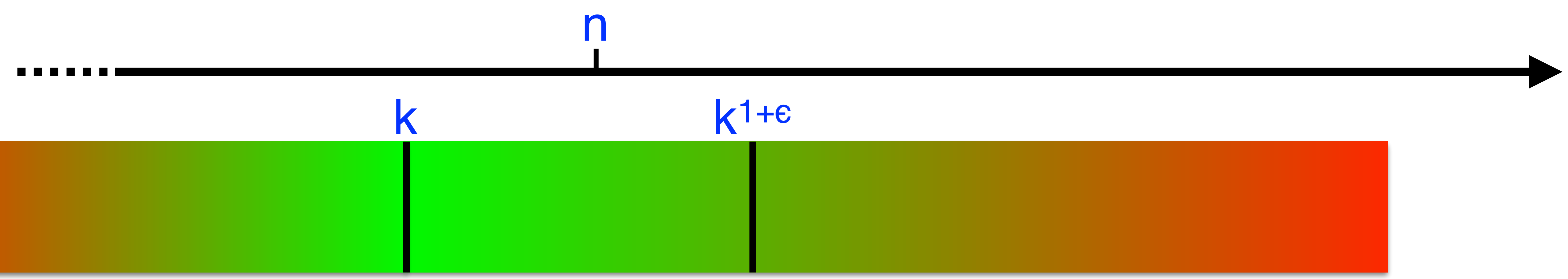

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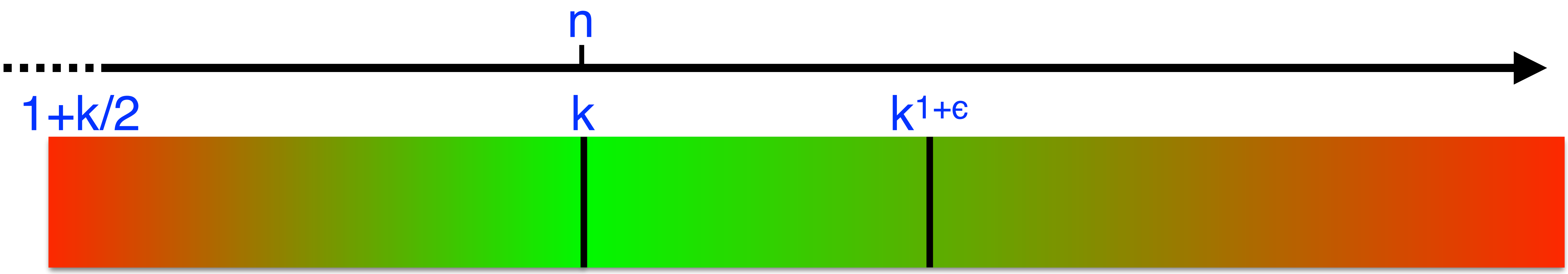
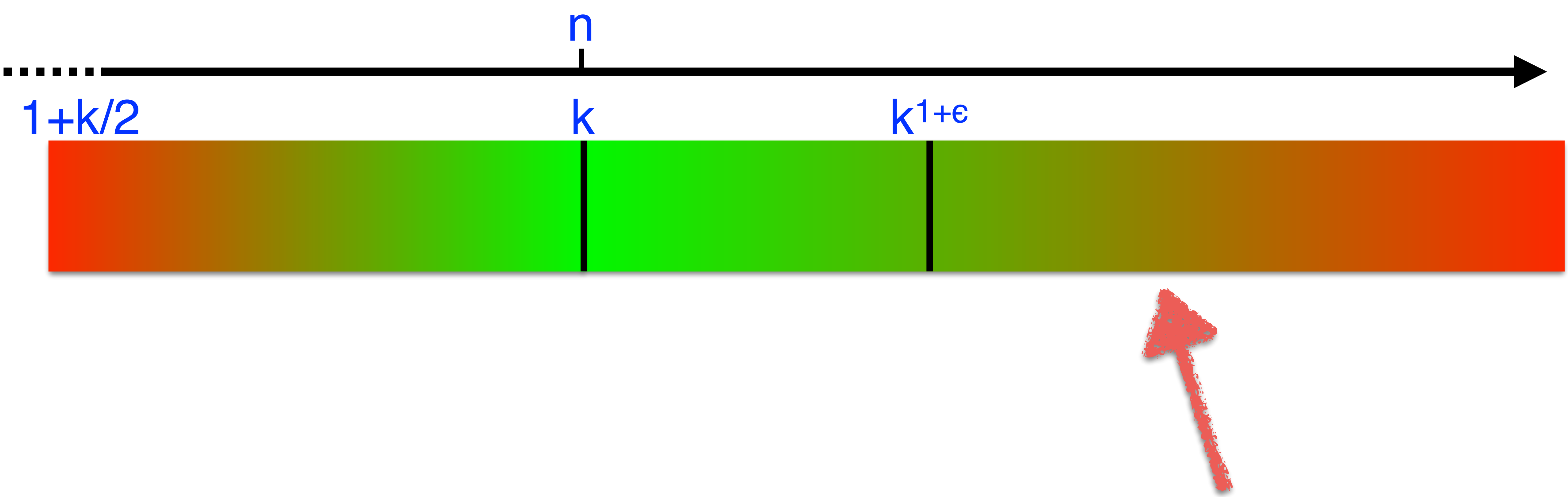


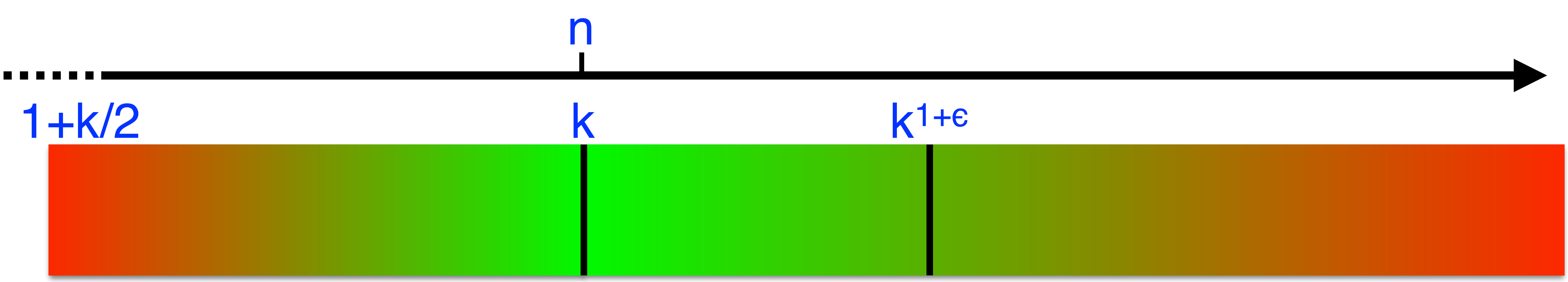
MMC Alarms



If n is "far" from k then not "many" nodes have "low" potential after phase 1,

so, blacks receive alarm from nodes with "high" potential "soon" after phase 1.

MMC Alarms



If n is “close” to k
from above
then blacks remove
“too much” potential.

If n is “far” from k then not
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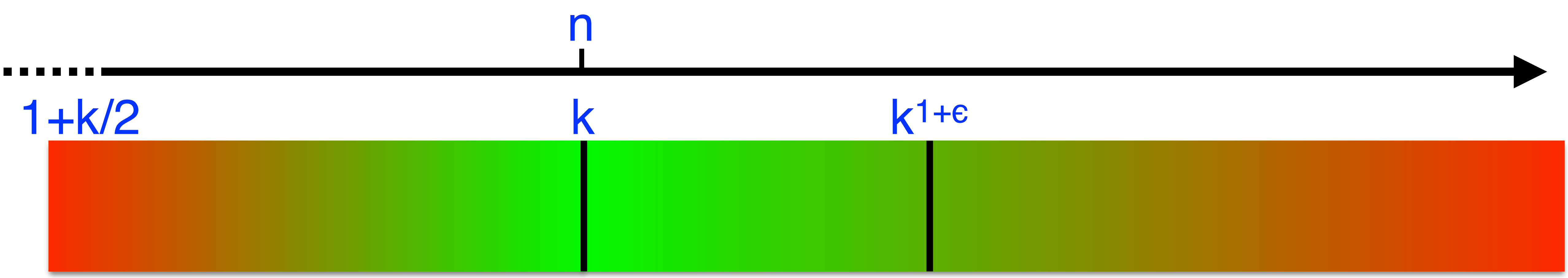
MMC Alarms



If n below k then blacks remove "too little" potential.

If n is "close" to k from above then blacks remove "too much" potential.

If n is "far" from k then not "many" nodes have "low" potential after phase 1,

so, blacks receive alarm from nodes with "high" potential "soon" after phase 1.

Randomized Counting

As we showed, if ℓ is unknown or zero, $\exists$ executions that do not stop

$\Rightarrow$ we need black nodes, and we need to know how many,

we aim (stochastically) for $\ell=1$.

Randomized Counting

LLMC Key Ingredients:

- consider consecutive powers of 2 as values of K
- for each K
- each node chooses to be black with probability inverse of K
- run MMC for $\ell = 1$ and $k \leqslant K$
- if $K \geqslant n$ and there is one black node $\Rightarrow$ done

Randomized Counting

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Challenges:

- How to detect that $K \geqslant n$?
- If $\ell = 0$ no count, but what if $\ell > 1$?

Randomized Counting

LLMC Key Ingredients:

- consider consecutive powers of 2 as values of K
- for each K
- each node chooses to be black with probability inverse of K
- run MMC for $\ell = 1$ and $k \leqslant K$
- if $K \geqslant n$ and there is one black node $\Rightarrow$ done

Two additional techniques:

- Run parallel threads:
  - if # threads with $\ell = 0$ is large enough, $K \geqslant n$ is likely
- Take max count over threads:
  - count with $\ell > 1$ is smaller than with $\ell = 1$

LLMC

procedure
$K \leftarrow \lceil\lceil 12/(\epsilon) \rceil\rceil$ // $\lceil\lceil x \rceil\rceil$: the smallest power of 2 bigger than x
$Count \leftarrow \emptyset$ // set of potentially "good" estimates computed in threads
$EmptyThreads \leftarrow 0$ // number of threads with no black node detected
while $Count = \emptyset$ or $EmptyThreads \leq f(K)/2$ **do**
$Count \leftarrow \emptyset$, $EmptyThreads \leftarrow 0$
$K \leftarrow 2K$
Initiate $f(K) = 64\frac{\log(K/\epsilon)}{\log(e/(e-2))}$ parallel threads
// parallel computation and messages sharing same resources/medium
for each thread **do**
for each node **do**
Select to be a black node with probability $1/g(K)$, where $g(K) = K/2$
end for
$k \leftarrow MMC(K, 1)$ // refer to Figure 2
if $k > 0$ **then**
$Count \leftarrow Count \cup \{k\}$
end if
if no black node detected **then**
Increase $EmptyThreads$ by 1
end if
end for
end while
return $\max(Count)$ // Output the maximum number in $Count$ as the size n.
end procedure

LLMC

if K too small ⇒

prob of black too high ⇒

black nodes too big ⇒

empty threads too small

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end if

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Increase $EmptyThreads$ by 1

end if

end for

end while

return $\max(Count)$ // Output the maximum number in $Count$ as the size n.

end procedure

when K gets close to n ⇒

even if ℓ =1, while K < n ⇒

it is k ⩽ K < n ⇒

no count

LLMC

if K too small ⇒

prob of black too high ⇒

black nodes too big ⇒

empty threads too small

```
procedure
    K ← ⌈⌈12/(ε)⌉⌉            // ⌈⌈x⌉⌉: the smallest power of 2 bigger than x
    Count ← ∅                 // set of potentially "good" estimates computed in threads
    EmptyThreads ← 0          // number of threads with no black node detected
    while Count = ∅ or EmptyThreads ≤ f(K)/2 do
        Count ← ∅, EmptyThreads ← 0
        K ← 2K
        Initiate f(K) = 64 log(K/ε)/log(e/(e−2)) parallel threads
          // parallel computation and messages sharing same resources/medium
        for each thread do
            for each node do
                Select to be a black node with probability 1/g(K), where g(K) = K/2
            end for
            k ← MMC(K, 1)                                    // refer to Figure 2
            if k > 0 then
                Count ← Count ∪ {k}
            end if
            if no black node detected then
                Increase EmptyThreads by 1
            end if
        end for
    end while
    return max(Count)  // Output the maximum number in Count as the size n.
end procedure
```

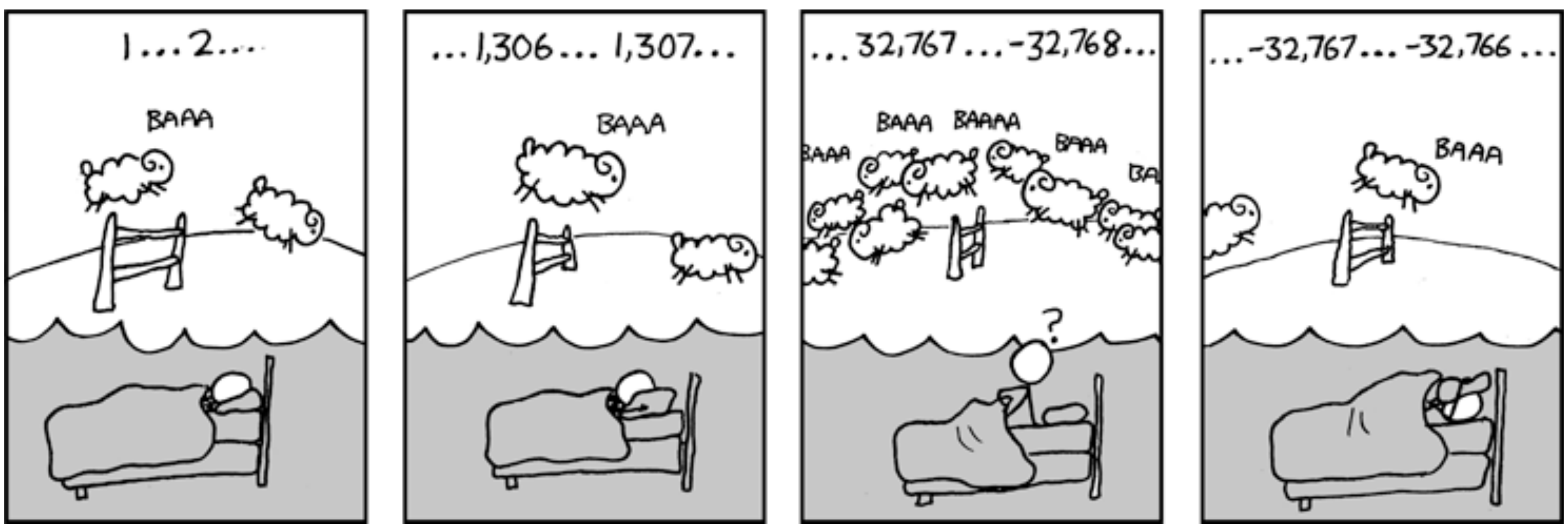
ICALP 2018 Open Questions

- Many distinguished nodes.
- Improve upper and/or lower bounds.
- Other computations in ADNs (beyond sum, avg, etc.).
- Asynchronous protocol.

ICALP 2018 2019 Open Questions

- Many distinguished nodes. ✔
- Improve upper and/or lower bounds. ✔
- Other computations in ADNs (beyond sum, avg, etc.).
- Asynchronous protocol.

Thank you!


1... 2...
BAAA
...1,306... 1,307...
BAAA
...32,767...-32,768...
BAAA BAAAA BAAA
?
...-32,767...-32,766...
BAAA