On the Complexity of Deterministic Distributed Wireless Link Scheduling

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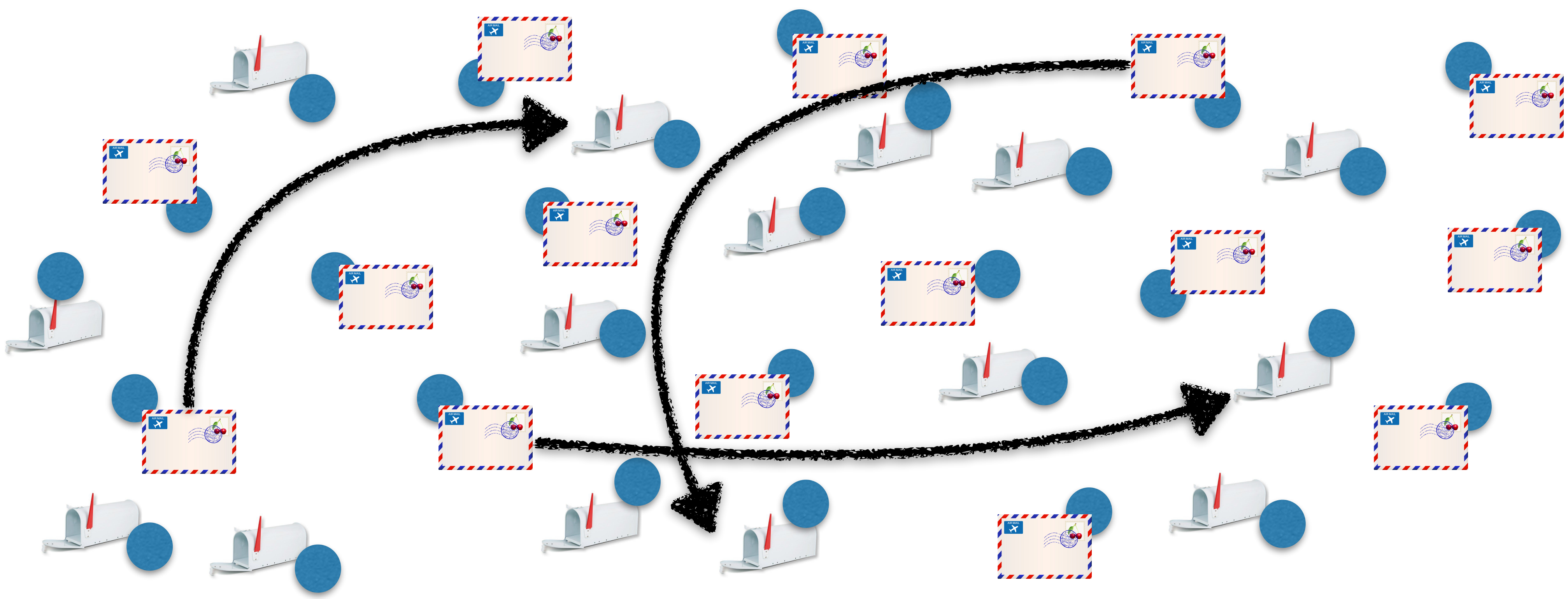
Link Scheduling

Link Scheduling is about

realization of requests between pairs of nodes

while minimizing makespan.



A request is the task of sending a message from some transmitter to some receiver.

Distributed Wireless Link Scheduling

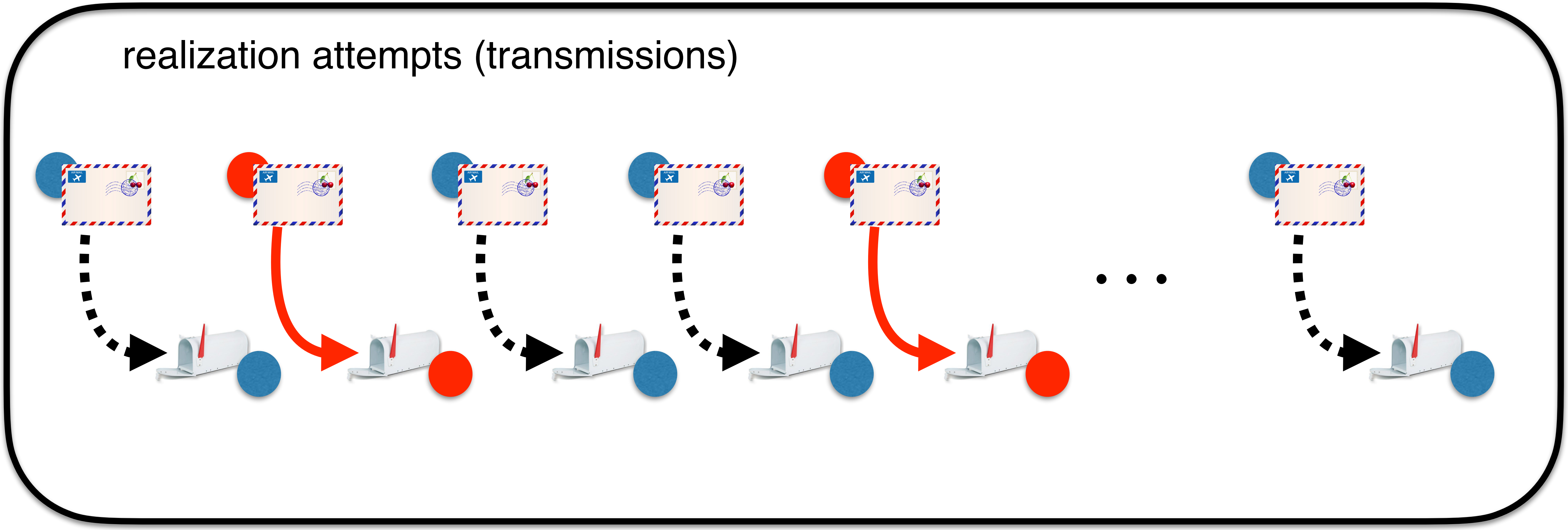
Main challenges:

- locality

  requests are known only locally by involved nodes

- dependencies among requests

  due to wireless interference



Distributed Wireless Link Scheduling

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- dependencies among requests

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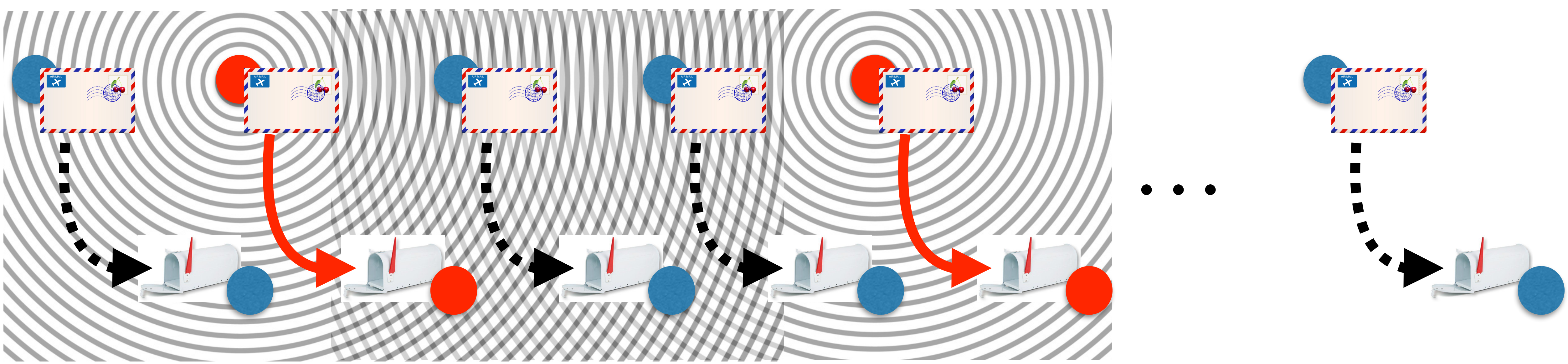
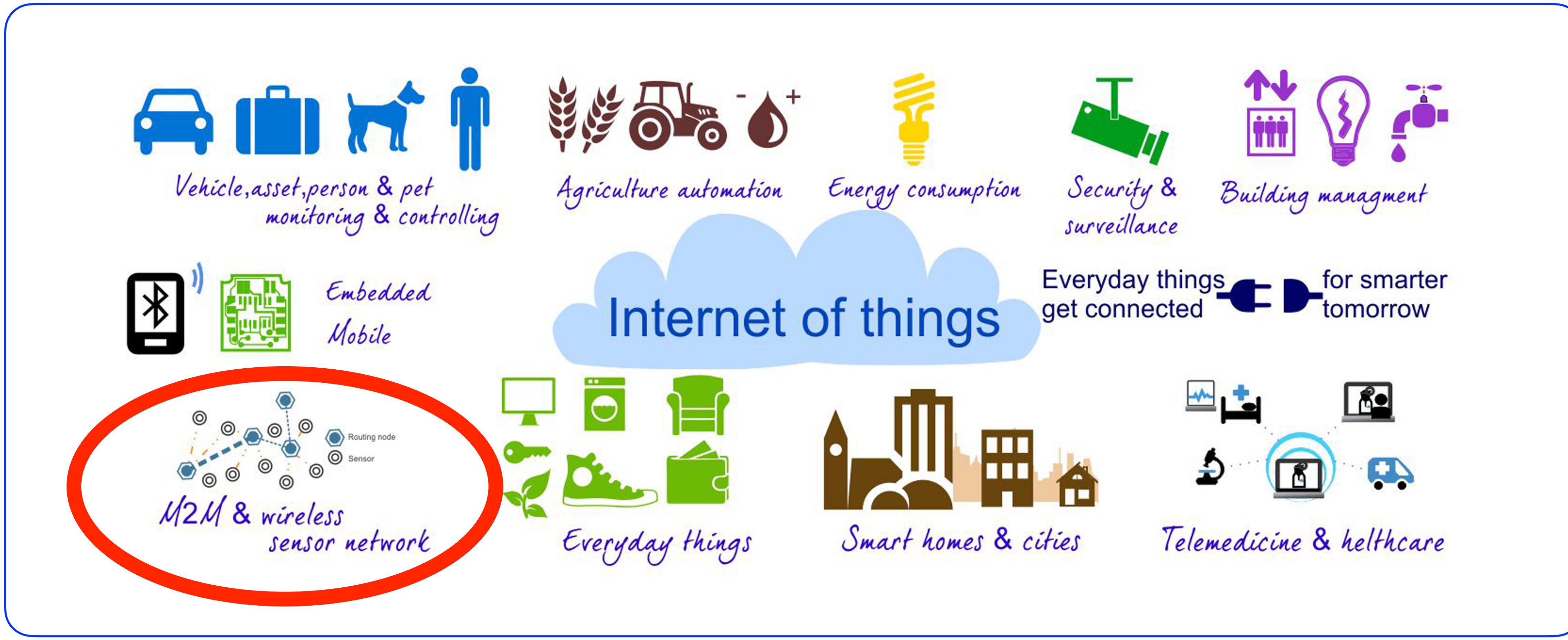
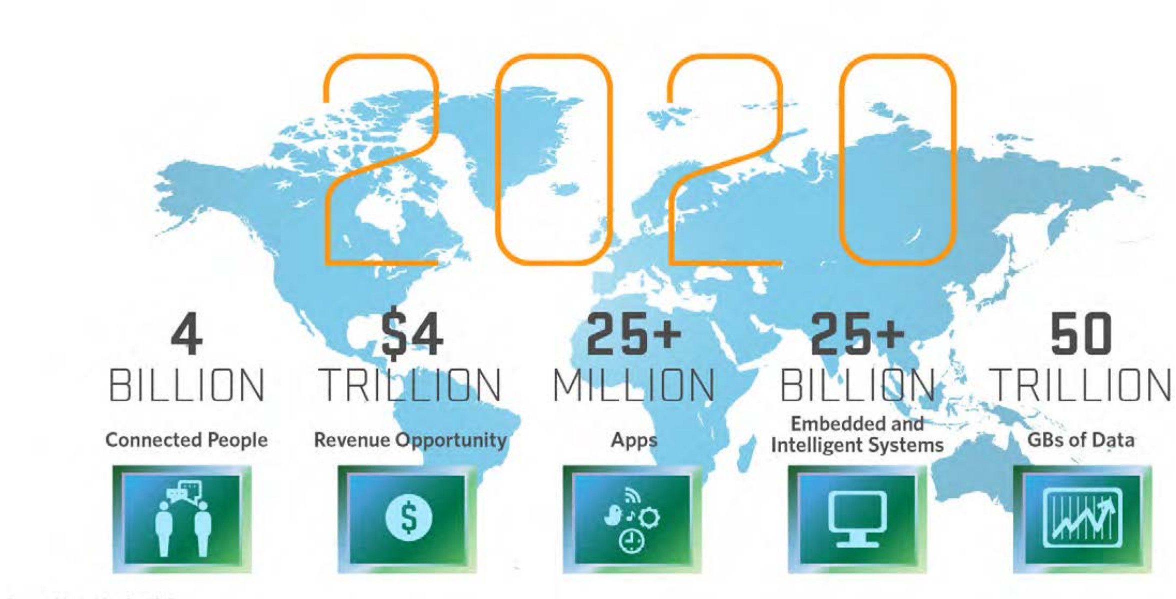
realization attempts (transmissions)

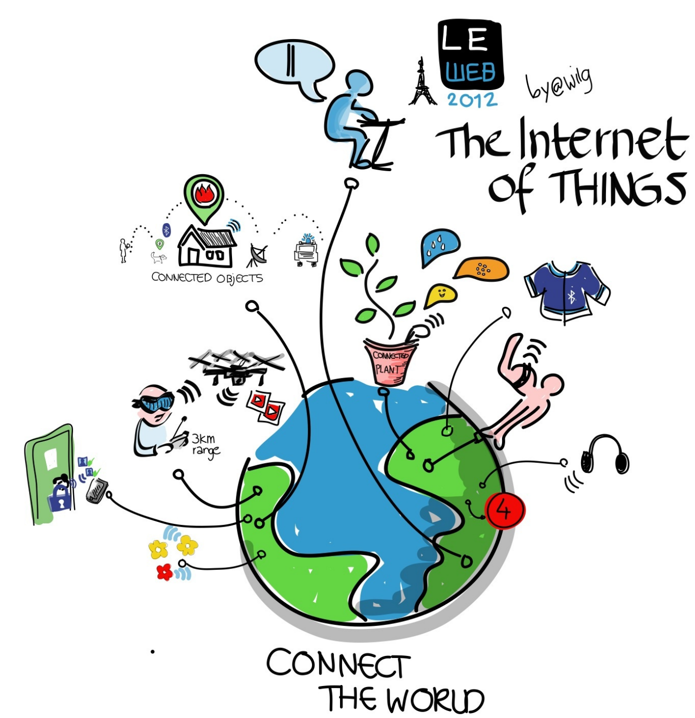


some requests attempted are not realized

App.: Ad-hoc Wireless Networks







Randomized or Deterministic?

Most Link Scheduling solutions rely on true randomness

It helps to break
those dependencies!

BUT

- Ad-hoc network nodes:

  access to truly-random bits is physically very limited!
- Massive networks:

  pseudorandom sequences may be too short!

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In this work we focus on Deterministic DWLS Protocols.

Distributed Wireless Link Scheduling Problem

Scenario :

- n network nodes called transmitters
- n network nodes called receivers
- Each transmitter holds a message to be delivered to some receiver
- Each (transmitter, receiver, message) is called a request
- Successful delivery of a message is called a realization of the request

Conditions :

- Realizations implemented through wireless communication

  $\Rightarrow$ interference among concurrent attempts of realization
- Adaptiveness: only to realization of own request.
- Unique ID's, only n is known
- Time slotted in rounds of communication

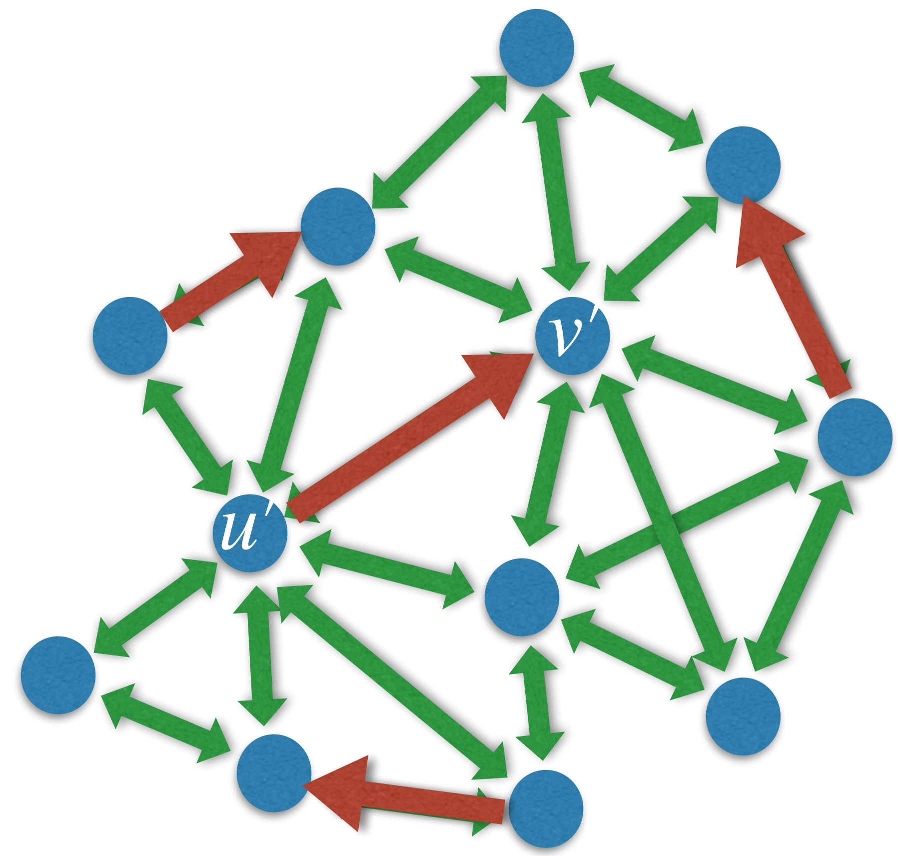
Goal : Realize all requests

Interference Models

Affectance Model [1,2,3]:

$a((u,v),(u',v'))$: real value in $[0,1]$

function quantifying interference
of communication through link (u,v)
on communication through link (u',v').



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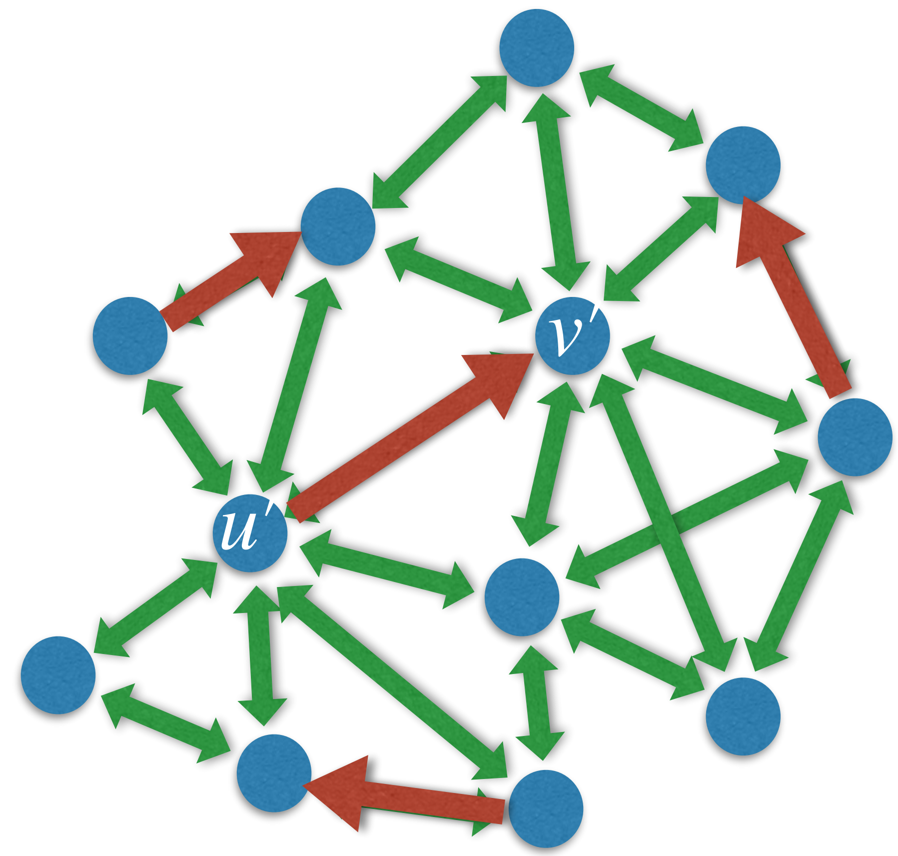

Radio Network Model [4]:

$a((u,v),(u',v'))$: either $\{0,1\}$, depending on $\{u,v'\} \notin E$ and $u \neq v'$

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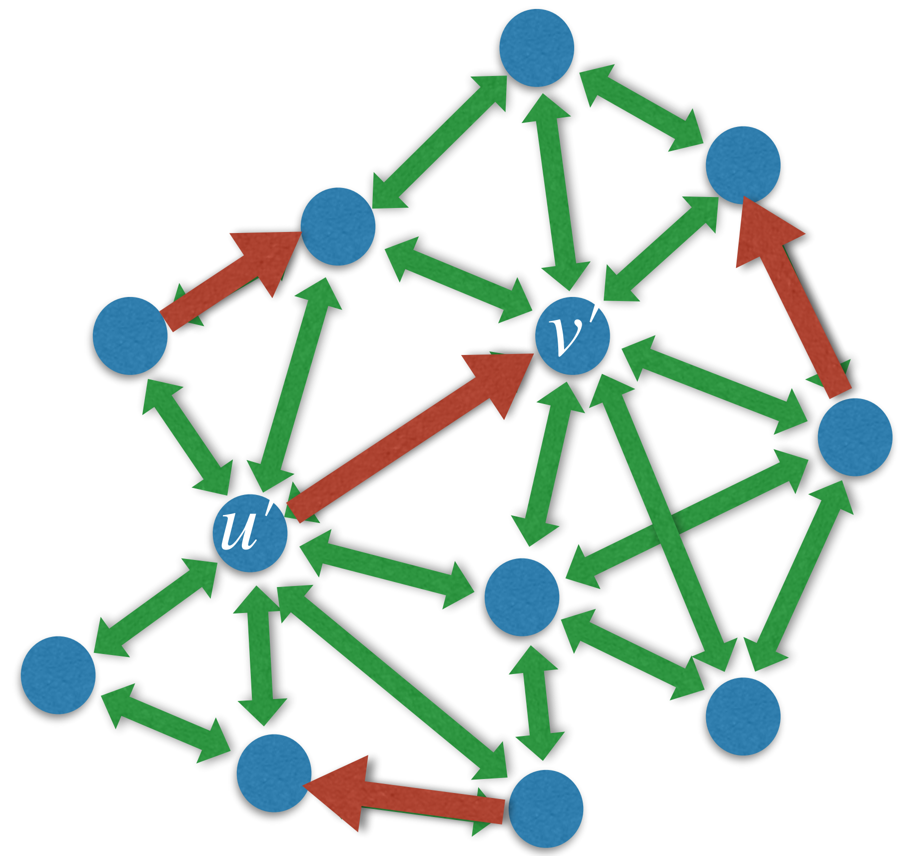

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SINR Model [5]:

$$a_p((u,v),(u',v')) : \min\left\{1, \frac{\beta p(u,v)}{d(u,v')^\alpha} \Big/ \left(\frac{p(u',v')}{d(u',v')^\alpha} - \beta N\right)\right\}$$

Previous work: uniform power, constant noise…

… combined with Euclidean distance and constant attenuation

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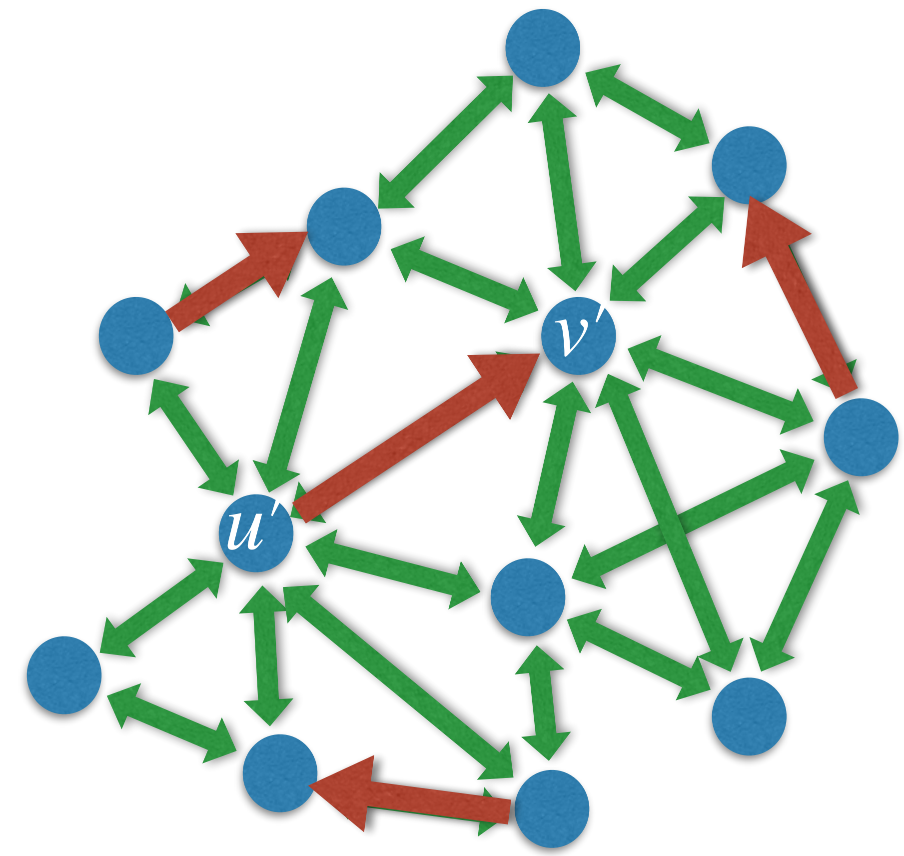

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Graph-metric SINR Model:

$$a((u,v),(u',v')) : \min\left\{1, \frac{\beta}{d(u,v')^{\alpha}}\right\}$$

α : Attenuation

β : Threshold

Uniform power (overcoming noise)

$d(\cdot,\cdot)$: distance in # hops

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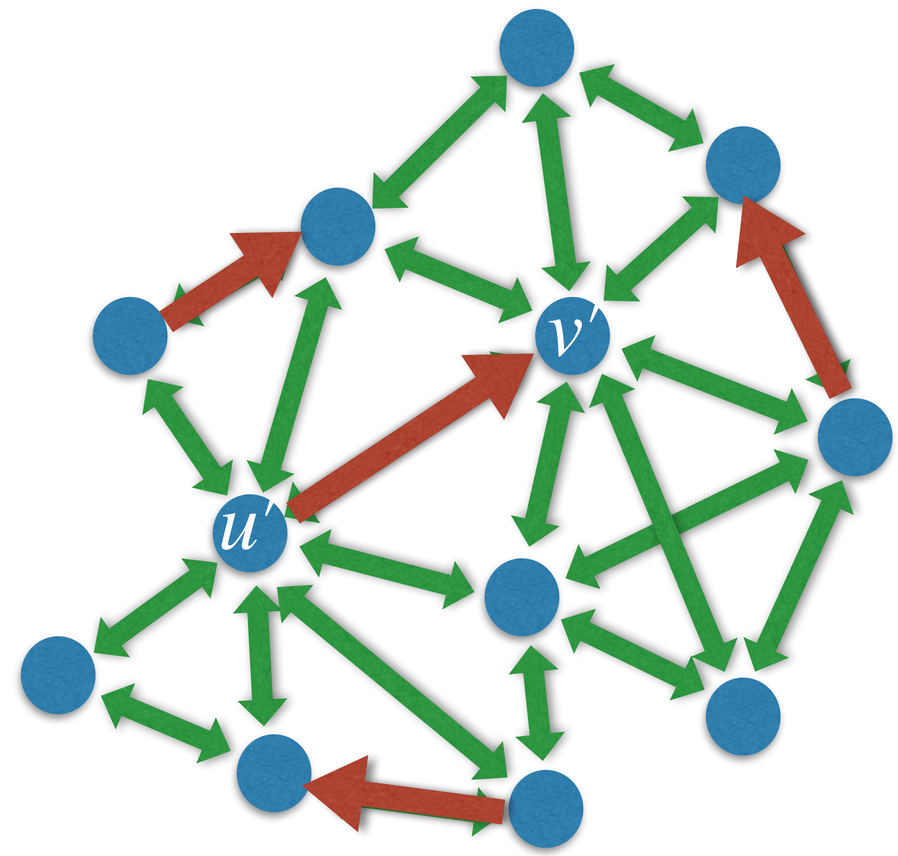

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- **Realization:**

  Request (u',v')
  is realized (the message from u' is received by v') at time j
  if and only if

  » u' transmits the message at time j and

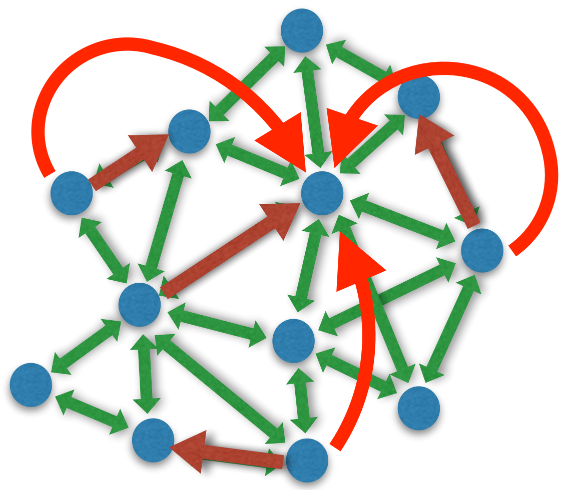
  » $$\sum_{(u,v)\in L(j):u\neq u'} a((u,v),(u',v')) < 1,$$

$L(j)$: subset of links carrying transmissions at time j.

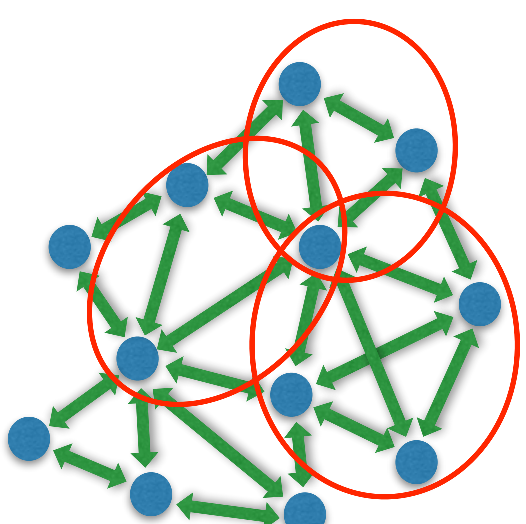
Performance Metrics

- Length of schedule: number of rounds to realize all requests given as a function of the number of requests n, the maximum average affectance $\mathscr{A}$ [1], and the metric growth ϕ of the underlying metric space.

Intuitively:



- $\mathscr{A}$ is the maximum cumulative affectance an average receiver can experience, for any set of broadcasting transmitters.



- The topology has metric growth ϕ if every 2-clique in the network can be covered by at most ϕ regular cliques.

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Contributions and Previous Work

Bound	Model features	Ref
$\Omega\left(\frac{\mathcal{A}}{\log n}\right)$	SINR Euclidean space	[19]
$O\left(\mathcal{A}\log n\right)$, ***whp***	SINR Euclidean space	[19]
$O(\mathcal{A}\log n)$, ***whp***	Arbitrary interference	[20]
$O(\min\{n, \mathcal{A}^2\log^3 n\})$	Arbitrary interference	[20]
$\Omega\left(\min\left\{n, \frac{\mathcal{A}^2}{\log^2 n}\right\}\right)$	RN, SINR, metric space	Thm 1
$O\left(\mathcal{A}\phi^6\log^4 n\right)$	RN, ϕ-bounded growth	Thm 2
$\omega\left(1+\min\left\{n, \frac{\min\{\mathcal{A},\phi\}\mathcal{A}}{\log^2 n}\right\}\right)$	RN, SINR, ϕ-bound. metric	Thm 3

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1st super-linear lower bound

nearly matches (up to polylog)

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below general lower bound for $A >> \phi$

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1st lower bound showing dependency on ϕ

both lower bounds hold even with realization acks

below general lower bound for $A >> \phi$

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RN Lower Bound

<u>Protocol</u>

Schedule of transmissions for each transmitter

Can be viewed as a sequence of ”queries” (subsets of transmitters)

include in query or not

Each query is a column

Depends on input graph (adaptive to realizations)

Each transmissions schedule is a row

Query	1	2	3	4	5	6	7	…
Transmitter 1	0	1	0	1	1	1	0	…
Transmitter 2	1	1	0	0	1	0	1	…
Transmitter 3	0	0	1	0	1	1	1	…
Transmitter 4	0	1	0	1	0	0	0	…
…	…	…	…	…	…	…	…	…
Transmitter n	1	1	0	1	0	1	1	…

RN Lower Bound

Protocol

Schedule of transmissions for each transmitter

Can be viewed as a sequence of "queries" (subsets of transmitters)

include in query or not

query $L_6(G_6) = \{t_1, t_3, \ldots, t_n\}$
query size $|L_6(G_6)|$

Depends on input graph (adaptive to realizations)

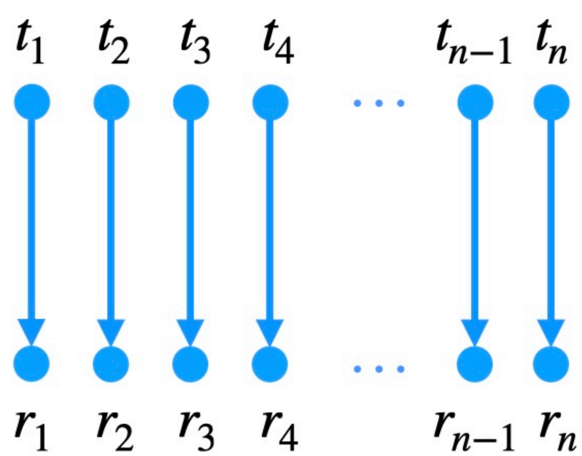
Query	1	2	3	4	5	6	7	…
Transmitter 1	0	1	0	1	1	1	0	…
Transmitter 2	1	1	0	0	1	0	1	…
Transmitter 3	0	0	1	0	1	1	1	…
Transmitter 4	0	1	0	1	0	0	0	…
…	…	…	…	…	…	…	…	…
Transmitter n	1	1	0	1	0	1	1	…

frequency = number of 1's in "small" queries

RN Lower Bound

Adversarial network

Built incrementally simulating the protocol query by query.



Initial graph $G_1 = (V, E_1)$ containing only the set of requests $E_1 = L$.

Then, for each query, the adversary adds more links to produce interference:

query size →
- singleton : do nothing, impossible to block.
- "small" :
  - → block "low" frequency requests
  - → "high" frequency requests allowed (hard to block with limited degree)
- "large" : block connecting to d other transmitters (chosen at random)

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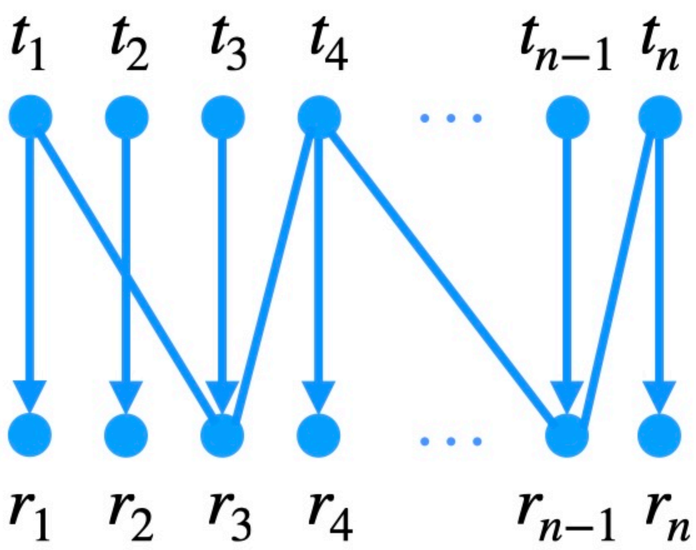

Graph $G_j = (V, E_j)$ at the beginning of some round j.

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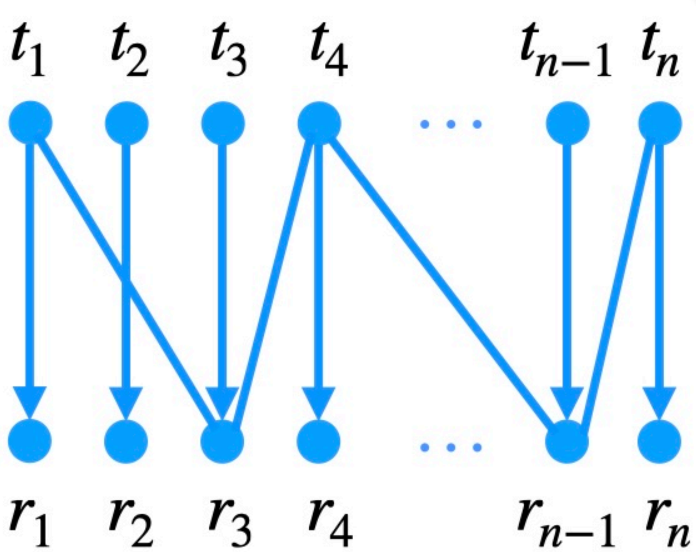

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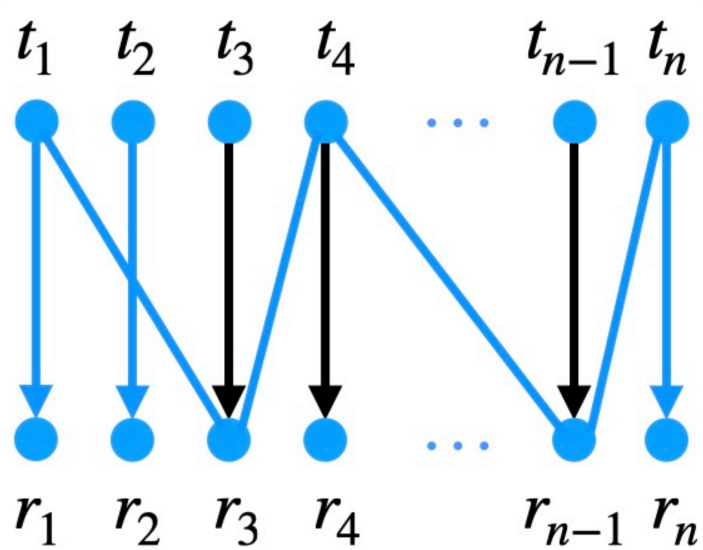
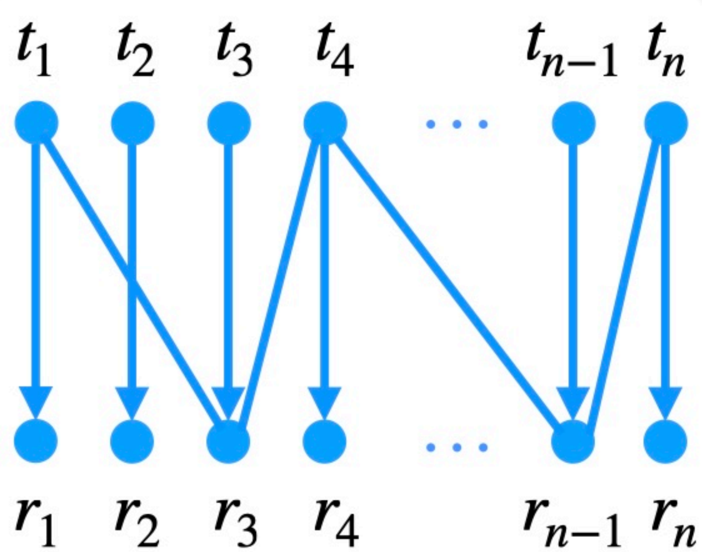


Query $L_j(G_j) = \{t_3, t_4, t_{n-1}\}$. Without additional links, (t_4, r_4) would be realized. Let $|L_j(G_j)|$ be "large".

RN Lower Bound

Adversarial network

Built incrementally simulating the protocol query by query.



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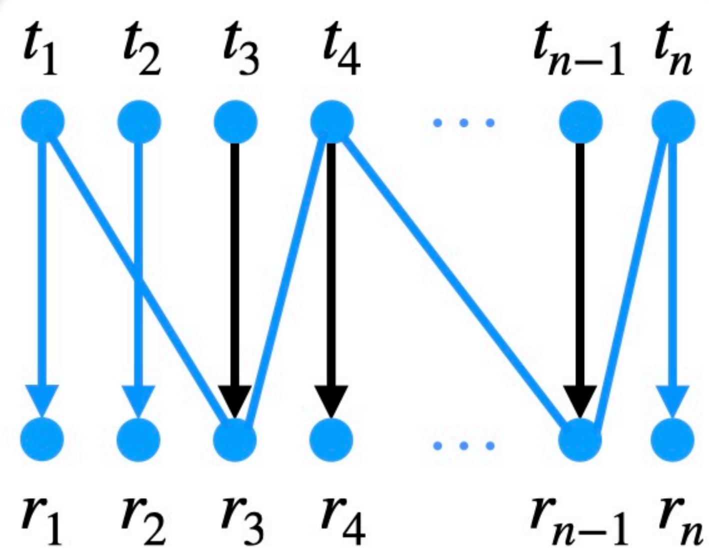

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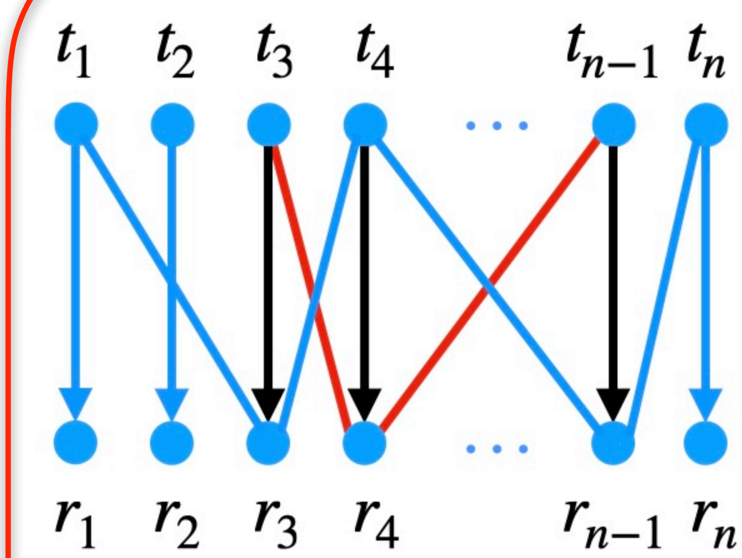

Graph $G_{j+1} = (V, E_{j+1})$ after adding (red) links from $d = 2$ (random) transmitters in $L_j(G_j)$ to interfere at r_4.

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Adversarial network

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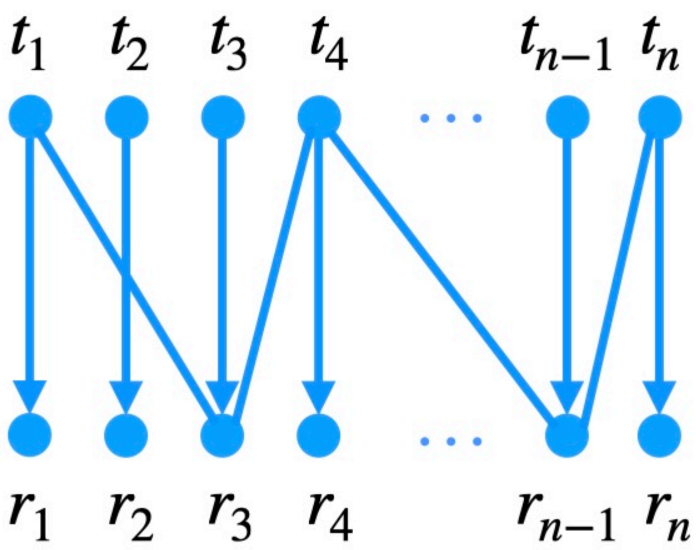

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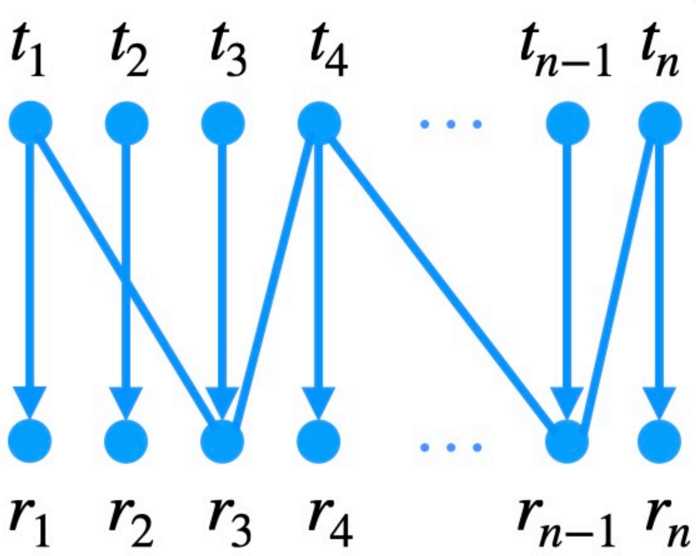

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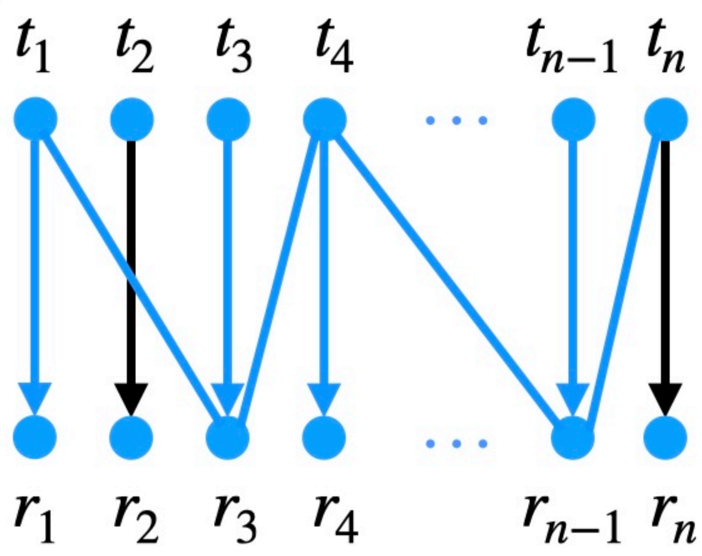

Query $L_j(G_j) = \{t_2, t_n\}$. Without additional links, both requests would be realized. Let t_2 be low frequency and t_n not.

RN Lower Bound

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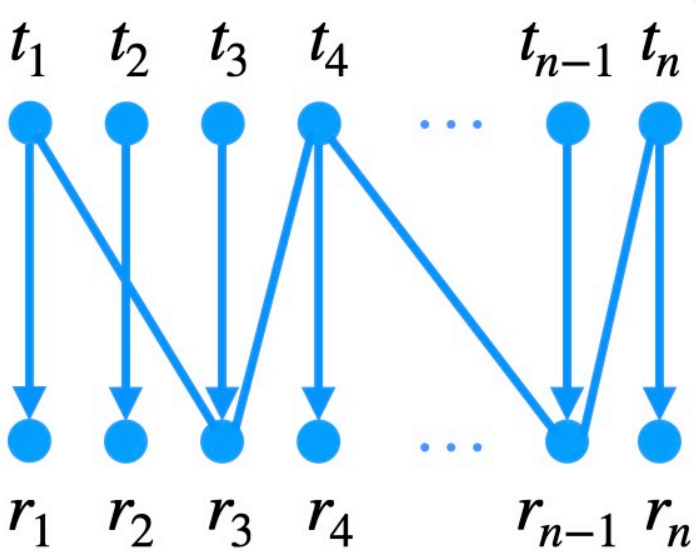

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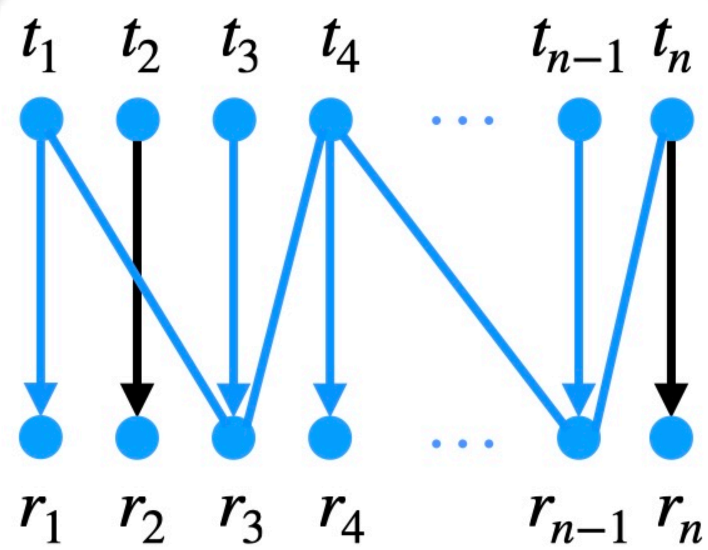

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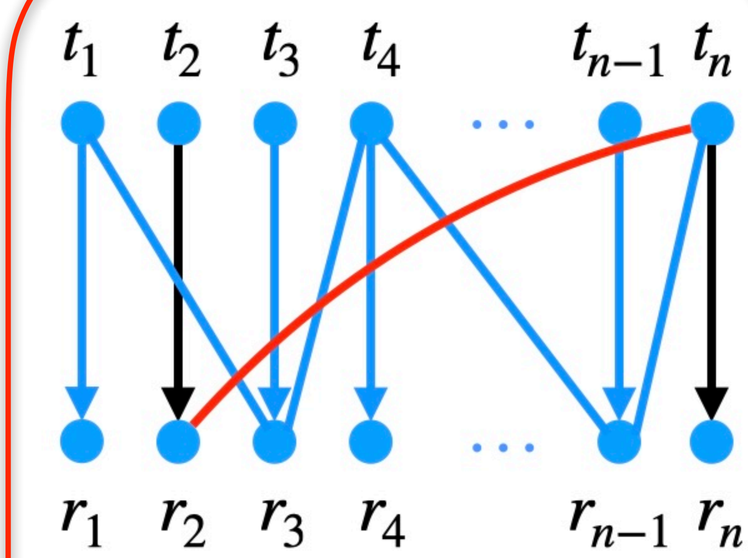

Graph $G_{j+1} = (V, E_{j+1})$ after adding link (t_n, r_2) to interfere at r_2. (t_n, r_n) is allowed to be realized.

RN Lower Bound

Adversarial network

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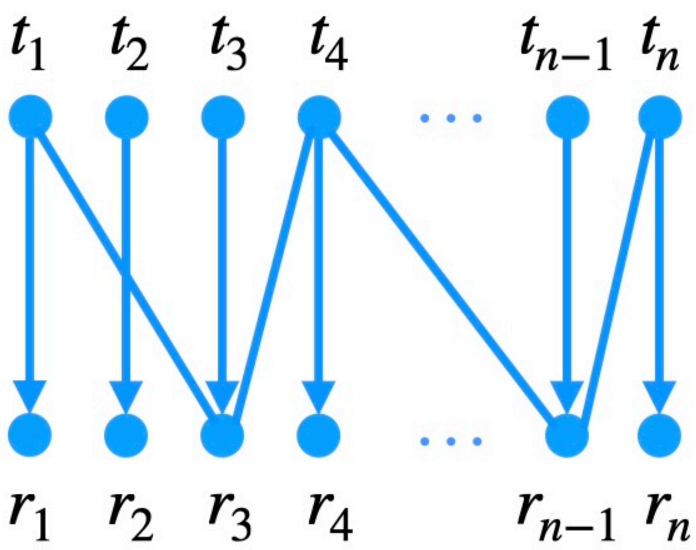

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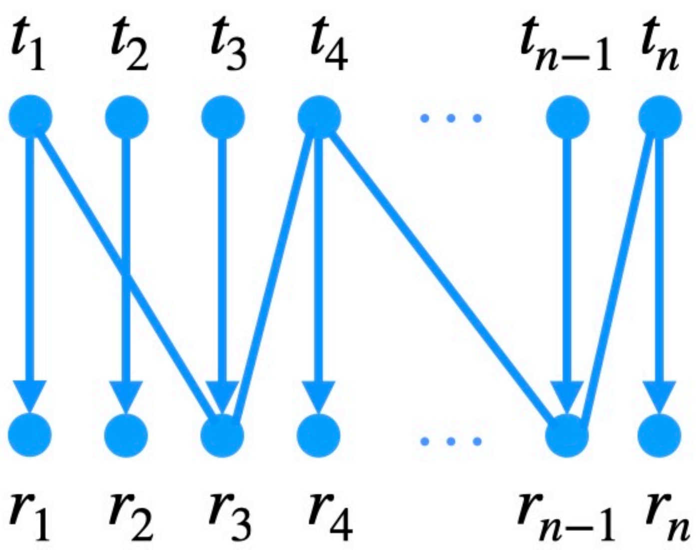

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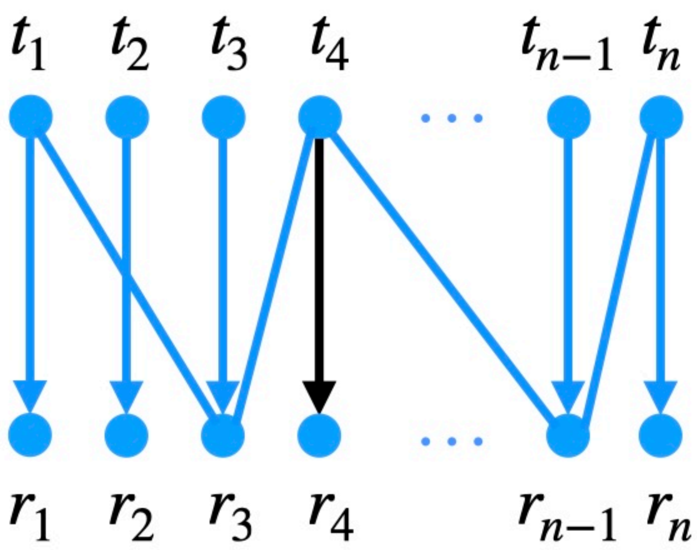

Query $L_j(G_j) = \{t_4\}$. Without additional links, (t_4, r_4) would be realized.

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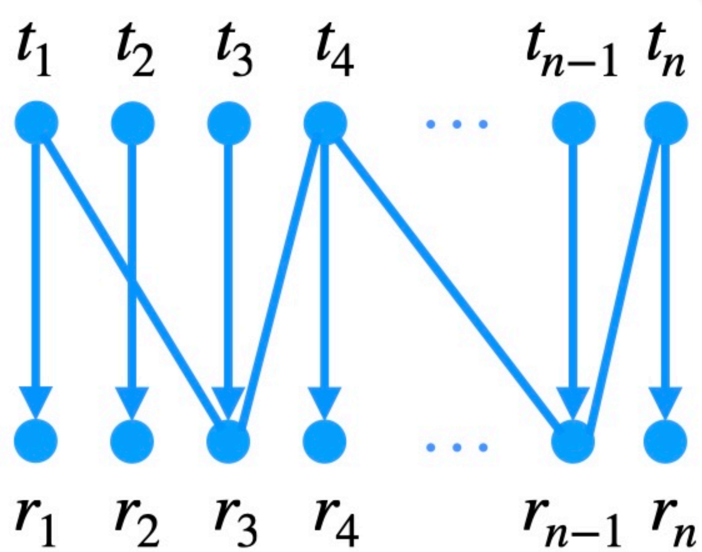

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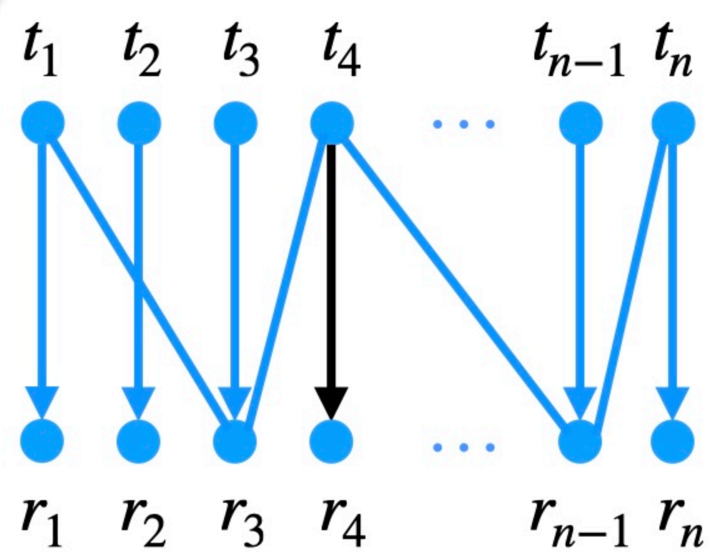

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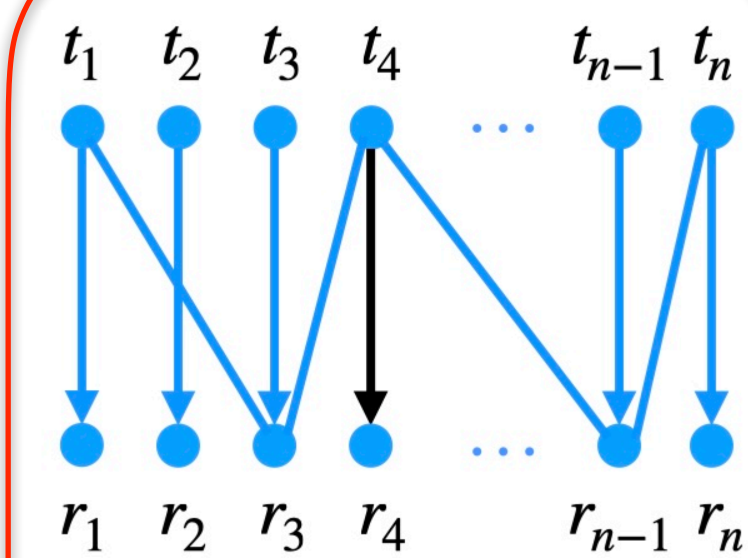

Graph $G_{j+1} = (V, E_{j+1})$ without additional links because it is not possible to interfere at r_4 with $|L_j(G_j)| = 1$.

RN Lower Bound

Proof sketch:

- Δ is an upper bound on A
- adversarial network has max degree Δ
- prove that within the claimed time function (of Δ)
  - low frequency requests in small queries not realized
  - requests in large queries are not realized whp
  - requests in singleton queries plus high frequency requests in small queries are a fraction of total
- applying probabilistic method, we show existence of adversarial network for each protocol

For SINR: similar, time differs by a constant only.

RN and SINR Lower Bounds

Theorem 1. *Consider any deterministic adaptive protocol* $\mathcal{P}$ *that solves* DLS *on a set of* n *requests embedded in a wireless network with maximum average affectance* $\mathcal{A}$ *under the RN model. Let* $\tau = \tau(n, \mathcal{A})$ *be the number of rounds required by* $\mathcal{P}$ *in the worst case. Then, there exists an adversarial network such that* $\tau \in \Omega\left(\min\left\{n, \frac{\mathcal{A}^2}{\log^2 n}\right\}\right)$.

The above holds also for the SINR model of interference with attenuation $\alpha \in \Omega(\log n/(\log\log n - \log\log \mathcal{A}))$ *in a graph metric space.*

<u>For bounded growth</u>: similar ideas, laying out nodes in multidimensional space to limit ϕ and different thresholds.

DWLS Algorithm for RNs

Independent for each request

Based on Selectors [1,2,3]

No knowledge of A or ϕ

Algorithm 1: DLS algorithm for each request (t, r).

```
/* Algorithm for transmitter t                              */
S(k, x) ← a (2n, k, x)-avoiding-selector for any
  k ≤ x ≤ n being powers of 2
for each j = 1, 2, 3, ... do
    for each k = 1, 2, 4, 8, 16, ..., n do
        for each x = k, 2k, 4k, 8k, 16k, ..., n do
            if t ∈ S(k, x)_j then
                transmit request (t, r) in round
                  2j · (1 + log k) · (1 + log(x/k)) − 1
                if acknowledgment is received from r in
                  round 2j · (1 + log k) · (1 + log(x/k))
                  then stop
/* Algorithm for receiver r                                 */
for each j = 1, 2, ... do
    if transmission with a request (t, r), for some t, is
      received in round 2j − 1 then
        transmit acknowledgement to t in round 2j
        stop
```

[1] De Bonis, Gasieniec and Vaccaro. Siam J. Comp. 2005.
[2] Chlebus and Kowalski. FCT 2005.
[3] Indyk. SODA 2002.

Avoiding Selectors

Transmit or not

OBLIVIOUS
Transmission Schedules

Round	1	2	3	4	5	6	7	...
Transmitter 1	0	1	0	1	1	1	0	...
Transmitter 2	1	1	0	0	1	0	1	...
Transmitter 3	0	0	1	0	1	1	1	...
Transmitter 4	0	1	0	1	0	0	0	...
...	...	...	...	...	...	...	...	...
Transmitter n	1	1	0	1	0	1	1	...

For any subset of nodes ...
"selects" some number of
elements while avoiding others.

[1] De Bonis, Gasieniec and Vaccaro. Siam J. Comp. 2005.
[2] Chlebus and Kowalski. FCT 2005.
[3] Indyk. SODA 2002.

Bounded-growth RN Upper Bound

Theorem 2. DLS *is a deterministic distributed algorithm that solves the Link Scheduling problem in* $O(\mathcal{A}\phi^6 \log^4 n)$ *rounds, for any set of requests of maximum average affectance at most* $\mathcal{A}$ *in any Radio Network model with* ϕ*-bounded-growth. This holds even without initial knowledge of the parameters* $\mathcal{A}, \phi$.

Proved showing how the selectors used are carefully combined to eventually realize all requests.

Open Directions

More sophisticated local communication, such as multicast?

Link scheduling with forwarding? (for problems where the order of realizations matter) Global point-to-point routing?

More adversarial environment with jamming (some nodes controlled by adversary could jam in some limited number of rounds)?

More efficient constructions of the used types of selectors?

Thank you!

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