# Sensor Network Gossiping or How to Break the Broadcast Lower Bound

Martín Farach-Colton<sup>1</sup> Miguel A. Mosteiro<sup>1,2</sup>

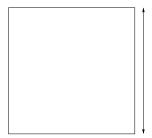
<sup>1</sup>Department of Computer Science Rutgers University

<sup>2</sup>LADyR (Distributed Algorithms and Networks Lab) Universidad Rey Juan Carlos

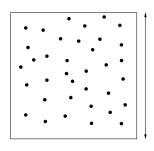
**ISAAC 2007** 



Radio Network = abstraction of a radio communication network



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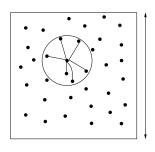
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- $k = 1 \rightarrow Broadcast$  [BGT'92.KM'98]
- k = n:  $\rightarrow Gossiping$  [CGLP'01,LP'02]
- k arbitrary:  $\rightarrow k$ -selection [K'05]

We study

Gossiping in Sensor Networks

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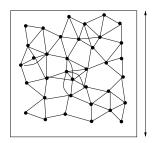
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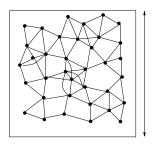
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Gossiping in Sensor Networks

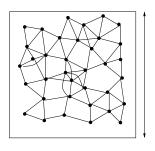
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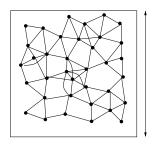
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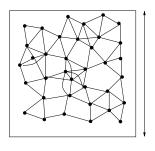
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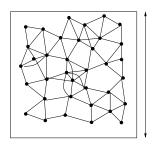
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Sensor Node Capabilities

- processing
- sensing
- communication

- range
- memory
- life cycle

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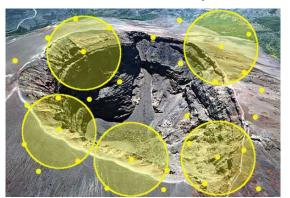
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### Related Work

Upper Bounds

• Symmetric Radio Networks:

```
BII'93 O(n \log^2 n) expected (BFS tree).
CGLP'01 same, w.h.p.
```

- Asymmetric connected Radio Networks:
- CGR'01  $O(n \log^3 n \log(n/\epsilon))$  with prob  $1 \epsilon$  and  $O(n \log^4 n)$  expected (limited broadcast doubles message copies per phase).
  - LP'02 same, reduced by a log factor (limited broadcast is randomized).
  - CR'03  $O(n \log^2 n)$  w.h.p. (linear randomized broadcast by special distribution).
- CGR'00  $O(n^{3/2} \log^2 n)$  (deterministic, selecting sequences).
  - ALL: globally synchronous, and  $\Omega(nm)$  memory size, all but first:  $\Omega(nm)$ message size.
  - Sensor Networks
    - R'07  $O(\sqrt{n} \log n)$  w.h.p. in RGGs (claimed optimal using KM's lower bound, but includes pre-coloring).

## Related Work

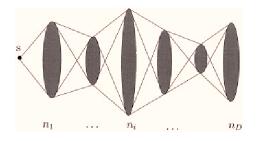
#### • Gossiping:

- CGLP'01 deterministic oblivious (no history):  $\geq n^2/2 n/2 + 1$  fair (same  $p_{trans}$ ) protocols:  $\forall n \leq q \leq n^2/2$ ,  $\exists$  asymmetric network s.t.  $\Omega(q)$  expected.
  - GP'02  $\Omega(n^2)$  asymmetric networks  $\Omega(n \log n)$  symmetric networks not embeddable in GG.
  - Broadcast (no preprocessing):
  - BDP'97  $\Omega(D \log n)$  globally synchronous, nodes know message history.
  - CMS'01  $\Omega(n \log D)$  symmetric networks, nodes are not synchronized.
    - KP'04  $\Omega(n^{1/4})$ , diameter 4.
    - KM'98  $\Omega(D \log(n/D))$  expected (best, more on this...)



# Related Work Broadcast Lower Bound

### [KM'98] proved $\Omega(D\log(n/D))$ expected, showing a layered structure



#### Crucial assumption:

"...any other processor is inactive"

"until receiving a message for the first time."

#### Crucial in proof

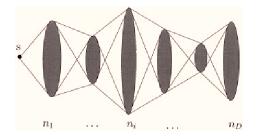
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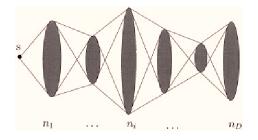
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### Node Constraints Model

Sensor Networks

# THE WEAK SENSOR MODEL [BGI 92, FCFM 05]

- Local Synchronism.
- Adversarial wake-up schedule.
- Low-info channel contention:
  - Radio TX on a shared Channel.
  - NO COLLISION DETECTION.
  - Non-simultaneous RX and TX.

- Constant memory size.
- Limited life cycle.
- SHORT TRANSMISSION RANGE.
- Discrete TX power range.
- One channel of communication.
- No position information.
- Unreliability.

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### Our Results

#### Sensor Network:

- $\bullet$  n nodes
- ullet range of transmission r
- diameter D
- ullet max degree  $\Delta$
- nodes only know n.
- $\bullet$  all nodes hold message of size m to disseminate.
- $\bullet$  O(nm) message and memory size.

### Gossiping algorithm:

- $O(\Delta + D)$  w.h.p. relaxed-WSM-compatible
- $\Omega(D)$  and  $\Omega(\Delta)$  are lower bounds  $\Rightarrow$  optimal.

#### Observations:

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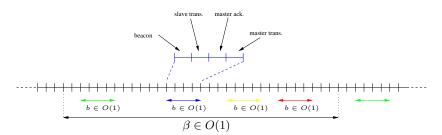
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- Partition nodes in masters and slaves
  - every slave is at  $d \leq ar$  from some master (0 < a < 1/3)
  - every pair of masters are at d > ar
    - $\rightarrow MIS(ar)$
- Every master reserves blocks of time steps for local use
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    - $\rightarrow$  window back-on/back-off +  $O(\log^2 n)$  times  $p_{trans} = 1/\log n$
- Every master disseminates local set (using reserved blocks)
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     masters add messages received from other masters to local set
     → flooding among masters



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### Assume phase synchronism

- ② Partition nodes in masters and slaves  $\Rightarrow$  MIS  $\rightarrow O(\log^2 n)$
- ② Every master reserves blocks of time steps for local use  $\Rightarrow$  Coloring  $\rightarrow O(\log n)$
- ② Every master maintains set of messages received ⇒ window back-on/back-off →  $O(\Delta + \log^2 n \log \Delta)$
- Every master disseminates local set  $\Rightarrow$  flooding among masters  $\rightarrow O(D)$

#### Overall

$$O(\log^2 n + \log n + \Delta + \log^2 n \log \Delta + D) \in O(\Delta + D)$$



Time efficiency

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