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Deterministic Local Problems in Radio Networks: On the Impact of Local Domination and a Bit of Advice

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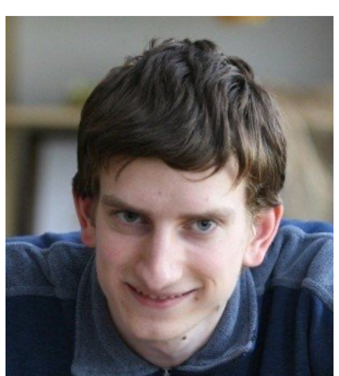

Paweł Garncarek
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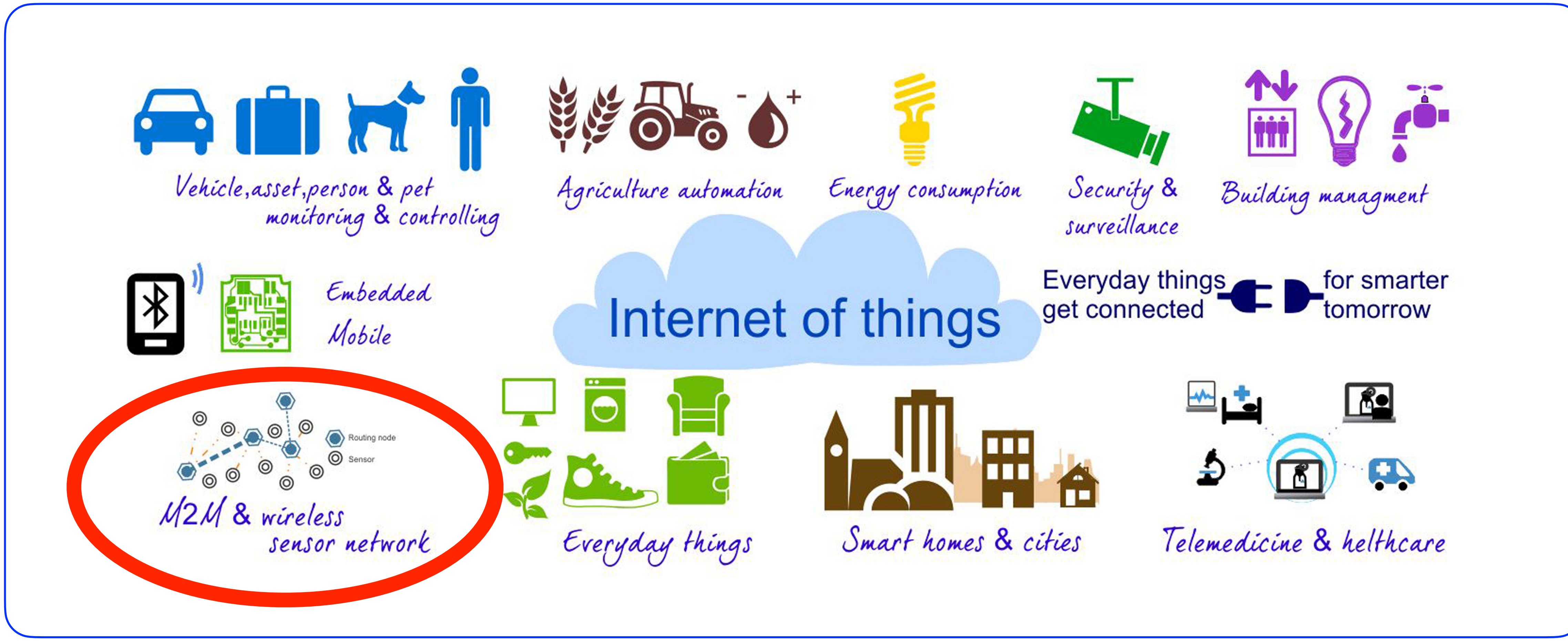
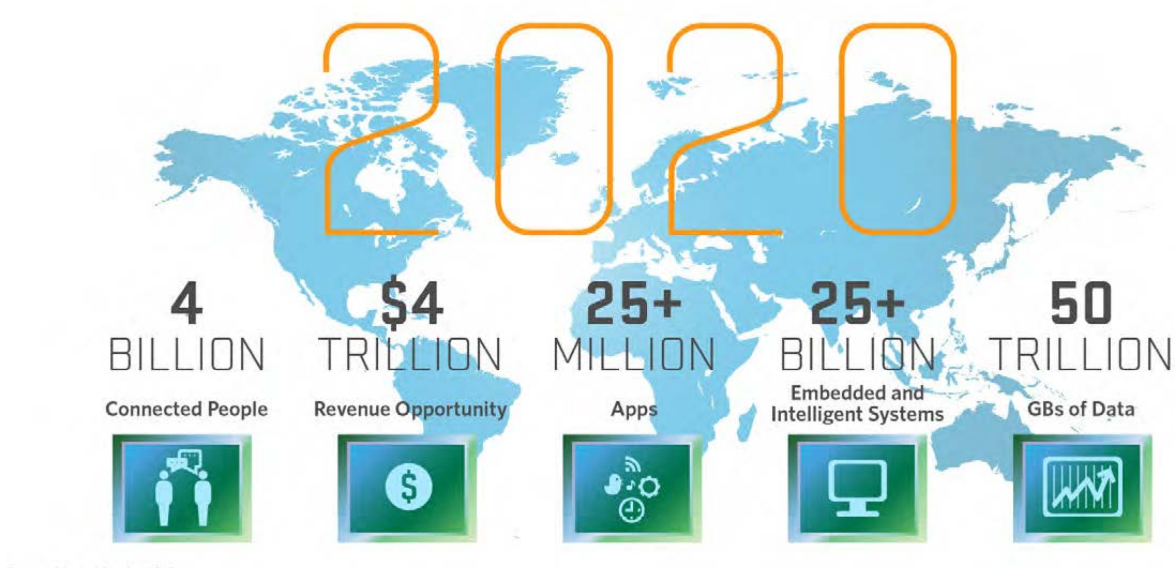
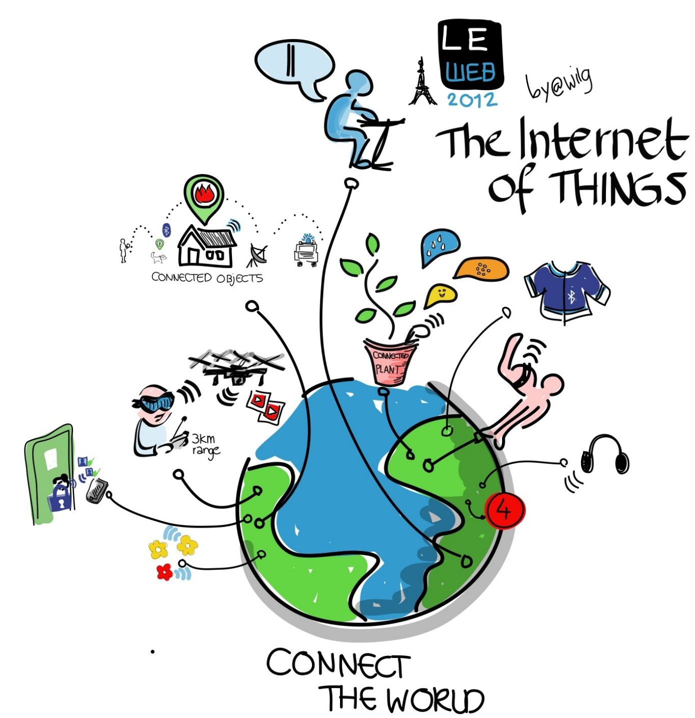
Augusta University

Shay Kutten
Technion

Ad-hoc Wireless Networks

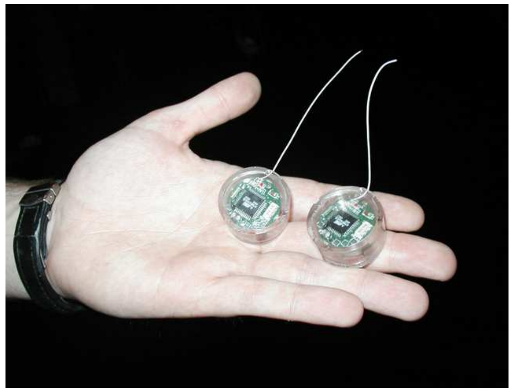






Ad-hoc Wireless Networks

Example: A Sensor Network



Intel Berkeley Research Lab

Capabilities

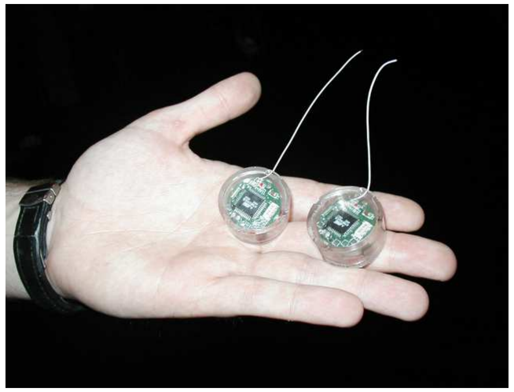
- processing
- sensing
- communication

Limitations

- range
- memory
- life cycle

Ad-hoc Wireless Networks

Example: A Sensor Network



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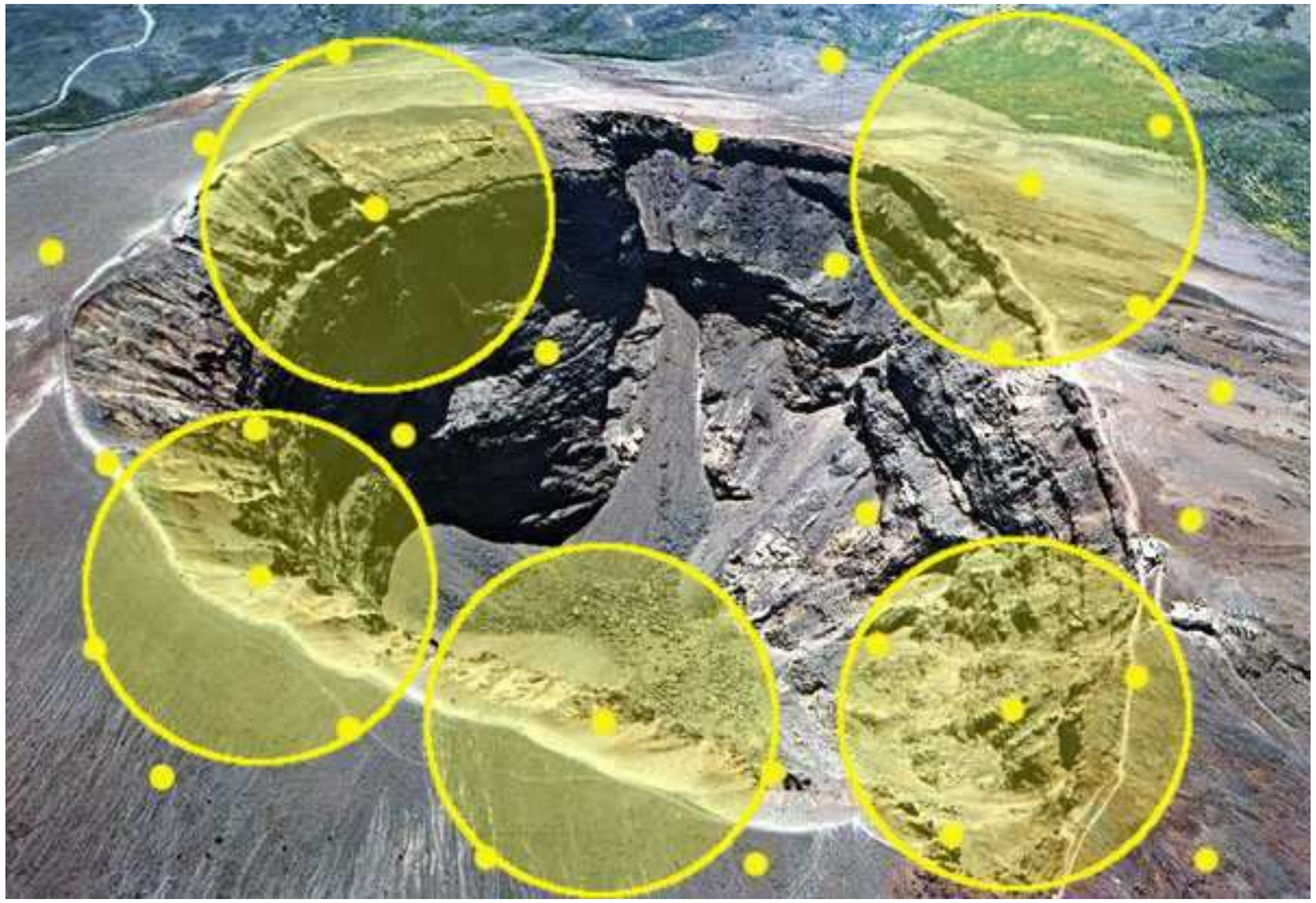
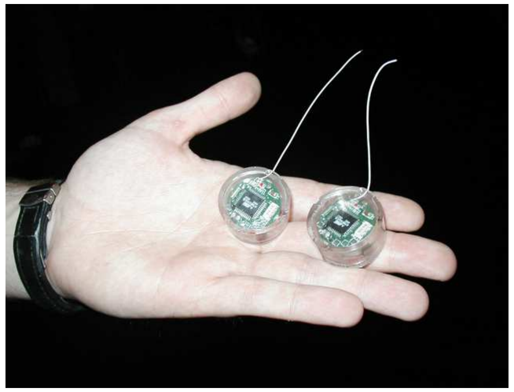


Ad-hoc Wireless Networks

Example: A Sensor Network



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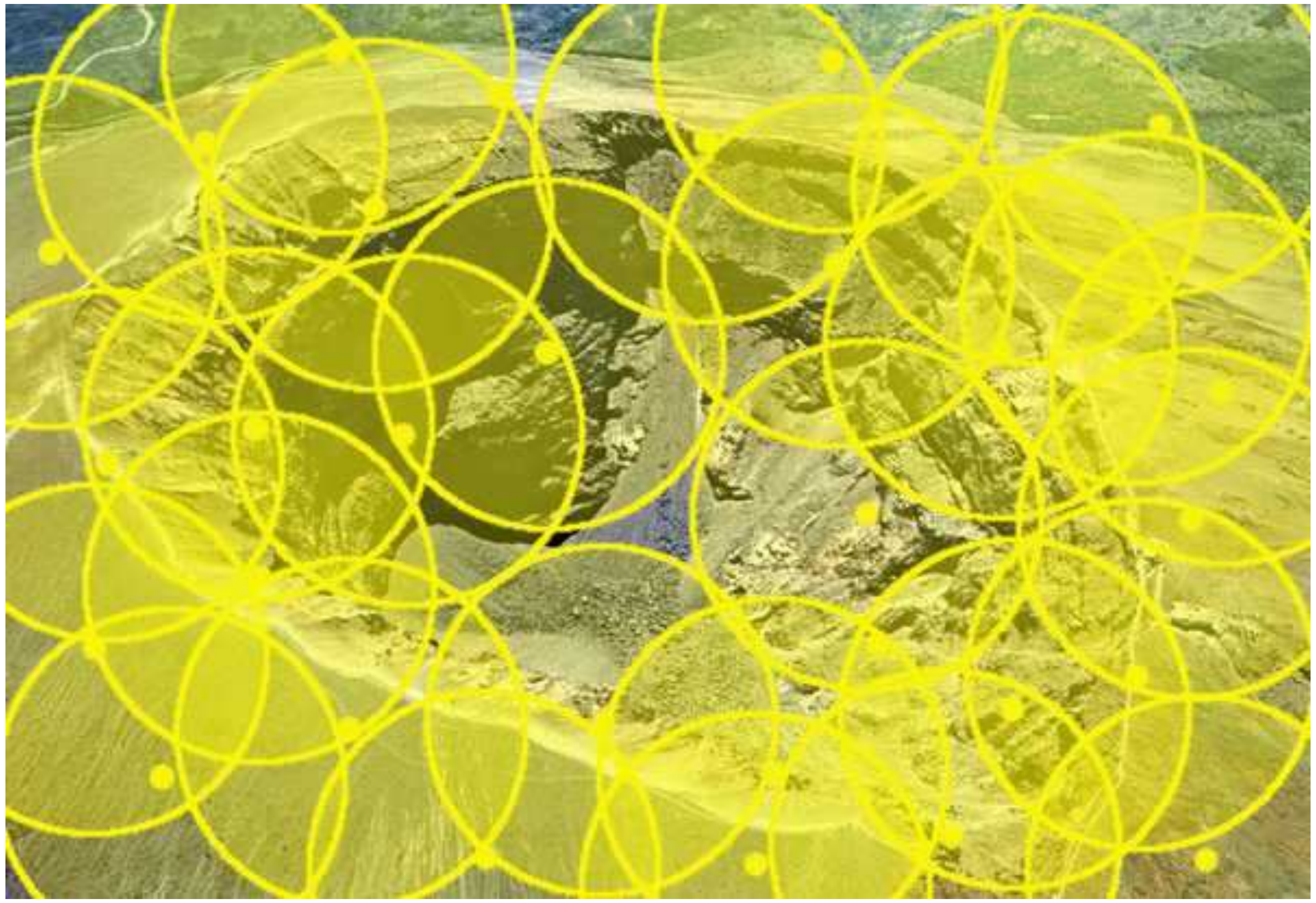
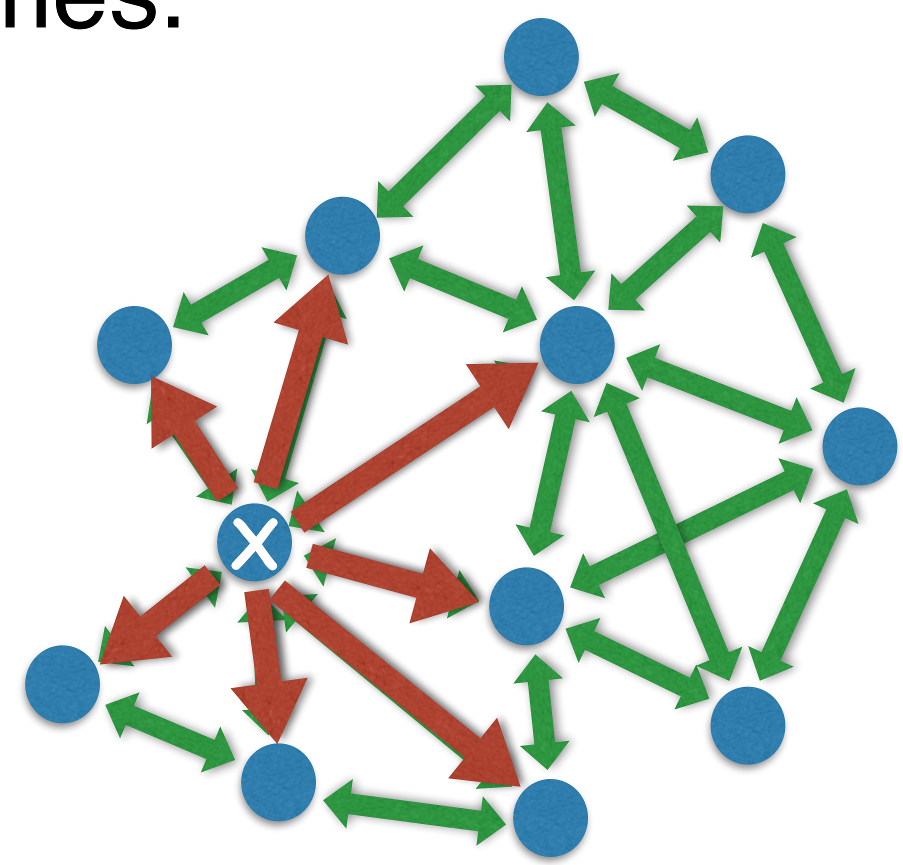


Local Broadcast Problem

Definition:

Every node that holds a message has to deliver it to all its neighbors, possibly at different times.



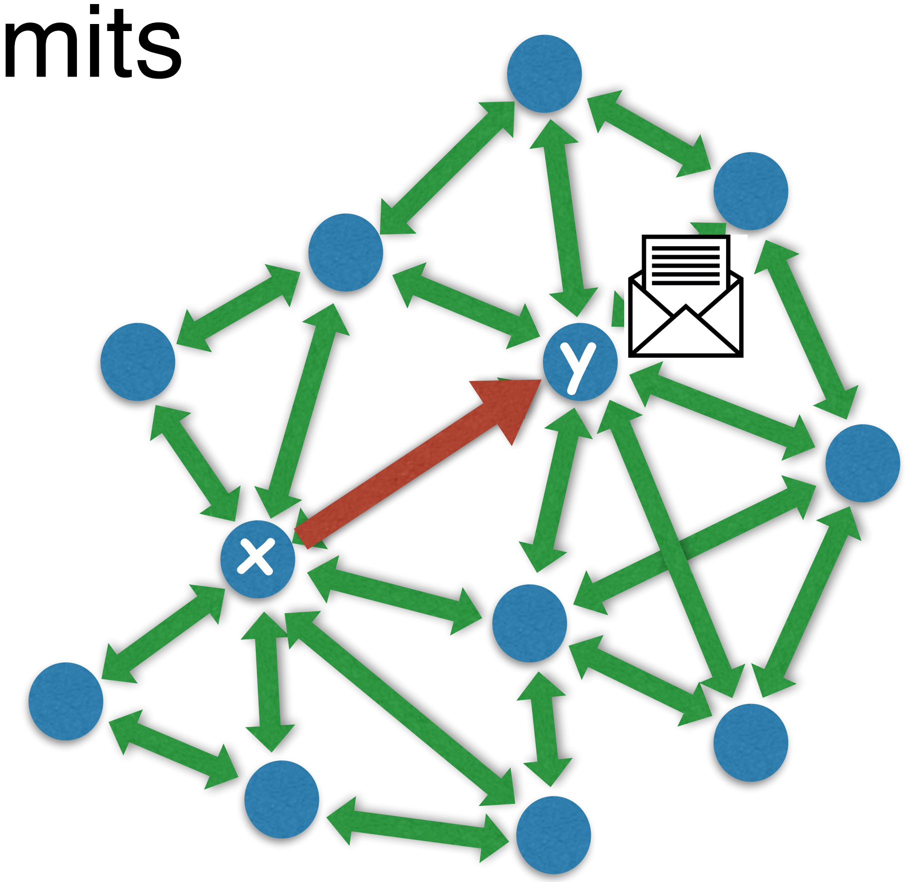
Fundamental primitive for more complex communication problems

Radio Network Interference Model

Successful transmission:

y receives from x if

- no other neighbor of y transmits
- y is not transmitting



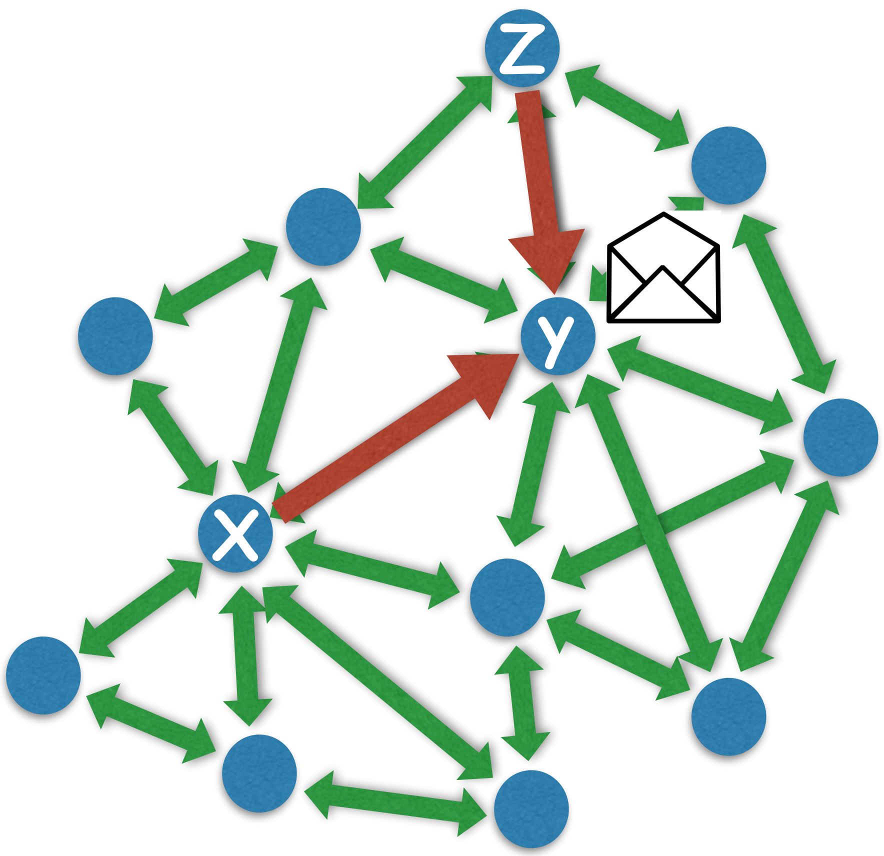
[1] Chlamtac and Kutten. *Trans. on Computers*. IEEE, 1987.

Radio Network Interference Model

Collision:

If x and z transmit at the same time

y receives nothing



We need algorithms to schedule the transmissions!

Other definitions

- Time is slotted in rounds of communication
- Δ: $\max\limits_{V}$ # of neighbors
- γ_2: $\min\limits_{\mathcal{S}}$ ($\max\limits_{V}$ # of dominators within 2 hops)
- Nodes do not know topology, only Δ, γ_2 and n
- Deterministic algorithms (no coin tossing)
- Distributed algorithms

We measure performance in rounds,
as a function of Δ, γ_2 and n.

Protocols Studied

- Oblivious …
- Quasi-adaptive …
- Adaptive …

Transmit or not

Round	1	2	3	4	5	6	7	…
v_1	0	1	0	1	1	1	0	…
v_2	1	1	0	0	1	0	1	…
v_3	0	0	1	0	1	1	1	…
v_4	0	1	0	1	0	0	0	…
…	…	…	…	…	…	…	…	…
v_n	1	1	0	1	0	1	1	…

Protocols with Advice

Nodes do not know the topology

What if they have some “advice”?
(know something but only local)

We present 3 algorithms trading adaptiveness for bits of advice.

All 3 algorithms run in $\tilde{O}(\Delta\gamma_2^2)$ rounds.

Plato and Aristotle knew better.

Oblivious Protocol with $O(\log(\Delta\gamma_2))$ Bits of Advice

Uses combinatorial structure called **Strong Selectors**

Transmit or not

Round	1	2	3	4	5	6	7	…
v_1	0	1	0	1	1	1	0	…
v_2	1	1	0	0	1	0	1	…
v_3	0	0	1	0	1	1	1	…
v_4	0	1	0	1	0	0	0	…
…	…	…	…	…	…	…	…	…
v_n	1	1	0	1	0	1	1	…

Can be seen as OBLIVIOUS Transmission Schedules

For any subset $S \subseteq V$, for every $v \in S$, exists some round that "selects" v.

[1] Erdos, Frankl, and Furedi. Israel J. Math, 1985.
[2] Kautz and Singleton. Trans. Inf. Thy., 1964.

Oblivious Protocol with $O(\log(\Delta\gamma_2))$ Bits of Advice

- Consider a dominating set DS that minimizes γ_2 ,
- a 4-distance coloring of DS of $O(\gamma_2^2)$ colors, and
- a labeling of dominated nodes wrt their dominators

The color corresponds to phases, and each phase to one selector. The color of w=dominator(v) indicates that this is the phase when v is active and should transmit.

Algorithm 1 LB_LAd– Local Broadcast algorithm with $O(\log(\Delta\gamma_2))$ Advice for a node v

Let $\mathcal{F} = \{S_1, \ldots, S_\ell\}$ be the fixed (shared between all nodes) (γ_2^2, γ_2)-strong selector, where $\ell = |\mathcal{F}|$

Receive (from oracle) color c of DS node w, in some distance 4 coloring, that v will be assigned to

Receive from the oracle the number x assigned to v

for $phase = 1, \ldots, \ell$ **do**

  for $round = 1, \ldots, \Delta + 1$ **do**

    if $w \in S_{phase}$ and $round = x$ **then**

      Transmit the message, ID of v and the color of w

    else

      Remain silent

    end if

  end for

end for

Quasi-adaptive Protocol with $O(\log(\gamma_2))$ Bits of Advice

Uses combinatorial structure called **Avoiding Selectors**

Round	1	2	3	4	5	6	7	...
v_1	0	1	0	1	1	1	0	...
v_2	1	1	0	0	1	0	1	...
v_3	0	0	1	0	1	1	1	...
v_4	0	1	0	1	0	0	0	...
...	...	...	...	...	...	...	...	...
v_n	1	1	0	1	0	1	1	...

Nodes can stop after local task is completed.

For any subset of nodes ... "selects" some elements while avoiding others.

[1] De Bonis, Gasieniec and Vaccaro. Siam J. Comp. 2005.
[2] Chlebus and Kowalski. FCT 2005.
[3] Indyk. SODA 2002.

Quasi-adaptive Protocol with $O(\log(\gamma_2))$ Bits of Advice

- Consider a dominating set DS that minimizes γ_2 , and
- a 4-distance coloring of DS of $O(\gamma_2^2)$ colors (no labeling).

Algorithm 2 LB_quasi – Local Broadcast algorithm with $O(\log \gamma_2)$ Advice for a node v

Let $\mathcal{F}_i = \{S_1^{(i)}, \ldots, S_{\ell_i}^{(i)}\}$ be a fixed (shared between all nodes) $(n, \Delta/2^{i-1}, \Delta/2^i)$-avoiding selector, where $\ell_i = |\mathcal{F}_i|$, for $i = 0, 1, \ldots, \log \Delta$.
Receive from the oracle the color $\text{col}(v) = c$ of DS node w such that v is assigned to w
for $phase = 1, \ldots, \log \Delta$ **do**
    for $stage = 1, \ldots, \ell_{phase}$ **do**
        for $block = 1, \ldots, \gamma_2^2 + 1$ **do**
            Round 1:
            if $v \in S_{stage}^{(phase)}$ and $\text{col}(v) = block$ and $v \notin DS$ **then**
                Transmit the message and the ID of v
            else
                Remain silent
            end if
            Round 2:
            if $\text{col}(v) = block$ and $v \in DS$ **then**
                if v received a message in the previous round **then**
                    send the received message back
                else
                    Send a dummy message
                end if
            end if
            if v sent a message in Round 1 and received it back in Round 2 **then**
                v switches off
            end if
        end for
    end for
end for

Thanks to avoiding selectors, no need for round robin (no labeling)

Thanks to acknowledgments, nodes switch off after all neighbors received.

Adaptive Protocol with 1 Bit of Advice

- Consider a dominating set DS that minimizes γ_2 (no labeling, no coloring).

Preprocessing for node v:

- receive 1 bit of advice indicating whether v belongs to DS or not
- compute assignment of dominators
- compute labeling (constant adaptivity)

Algorithm 5 Local broadcast after assignment of DS nodes and numbering – Algorithm for node v

Let $\mathcal{F} = \{S_1, \ldots, S_\ell\}$ be the fixed shared (n, γ_2)-strong selector, where $\ell = |\mathcal{F}|$
for $i = 1, \ldots, \ell$ **do**
    for $j = 1, \ldots, \Delta + 1$ **do**
        $DS_v \leftarrow$ the DS node assigned to v
        if $DS_v \in \mathcal{F}_i$ and the local number of v is equal to j **then**
            v transmits
        end if
    end for
end for

Lower Bounds?

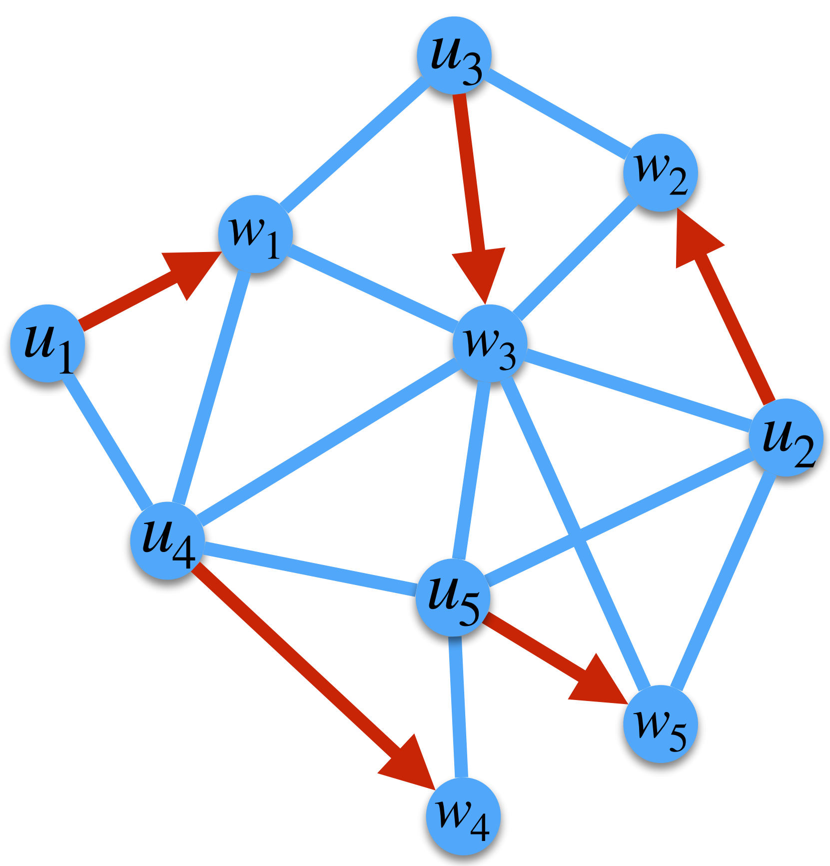
Lower Bound

The Directed-matching Hitting Game

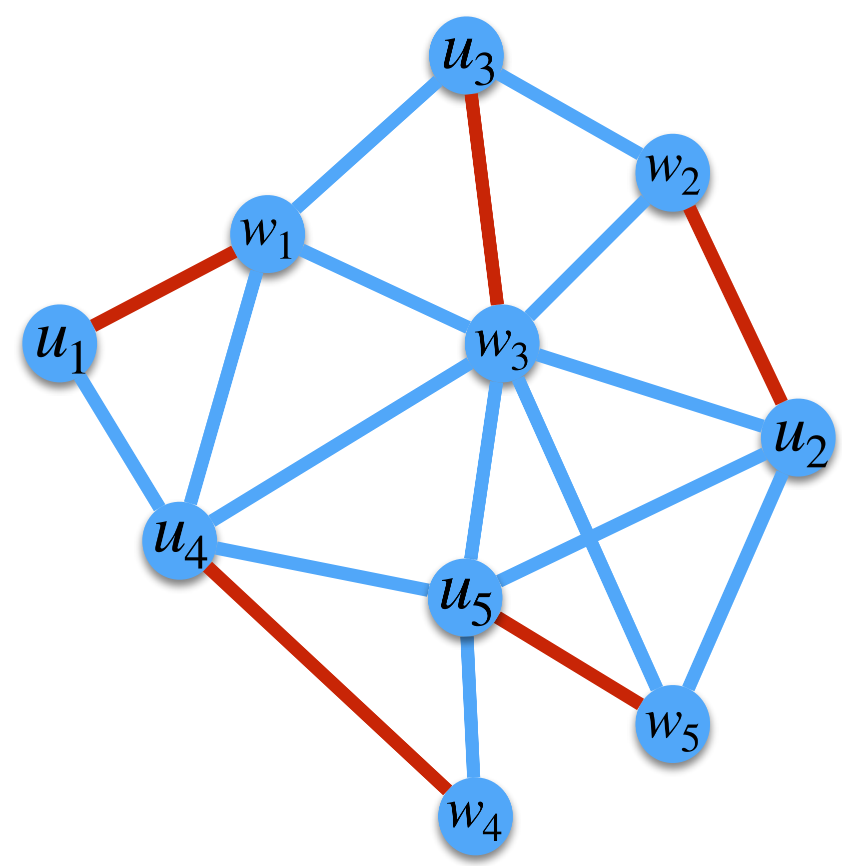
Technical tool to prove our lower bound.

Definitions:

- A set of ordered pairs $M = \{(u_1, w_1), (u_2, w_2), \dots\}$ is a *directed matching* on $G = (V, E)$ if the set of edges $M' = \{\{u_1, w_1\}, \{u_2, w_2\}, \dots\}$ is a matching on G.



$M = \{(u_1, w_1), \dots, (u_5, w_5)\}$



$M' = \{\{u_1, w_1\}, \dots, \{u_5, w_5\}\}$

Lower Bound

The Directed-matching Hitting Game

Technical tool to prove our lower bound.

Definitions:

- Any subset of nodes $\{u_i \,|\, (u_i, w_i) \in M\}$ is called a *query*.
- A sequence of queries $\langle Q_1, Q_2, \ldots \rangle$ is called a *protocol*.

Indicates $u_1 \in Q_6$

Round	1	2	3	4	5	6	7	...
u_1	0	1	0	1	1	1	0	…
u_2	1	1	0	0	1	0	1	…
u_3	0	0	1	0	1	1	1	…
u_4	0	1	0	1	0	0	0	…
...	…	…	…	…	…	…	…	…
$u_{n/2}$	1	1	0	1	0	1	1	…

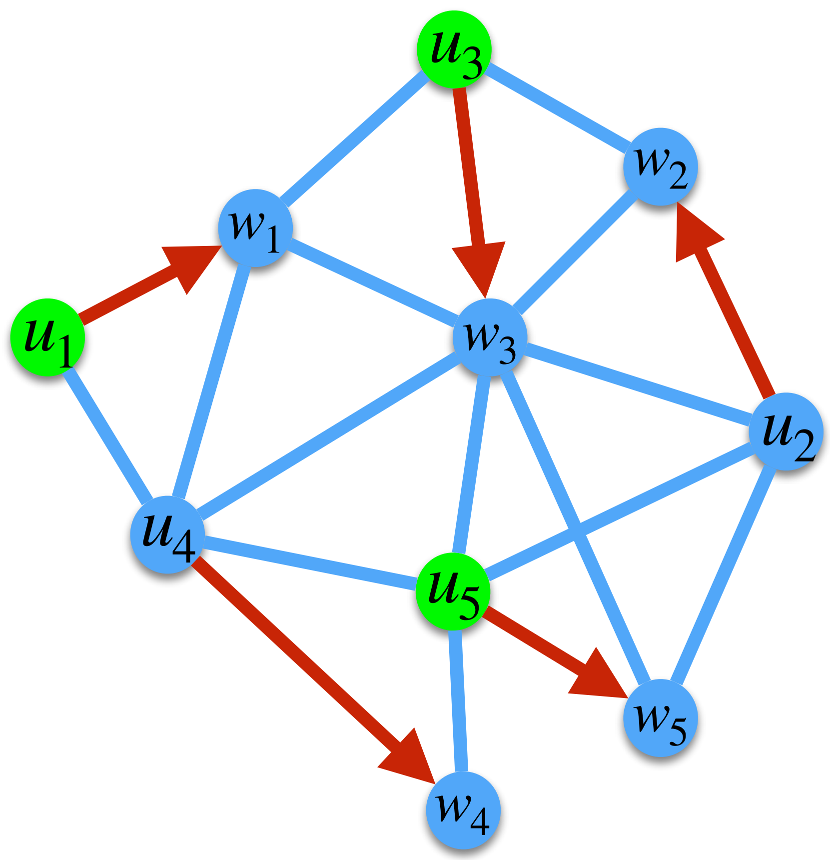
Lower Bound

The Directed-matching Hitting Game

Technical tool to prove our lower bound.

Definitions:

- A *query* $Q \subseteq V$ *hits the ordered pair* $(u, w) \in M$ iff $u \in Q$ and for all other $v \in Q$ it is $\{v, w\} \notin E$.



$Q = \{u_1, u_3, u_5\}$ hits (u_5, w_5)

Same as successful transmission in RNs!

Lower Bound

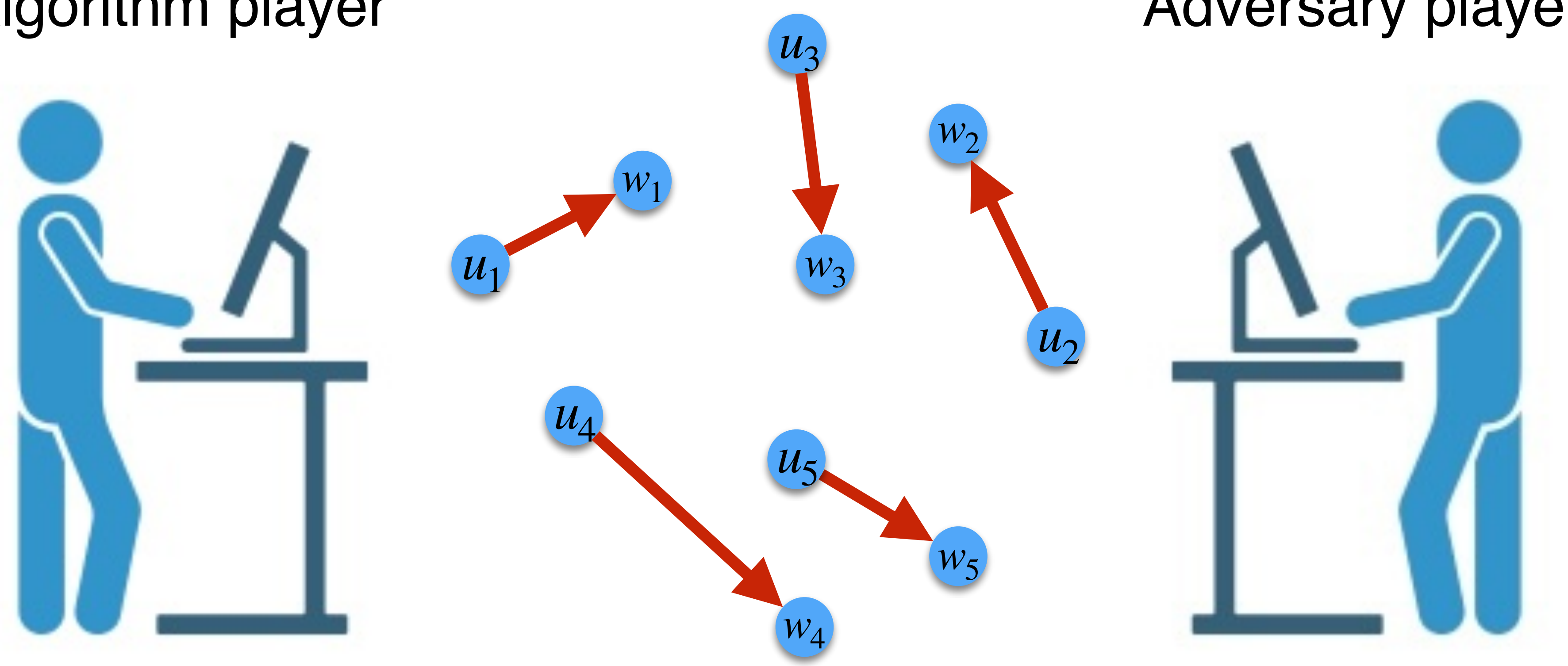
<u>The Directed-matching Hitting Game</u>

Parameters: number of nodes n, max degree Δ, and 2-local domination γ_2.

Initialization

Algorithm player

Adversary player



Chooses a protocol (sequence of queries) $\langle Q_1, Q_2, \ldots \rangle$

<u>Objective</u>: minimize rounds to hit all pairs.

Chooses directed matching $M = \{(u_1, w_1), (u_2, w_2), \ldots, (u_{n/2}, w_{n/2})\}$

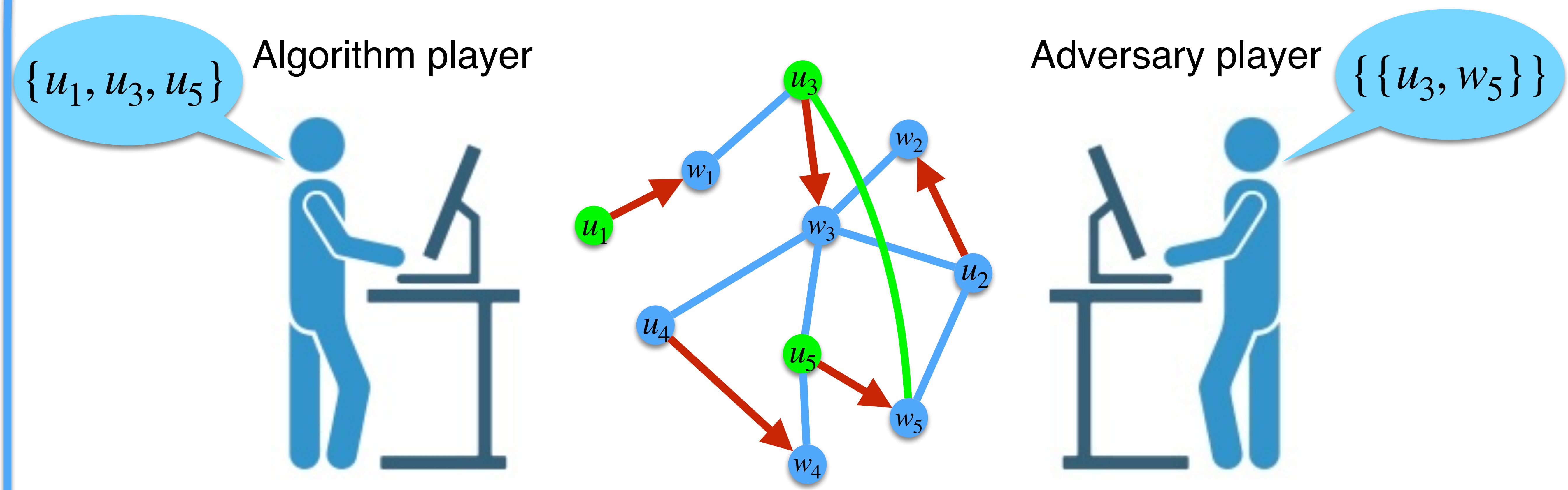
<u>Objective</u>: maximize rounds to hit all pairs.

Lower Bound

The Directed-matching Hitting Game

Parameters: number of nodes n, max degree Δ, and 2-local domination γ_2.

For each round r



Announces query $Q_r \backslash H_r$ where
$H_r = \{u_i \in Q_r \mid (u_i, w_i)$ hit before$\}$

Adds to the graph edges (u_i, w_j)
such that $u_i \notin H_r$, restricted to Δ and γ_2.

Lower Bound

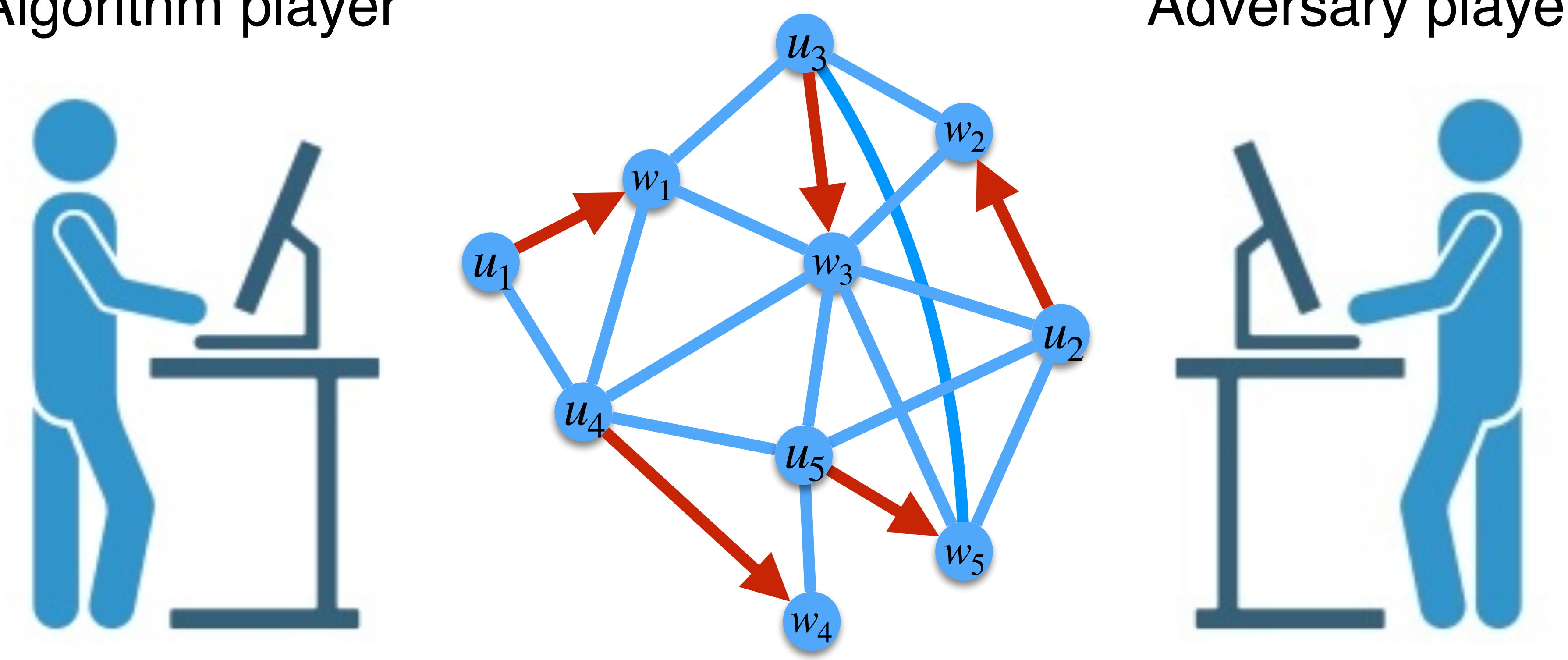
<u>The Directed-matching Hitting Game</u>

Parameters: number of nodes n, max degree Δ, and 2-local domination γ_2.

Ending conditions

Algorithm player

Adversary player



Game ends at round τ if every edge of the directed matching M has been hit by queries $\langle Q_1, Q_2, \ldots, Q_\tau \rangle$.

Lower Bound

The Directed-matching Hitting Game

▶ **Theorem 16.** *Consider a directed-matching hitting game with parameters $\gamma_2 > 1$, $\Delta > 1$ and $n \geq 3$. For each algorithm player $\mathcal{P}_A$, there exists an adversary player $\mathcal{P}_B$ such that the number of rounds τ needed to finish the game is* $\tau \in \Omega\left(\min\left\{\left(\frac{\min\{\Delta,\gamma_2\}}{\log n}\right)^2, n\right\}\right)$.

Proof sketch: we show

- an adversary strategy that prevents some hits,
- that such strategy fulfills the restrictions of the game,
- that within the claimed time bound only a fraction of the matching is hit,

all with positive probability ⇒ by the probabilistic method the claim follows.

Let's look at the adversary strategy…

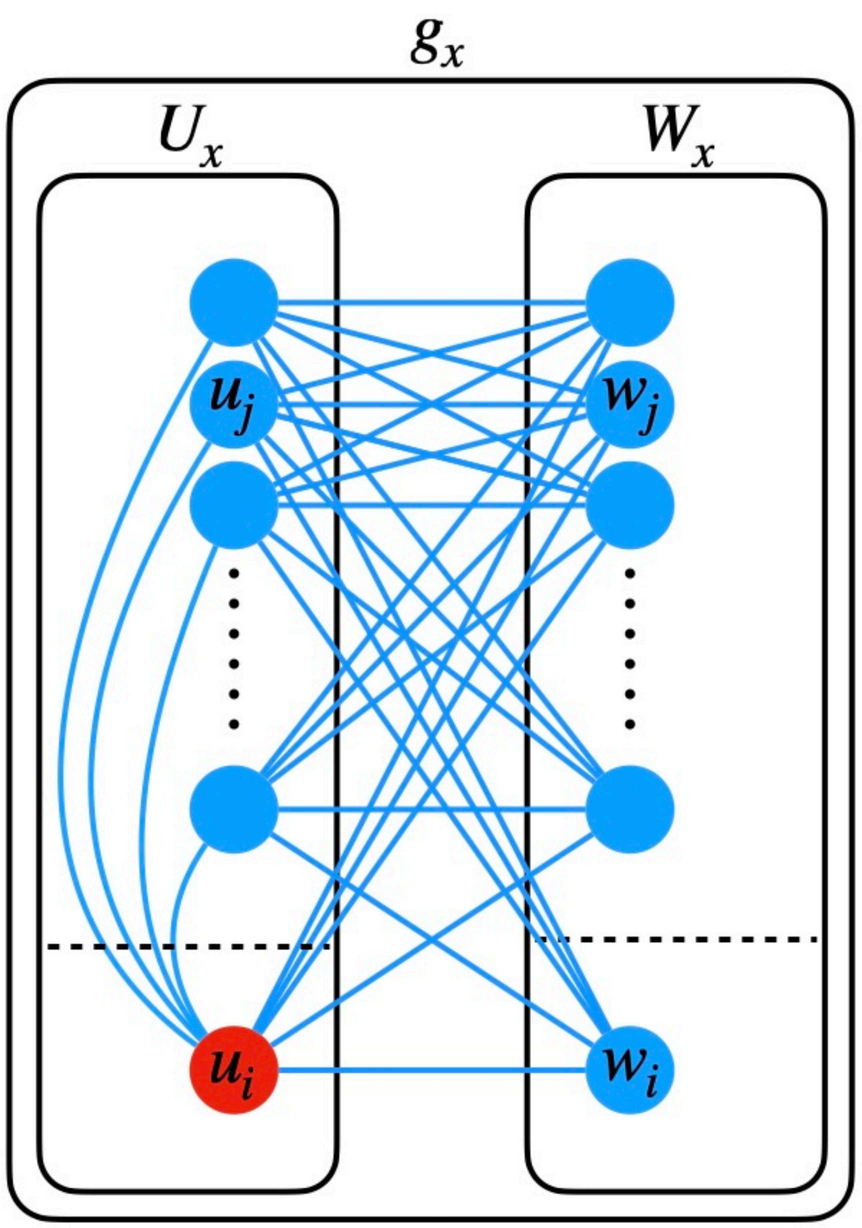
Lower Bound

The Directed-matching Hitting Game

Adversary strategy:

- Initially:
  - partition at random the matching in subsets called “gadgets”.
  - add edges to each gadget to make it bipartite complete.
  - add edges to each gadget so that a chosen node dominates the gadget.

These edges will make large queries of the algorithm ineffective for hitting because, being the query large, the likelihood of having a hit will be low.



Lower Bound

The Directed-matching Hitting Game

Adversary strategy:

- For each round:

query size →

- singleton? : do nothing, impossible to block hit.
- "small"? : "frequency"
  - "low" : add edges
  - "high" : allow hit (hard to block with limited Δ)
- "large"? : initial gadget edges block hits whp

small / large : $\lessgtr \sigma = 1 + \dfrac{c_1 n \log n}{\min\{\Delta/2, \gamma_2 - 1\}}$

frequency : # of occurrences in small queries so far.

low / high : $\lessgtr \min\{\Delta/2, \gamma_2 - 1\}$

Lower Bound

▶ **Theorem 17.** *Consider any deterministic quasi-adaptive protocol $\mathcal{P}$ that solves Local Broadcast in the RN model. Let τ be the number of rounds needed by $\mathcal{P}$ in the worst case. Then, for each $\mathcal{P}$, there exists an adversarial input network with set V of n nodes, 2-local domination number at most γ_2, and maximum degree Δ such that* $\tau \in \Omega\left(\min\left\{\left(\frac{\min\{\Delta,\gamma_2\}}{\log n}\right)^2, n\right\}\right)$.

Proof sketch:

- For contradiction we assume that there exists such protocol P that completes Local Broadcast in less time.
- We show that then we can use P as the algorithm player in the Hitting Game, and use the network built by the adversary as input of Local Broadcast reaching a contradiction with Theorem 16.

Conclusion

Section # or reference	Communication rounds	Protocol class	Bits of advice
[8]:Thm 13	$O(\Delta^2 \min\{\log n, \log_\Delta^2 n\})$	oblivious, $\gamma_2 \geq 1$	none
[12]:Thm 1.6	$\Omega(\min\{\Delta^2 \log_\Delta n, n\})$	oblivious, $\gamma_2 \geq 1$	none
3.2	$O(\Delta\gamma_2^2 \log \gamma_2)$	oblivious	$O(\log(\Delta\gamma_2))$
3.3	$O(\Delta\gamma_2^2 \log n)$	quasi-adaptive	$O(\log \gamma_2)$
4.2	$\Omega\left(\min\left\{\left(\frac{\min\{\Delta,\gamma_2\}}{\log n}\right)^2, n\right\}\right)$	quasi-adaptive	none
3.4	$O(\Delta\gamma_2^2 \log^2 n)$	adaptive	1
[14]:Cor 11	$\Omega(\Delta \log n)$	any, $\gamma_2 \geq 1$	none

Open Directions

What is the inherent complexity of other local tasks?

Which global tasks can be performed without using Local Broadcast as a primitive, and thus, possibly have even a lower complexity?

Can one derive better algorithms for the case that only some $x < n$ nodes wish to locally broadcast? Must x be known to help?

We provided three points in the trade-off between adaptivity and advice. What is the complete trade-off?

Thank you!

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