

# Designing Mechanisms for Reliable Internet-based Computing

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# Motivation <sup>(1)</sup>

- Demand for processing complex computational jobs
  - One-processor machines have limited computational resources
  - Powerful parallel machines are expensive
- Internet is emerging as an alternative platform for HPC
  - @home projects (e.g., SETI [Korpela Werthimer Anderson Cobb Lebofsky 01] )
  - Volunteer computing
  - CPU scavenging
  - Convergence of P2P and Grid computing [Foster Iamnitchi 03]

## Internet-based Computing or P2P Computing – P2PC

# Motivation <sup>(2)</sup>

- Internet-based Computing
  - A machine acts as a server: **Master**
  - Distributes jobs, across the Internet, to client computers: **Workers**
  - Workers execute and report back the results
- Great potential but
  - Limited use due to **cheaters** [Kahney 01]
  - Cheater fabricates a bogus result and return it
- Possible solution: Redundant task allocation [Anderson 04, Fernandez et al 06, Konwar et al 06]
  - Master assigns same task to several workers and
  - Compares their returned results (voting)

# Our Solution/Approach

- Consider Internet-based Computing from a **game-theoretic** point of view: Model computations as games
  - Master chooses whether to **verify** the returned result
  - Worker chooses whether to be **honest** or a **cheater**
- Design cost-sensitive mechanisms that **incentive** the workers to be honest
- Objective
  - **Maximize** the probability of the master for obtaining the correct result **and**
  - **Maximize** master benefit

# Background <sup>(1)</sup>

A game consists of a set of players, a set of strategies available to those players, and a specification of payoffs (utilities) for each combination of strategies [\[wikipedia\]](#)

- Game Theory in Distributed Computing [\[Halpern 08\]](#)
  - Internet routing [\[Koutsoupias Papadimitriou 99, Mavronicolas Spirakis 01\]](#)
  - Resource location and sharing [\[Halldorsson Halpern Li Mirrokni 04\]](#)
  - Containment of Viruses spreading [\[Moscibroda Schmid Wattenhofer 06\]](#)
  - Secret sharing [\[Halpern Teague 04\]](#)

# Background <sup>(2)</sup>

- Traditional Distributed Computing
  - A priori behavior of processors: either good or bad
- Game Theory:
  - Processors (players) act on their **self-interest**
  - Rational [Golle Mironov 01] : seek to increase their utility
  - Protocol is given as a game, and the objective is to identify the *Nash equilibria* [Nash 50]  
  
NE: players don't increase their expected utility by choosing a different strategy, if other players don't change

# Background <sup>(3)</sup>

- Algorithmic Mechanism Design [Nisan Ronen 01]
  - Games are designed to provide necessary incentives s.t. players act “correctly”
    - Behave well: Reward
    - Otherwise: Penalize
  - The design objective is to **force** a desired behavior (unique NE)
- Close connection between [Shneidman Parkes 03]
  - Rational players in Algorithmic Mechanism Design
  - Workers in realistic P2P (P2PC) systems

# Framework <sup>(1)</sup>

## ○ Master

- ❑ Assigns a task to workers and collects responses
- ❑ Can verify (audit) the values returned by the workers
  - Verification is cheaper than computing
  - The correct result might not be obtained

## ○ Workers

- ❑ Rational: seek to maximize their benefit
- ❑ Honest: Computes the task and returns correct value
- ❑ Cheater: Fabricates and returns a bogus result



# Framework <sup>(2)</sup>

- We do not consider non-intentional errors produced by hardware or software problems
- **Weak collusion (no Sybil attacks)**: Workers decide independently, but all cheaters collude in returning the same incorrect answer
- The probability of guessing the correct value (without computing the task) is negligible

# General Protocol

- Master assigns a task to  $n$  workers
- Worker  $i$  cheats with probability  $p_C^{(i)}$
- Master verifies the responses with probability  $p_V$
- If master verifies
  - rewards honest workers and
  - penalizes the cheaters
- If master does not verify
  - Accepts value returned by majority of workers
  - Rewards majority ( $R_m$ ), none ( $R_0$ ) or all ( $R_a$ )
  - Does not penalize anyone (“In dubio pro reo”)

# Contributions <sup>(1)</sup>

- Identify a collection of **realistic** payoff parameters

$WP_C$	worker's punishment for being caught cheating
$WC_T$	worker's cost for computing the task
$WB_A$	worker's benefit from master's acceptance
$MP_W$	master's punishment for accepting a wrong answer
$MC_A$	master's cost for accepting the worker's answer
$MC_V$	master's cost for verifying worker's answers
$MB_R$	master's benefit from accepting the right answer

Note that it is possible that

$$WB_A \neq MC_A$$

# Contributions <sup>(2)</sup>

- Define the following games:
  - Between the master and a worker  $1:1$
  - $n$  games between the master and a worker  $1:1^n$
  - Between  $n$  workers (master out of the game)  $0:n$
  - Between the master and  $n$  workers  $1:n$

With the 3 reward models, we have 12 games in total!!
- Analysis of the 12 games under general payoff models
  - Characterize the conditions for a unique NE
  - Mechanisms that the master can run to trade cost and reliability

# Contributions <sup>(3)</sup>

- Design mechanism for two specific realistic system scenarios
  - A system of volunteering computing like SETI
    - Best is single-worker allocation with game  $(0:n, R_0)$
    - Always correct result, almost no verification, almost optimal master utility
  - Company that buys computing cycles from Internet computers and sells them to customers
    - No single optimal game
    - E.g., If only  $n$  chosen, best is single-worker allocation with game  $(0:n, R_a)$  or  $(0:n, R_0)$

GAMES

# Game Definition

$W = \{1, 2, \dots, n\}$	set of assigned workers
$M$	master processor
$\mathcal{S}_i$	set of pure strategies available to player $i$
$\{\mathcal{C}, \overline{\mathcal{C}}\}$	set of pure strategies of a worker
$\{\mathcal{V}, \overline{\mathcal{V}}\}$	set of pure strategies of the master
$s$	strategy profile (a mapping from players to pure strategies)
$s_i$	strategy used by player $i$ in the strategy profile $s$
$s_{-i}$	strategy used by each player but $i$ in the strategy profile $s$
$w_s^{(i)}$	payoff of worker $i$ for the strategy profile $s$
$m_s$	payoff of the master for the strategy profile $s$
$p_{s_i}^{(i)}$	probability that worker $i$ uses strategy $s_i$
$p_{s_M}$	probability that the master uses strategy $s_M$
$\sigma$	mixed strategy profile
$\sigma_i$	probability distribution over pure strategies used by player $i$ in $\sigma$
$\sigma_{-i}$	probability distribution over pure strategies by all players but $i$ in $\sigma$
$U_i(s_i, \sigma_{-i})$	expected utility of worker $i$ with mixed strategy profile $\sigma$
$U_M(s_M, \sigma_{-M})$	expected utility of master with mixed strategy profile $\sigma$
$\text{supp}(\sigma_i)$	set of strategies of player $i$ with probability $> 0$ in $\sigma$

# Equilibrium Definition

- For a finite game, a mixed strategy profile  $\sigma^*$  is a **mixed-strategy Nash Equilibrium (MSNE)**, iff, for each player  $i$

$$U_i(s_i, \sigma_{-i}^*) = U_i(s_j, \sigma_{-i}^*), \forall s_i, s_j \in \text{supp}(\sigma_i^*),$$

$$U_i(s_i, \sigma_{-i}^*) \geq U_i(s_k, \sigma_{-i}^*),$$

$$\forall s_i, s_k : s_i \in \text{supp}(\sigma_i^*), s_k \notin \text{supp}(\sigma_i^*).$$

A strategy choice by each game participant s.t none has incentive to change it.



# Methodology

- For each game in  $\{1:1, 1:1^n, 0:n, 1:n\}$ 
  - Identify the parameter conditions for which there is a MSNE
    - Instantiate the two equations of the MSNE definition
    - Assume a general payoff model
  - From the above, obtain the conditions on the parameters (payoffs and probabilities) that make such a MSNE unique
  - Plug the specific reward models ( $R_m, R_0, R_a$ ) on the conditions to obtain the trade-offs between cost and reliability

# Game 1:1, 1 master - 1 worker

Expected utility of the **master** in any equilibrium

$$U_M = p_C p_V m_{CV} + (1 - p_C) p_V m_{\bar{C}V} + p_C (1 - p_V) m_{C\bar{V}} + (1 - p_C) (1 - p_V) m_{\bar{C}\bar{V}}$$

Expected utility of the **worker** in any equilibrium

$$U_W = p_C p_V w_{CV} + p_C (1 - p_V) w_{C\bar{V}} + (1 - p_C) p_V w_{\bar{C}V} + (1 - p_C) (1 - p_V) w_{\bar{C}\bar{V}}$$

Probability of **master** accepting a **wrong** value:

$$P_{wrong} = (1 - p_V) p_C$$

# Game 1:1, 1 master - 1 worker

- Depending on the range of values that  $p_C$  and  $p_V$  take, we may have a MSNE or a pure NE
- Both  $p_C$  and  $p_V$  can take values either 0, 1, or in  $(0,1)$  (9 cases)
- Example

If  $p_C \in (0, 1), p_V \in (0, 1)$ , there is a MSNE if, simultaneously,

$$U_M(\mathcal{V}, p_C) = U_M(\bar{\mathcal{V}}, p_C) \quad U_W(\mathcal{C}, p_V) = U_W(\bar{\mathcal{C}}, p_V)$$

and hence

$$p_C = \frac{m_{\bar{\mathcal{C}}\mathcal{V}} - m_{\bar{\mathcal{C}}\bar{\mathcal{V}}}}{m_{\mathcal{C}\mathcal{V}} - m_{\bar{\mathcal{C}}\mathcal{V}} - m_{\mathcal{C}\bar{\mathcal{V}}} + m_{\bar{\mathcal{C}}\bar{\mathcal{V}}}} \quad p_V = \frac{w_{\bar{\mathcal{C}}\mathcal{V}} - w_{\mathcal{C}\bar{\mathcal{V}}}}{w_{\mathcal{C}\mathcal{V}} - w_{\mathcal{C}\bar{\mathcal{V}}} - w_{\bar{\mathcal{C}}\mathcal{V}} + w_{\bar{\mathcal{C}}\bar{\mathcal{V}}}}$$

# Reward Models

- $R_m$ : Rewards only majority

$$m_{C_V} = -MC_V$$

$$w_{C_V} = -WP_C$$

$$m_{\bar{C}_V} = MB_{\mathcal{R}} - MC_V - MC_A$$

$$w_{\bar{C}_V} = WB_A - WC_T$$

$$m_{C_{\bar{V}}} = -MP_{\mathcal{W}} - MC_A$$

$$w_{C_{\bar{V}}} = WB_A$$

$$m_{\bar{C}_{\bar{V}}} = MB_{\mathcal{R}} - MC_A$$

$$w_{\bar{C}_{\bar{V}}} = WB_A - WC_T$$

- $R_a$ : Rewards all workers (same as above)

- $R_0$ : Does not reward any worker

$$m_{C_V} = -MC_V$$

$$w_{C_V} = -WP_C$$

$$m_{\bar{C}_V} = MB_{\mathcal{R}} - MC_V - MC_A$$

$$w_{\bar{C}_V} = WB_A - WC_T$$

$$m_{C_{\bar{V}}} = -MP_{\mathcal{W}}$$

$$w_{C_{\bar{V}}} = 0$$

$$m_{\bar{C}_{\bar{V}}} = MB_{\mathcal{R}}$$

$$w_{\bar{C}_{\bar{V}}} = -WC_T$$

# Game $1:1^n$ , $n$ Games $1:1$

- Master runs  $n$  instances of Game  $1:1$ , one with each of the  $n$  workers
- Chooses to verify or not with prob  $p_V$  only once
- If no verification, rewards all or none like in the  $1:1$  game
- Key difference now is that redundancy can be used to reduce the prob. of accepting a wrong value.

$$\mathbf{P}_C = \sum_{\substack{(F,T) \in \mathcal{W} \\ |F| > |T|}} \prod_{j \in F} p_C^{(j)} \prod_{k \in T} (1 - p_C^{(k)})$$

$$\mathbf{P}_{wrong} = (1 - p_V) \mathbf{P}_C$$

# Game $1:1^n, R_m, R_a$ (Game $1:1$ for $n=1$ )

Equilibrium $p_C, p_V$	Conditions	$\mathbf{P}_{wrong}$	$U_M$	$U_W$
$\frac{MC_V}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$		$(1 - p_V)\mathbf{P}_C$	$p_V((1 - p_C^n)MB_R - MC_V - (1 - p_C)nMC_A) + (1 - p_V)(MB_R(1 - \mathbf{P}_C) - MP_W\mathbf{P}_C - nMC_A)$	$WB_A - WC_T$
$0, \frac{WC_T}{WB_A + WP_C} \leq p_V < 1$ $0 < p_V$	$MC_V = 0$	0	$MB_R - nMC_A$	$WB_A - WC_T$
$1, 0 < p_V \leq \frac{WC_T}{WB_A + WP_C}$ $p_V < 1$	$MC_V = MP_W + MC_A$	$1 - p_V$	$-p_V MC_V - (1 - p_V)(MP_W + nMC_A)$	$(1 - p_V)WB_A - p_V WP_C$
$0 \leq p_C \leq \frac{MC_V}{MC_A + MP_W}, 0$ $p_C < 1$	$WC_T = 0$	$\mathbf{P}_C$	$MB_R(1 - \mathbf{P}_C) - MP_W\mathbf{P}_C - nMC_A$	$WB_A$
$\frac{MC_V}{MC_A + MP_W} \leq p_C < 1$ $0 < p_C$	$WC_T = WB_A + WP_C$	0	$(1 - \prod_{j \in W} p_C^{(j)})MB_R - MC_V - \sum_{(W_F, W_T) \in W} \prod_{j \in W_F} p_C^{(j)} \cdot \prod_{k \in W_T} (1 - p_C^{(k)}) W_T MC_A$	$-WP_C$
1, 1	$MC_V \leq MP_W + MC_A$ $WC_T \geq WB_A + WP_C$	0	$-MC_V$	$-WP_C$
0, 1	$MC_V = 0$ $WC_T \leq WB_A + WP_C$	0	$MB_R - nMC_A$	$WB_A - WC_T$
1, 0	$MC_V \geq MP_W + MC_A$	1	$-MP_W - nMC_A$	$WB_A$

# Game $1:1^n, R_0$ (Game $1:1$ for $n=1$ )

Equilibrium $p_C, p_V$	Conditions	$P_{wrong}$	$U_M$	$U_W$
$\frac{MC_V + MC_A}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$		$(1 - p_V)P_C$	$p_V((1 - p_C^n)MB_R - MC_V - (1 - p_C)nMC_A) + (1 - p_V)(MB_R(1 - P_C) - MP_W P_C)$	$-p_V WP_C$
$0, \frac{WC_T}{WB_A + WP_C} \leq p_V < 1$ $0 < p_V$	$MC_A = MC_V = 0$	0	$MB_R$	$p_V WB_A - WC_T$
$1, 0 < p_V \leq \frac{WC_T}{WB_A + WP_C}$ $p_V < 1$	$MC_V = MP_W$	$1 - p_V$	$-MC_V$	$-p_V WP_C$
$0 \leq p_C \leq \frac{MC_V + MC_A}{MC_A + MP_W}, 0$ $p_C < 1$	$WC_T = 0$	$P_C$	$MB_R(1 - P_C) - MP_W P_C$	0
$\frac{MC_V + MC_A}{MC_A + MP_W} \leq p_C < 1, 1$ $0 < p_C$	$WC_T = WB_A + WP_C$	0	$(1 - \prod_{j \in W} p_C^{(j)})MB_R - MC_V - \sum_{(W_F, W_T) \in W} \prod_{j \in W_F} p_C^{(j)} \cdot \prod_{k \in W_T} (1 - p_C^{(k)})  W_T  MC_A$	$-WP_C$
1, 1	$MC_V \leq MP_W$ $WC_T \geq WB_A + WP_C$	0	$-MC_V$	$-WP_C$
0, 1	$MC_V = MC_A = 0$ $WC_T \leq WB_A + WP_C$	0	$MB_R$	$WB_A - WC_T$
1, 0	$MC_V \geq MP_W$	1	$-MP_W$	0

# Games $0:n$ and $1:n$

- In Game  $0:n$  the master participates only indirectly (by fixing  $p_V$ )
  - Under our assumptions, we prove that there are only pure NE
  - We force the unique equilibrium in which no worker cheats
- Game  $1:n$  is similar to  $0:n$ , but
  - The master is part of the game
  - This imposes additional restrictions on the NE
  - The unique NE in which no worker cheats requires  $MC_V=0$



# Mechanism Design

# SETI-like Scenario

- Here we assume
  - $WB_A > WC_T = 0$  (CPU scavenging, worker's incentive)
  - $MB_R > MC_A > 0$  (master's incentive to compute)
  - $MP_W > MC_V > 0$  (master's incentive to verify)
- Under these constraints,
  - In both Games  $1:1$  and  $1:1^n$  one single MSNE remains
  - There is no unique NE for Game  $1:n$

# Results for SETI-like

(Game, Model)	Equilibrium $p_C, p_V$	$P_{wrong}$	$U_M$	$U_W$
$(1:1, \mathcal{R}_m), (1:1, \mathcal{R}_a)$	$0 \leq p_C \leq \frac{MC_V}{MC_A + MP_W}, p_C < 1, p_V = 0$	$p_C$	$MB_{\mathcal{R}} - p_C(MB_{\mathcal{R}} + MP_W) - MC_A$	$WB_A$
$(1:1, \mathcal{R}_\emptyset)$	$0 \leq p_C \leq \frac{MC_V + MC_A}{MC_A + MP_W}, p_C < 1, p_V = 0$	$p_C$	$MB_{\mathcal{R}} - p_C(MB_{\mathcal{R}} + MP_W)$	0
$(1:1^n, \mathcal{R}_m), (1:1^n, \mathcal{R}_a)$	$0 \leq p_C \leq \frac{MC_V}{MC_A + MP_W}, p_C < 1, p_V = 0$	$P_C$	$MB_{\mathcal{R}} - P_C(MB_{\mathcal{R}} + MP_W) - nMC_A$	$WB_A$
$(1:1^n, \mathcal{R}_\emptyset)$	$0 \leq p_C \leq \frac{MC_V + MC_A}{MC_A + MP_W}, p_C < 1, p_V = 0$	$P_C$	$MB_{\mathcal{R}} - P_C(MB_{\mathcal{R}} + MP_W)$	0
$(0:n, \mathcal{R}_m)$	$p_C = 0, \frac{WB_A}{WP_C + 2WB_A} < p_V \leq 1$	0	$MB_{\mathcal{R}} - p_V MC_V - nMC_A$	$WB_A$
$(0:n, \mathcal{R}_a)$	$p_C = 0, 0 < p_V \leq 1$	0	$MB_{\mathcal{R}} - p_V MC_V - nMC_A$	$WB_A$
$(0:n, \mathcal{R}_\emptyset)$	$p_C = 0, 0 < p_V \leq 1$	0	$MB_{\mathcal{R}} - p_V(MC_V + nMC_A)$	$p_V WB_A$

- The best choice is Game  $0:n$  with reward model  $R_0$  (no verification, no payment) and non-redundant allocation
- To obtain always the correct answer it is enough for the master to verify with arbitrarily small probability
- The master's utility is arbitrarily close to optimal

# Contractor Scenario

- A company buys computational power from Internet users and sells it to computation-needing consumers (commercial P2PC)
- Workers must have incentive to participate:  
 $U_W > 0$
- Additionally
  - $WB_A = MC_A$  (worker's incentive)
  - $MB_R > MC_A$  (master's incentive to compute)
  - $WC_T > 0$  (computing cost)
  - $MP_W > MC_V > 0$  (master's incentive to verify)

# Results for Contractor

(Game, Model)	Equilibrium $p_C, p_V$	$\mathbf{P}_{wrong}$	$U_M$	$U_W$
$(1:1, \mathcal{R}_m), (1:1, \mathcal{R}_a)$	$\frac{MC_V}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$	$(1 - p_V)p_C$	$MB_R - p_C(MB_R + MP_W) - MC_A$	$WB_A - WC_T$
$(1:1, \mathcal{R}_\emptyset)$	$\frac{MC_V + MC_A}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$	$(1 - p_V)p_C$	$MB_R - p_C(MB_R + MP_W)$	$-p_V WP_C$
$(1:1^n, \mathcal{R}_m), (1:1^n, \mathcal{R}_a)$	$\frac{MC_V}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$	$(1 - p_V)\mathbf{P}_C$	$(p_V(1 - p_C^n) + (1 - p_V)(1 - \mathbf{P}_C))MB_R$ $-p_V MC_V - (1 - p_V)\mathbf{P}_C MP_W$ $-(1 - p_V p_C)nMC_A$	$WB_A - WC_T$
$(1:1^n, \mathcal{R}_\emptyset)$	$\frac{MC_V + MC_A}{MC_A + MP_W}, \frac{WC_T}{WB_A + WP_C}$	$(1 - p_V)\mathbf{P}_C$	$(p_V(1 - p_C^n) + (1 - p_V)(1 - \mathbf{P}_C))MB_R$ $-p_V MC_V - (1 - p_V)\mathbf{P}_C MP_W$ $-p_V(1 - p_C)nMC_A$	$-p_V WP_C$
$(0:n, \mathcal{R}_m)$	$0, \frac{WB_A + WC_T}{WP_C + 2WB_A} < p_V \leq 1$	0	$MB_R - p_V MC_V - nMC_A$	$WB_A - WC_T$
$(0:n, \mathcal{R}_a)$	$0, \frac{WC_T}{WP_C + WB_A} < p_V \leq 1$	0	$MB_R - p_V MC_V - nMC_A$	$WB_A - WC_T$
$(0:n, \mathcal{R}_\emptyset)$	$0, \frac{WC_T}{WP_C + WB_A} < p_V \leq 1$	0	$MB_R - p_V(MC_V + nMC_A)$	$p_V WB_A - WC_T$

- To obtain **always the correct answer**, verification probability  $p_V > WC_T / (WP_C + WB_A)$  can be large
- No single optimal game
  - Either Game  $(0:n, \mathcal{R}_a)$  or  $(0:n, \mathcal{R}_\emptyset)$  is the best if only  $n(=1)$  or  $WP_C$  can be changed
  - If master can only change  $WB_A = MC_A$ , sometimes Game  $(1:1^n, \mathcal{R}_m)$  is the best

# Summary

- ❑ We considered master-worker Internet-based computations from a game-theoretic point of view
- ❑ We defined the general model and cost-parameters
- ❑ Proposed and analyzed several games that the master can choose to play to achieve high reliability at low cost
- ❑ Designed appropriate mechanisms for two realistic scenarios
- ❑ No optimal game for every scenario or set of parameters.

# Future Work

- Consider other realistic scenarios where our work can be applied
- Consider other forms of collusion (e.g., Sybil attacks) and investigate how the trade-offs are affected
- Remove the complete knowledge assumption (implicit in NE analyses)
- Study games with several rounds (reputation)
- Consider irrational, bounded-rational, and faulty players.





Thank you!

# LAG vs Our Work

- Closer work to ours [Yurkewych Levine Rosenberg 05]
  - ❑ Master audits the results returned by rational workers with a tunable probability
  - ❑ Bounds for that probability are computed to provide incentives to workers to be honest in three scenarios
    - Redundant allocation with collusion
      - Cooperation among workers concealed from the master
    - Redundant allocation without collusion
    - Single-worker allocation
  - ❑ Their conclusion: Single-worker allocation is a cost-effective mechanism, especially in the presence of collusion
- Our model comprises a **weaker** type of collusion but
  - ❑ We study more algorithms and games
  - ❑ Consider a richer payoff model and probabilistic cheating
  - ❑ We have one-round protocols and
  - ❑ Show useful trade-offs between the benefit (cost) of the master and the probability of accepting a wrong result (reliability)

# Prior/Related Work

- Internet Auditing Game [Yurkewych Levine Rosenberg 05]
  - Three master-worker scenarios
    - Redundant allocation with/without collusion  
(Cooperation among workers concealed from the master)
    - Single-worker allocation
  - Master (out of the game) audits the results or accepts majority
  - (Fixed) probability  $p_V$  of auditing
  - (Fixed) payments  $R$  for accepted results
  - (Fixed) penalty  $P$  for rejected results
  - Result: Bounds on  $R$ ,  $P$ , and  $p_V$  to prevent cheating
  - Conclusions:
    - Lots of auditing ( $p_V > 1/2$ ) with collusion
    - Redundancy only useful if no collusion
    - Single-worker allocation is cost-effective