

Algebraic Computations in Anonymous VANET

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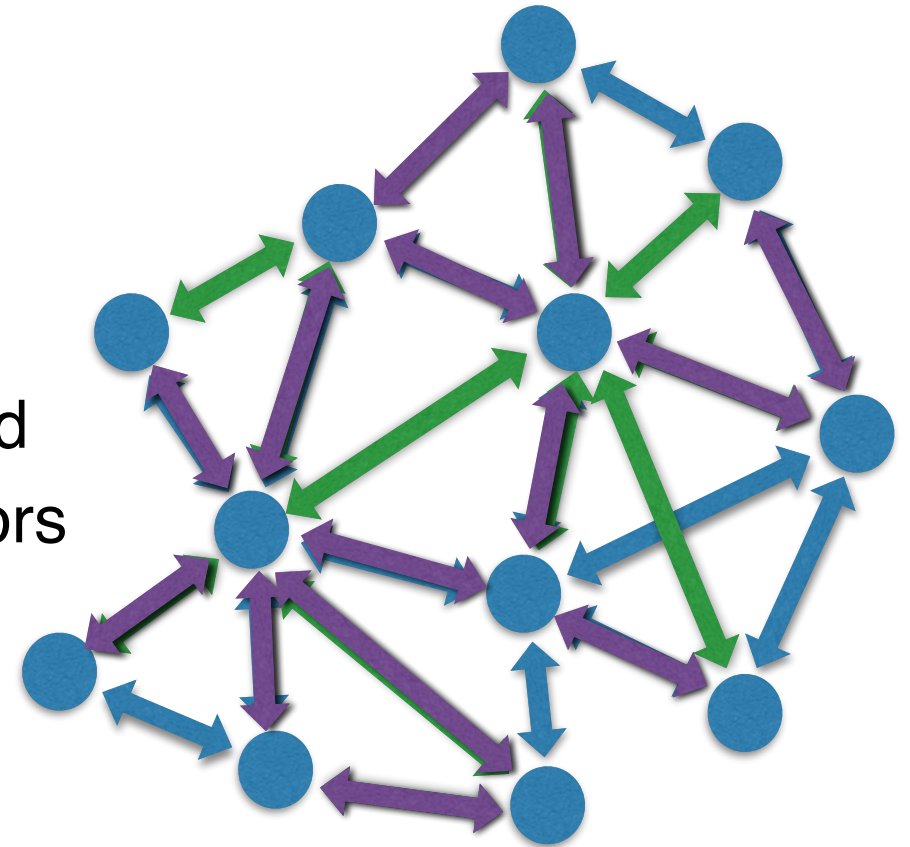


NETYS 2024



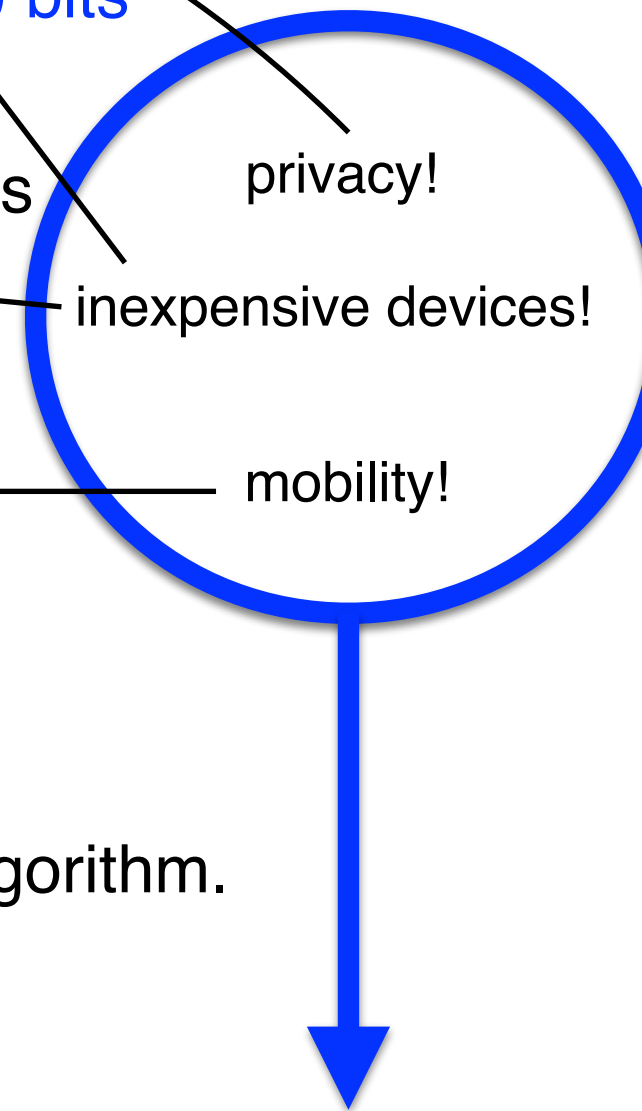
Anonymous Dynamic Networks (ADN)

- **Fixed set of n nodes**
 - No identifiers or labels
- **Synchronous communication** : At each round
 - a node broadcasts a message to its neighbors
 - receives the messages of its neighbors
 - executes some local computation
- **1-interval connectivity**
 - communication links may change from round to round, but
 - at each round the network is connected
- **Distinguished nodes**
 - $0 < \ell < n$ “counted” network nodes with ℓ known to the algorithm.
(Counting not solvable with $\ell = 0$ or ℓ unknown.)



Congested ADN with Opportunistic Connectivity

- **Fixed set of n nodes**
 - No identifiers or labels, **internal memory limited to $O(\log n)$ bits**
- **Synchronous communication** : At each round
 - a node broadcasts a **$O(\log n)$ bits** message to its neighbors
 - receives the messages of its neighbors
 - executes some local computation
- **T-connected**
 - communication links may change from round to round, but
 - **the union of T snapshot graphs is connected.**
- **Distinguished nodes**
 - $0 < \ell < n$ “counted” network nodes with ℓ known to the algorithm.
(Counting not solvable with $\ell = 0$ or ℓ unknown.)



Computation framework suitable for VANET:

Anonymous Vehicular Adhoc Networks (A-VANET)

Restricted Methodical Counting

[Kowalski-Mosteiro,22]

Algebraic computations require to know or compute n (Counting Problem).
Moreover, many are doable with a counting algorithm (e.g. AVG, SUM).
We focus on the **Counting Problem**.

RMC key ideas:

- ℓ counted network nodes and $n - \ell$ uncounted network nodes.
- try network size estimates $k = 2^i(\ell + 1)$ and binary search after estimate $k > n$.
- all nodes share some “potential” values for some function of k iterations.
- counted nodes remove potential every now and then to evaluate k .
- carefully designed alarms allow to detect correct or wrong k .

RMC Epoch Example

epochs:

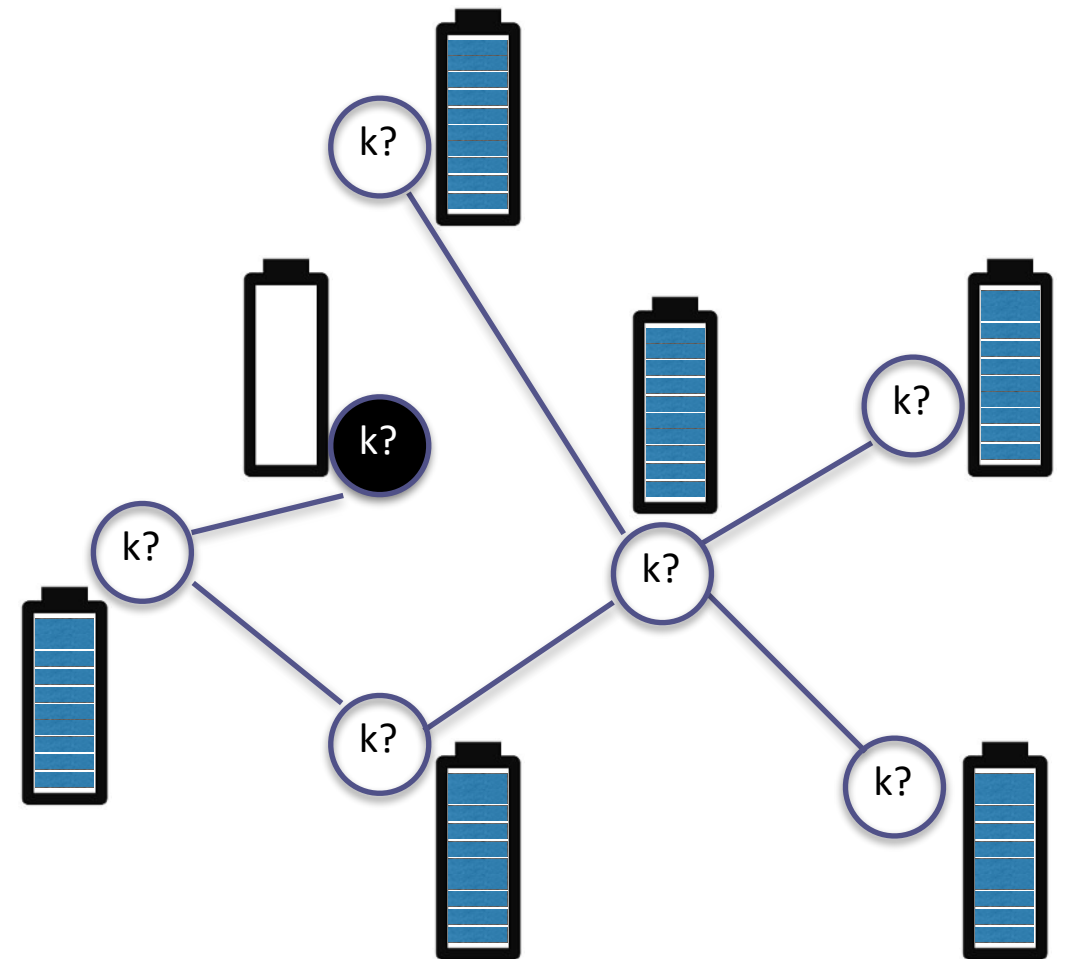
- one for each estimate k
- initialize potentials: $\Phi_{uncounted} = \ell, \Phi_{counted} = 0$

$p(k)$ phases:

(to let counted remove “enough” potential ρ)

$r(k)$ rounds:

(to “average” the current potentials Φ)



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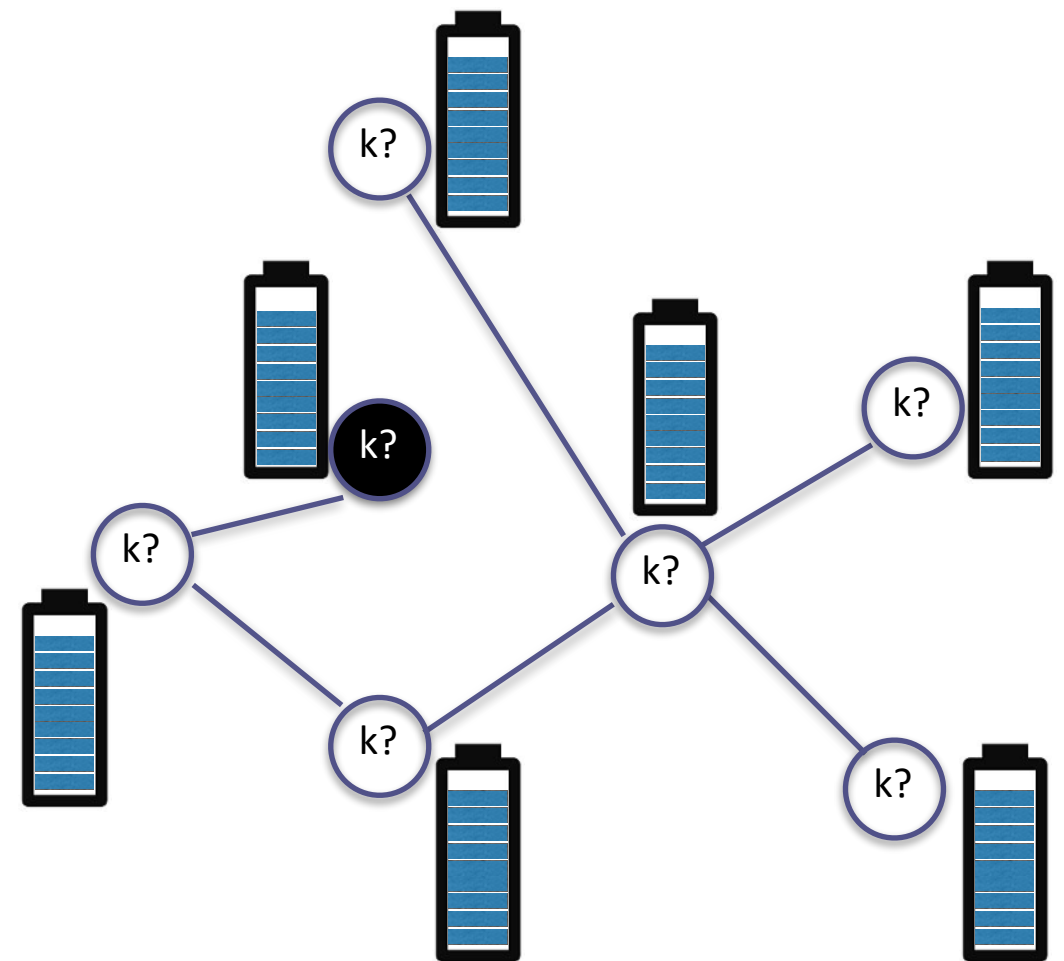
(to let counted remove “enough” potential ρ)

$r(k)$ rounds:

(to “average” the current potentials Φ)

mass distribution:

- broadcast Φ and receive neighbors' Φ_i
- $\Phi = \Phi + \sum_{i \in N} \Phi_i / d(k) - |N| \Phi / d(k)$
truncated to $O(\log n)$ bits
- counted remove potential: $\rho = \rho + \Phi, \Phi = 0$



$\rho =$



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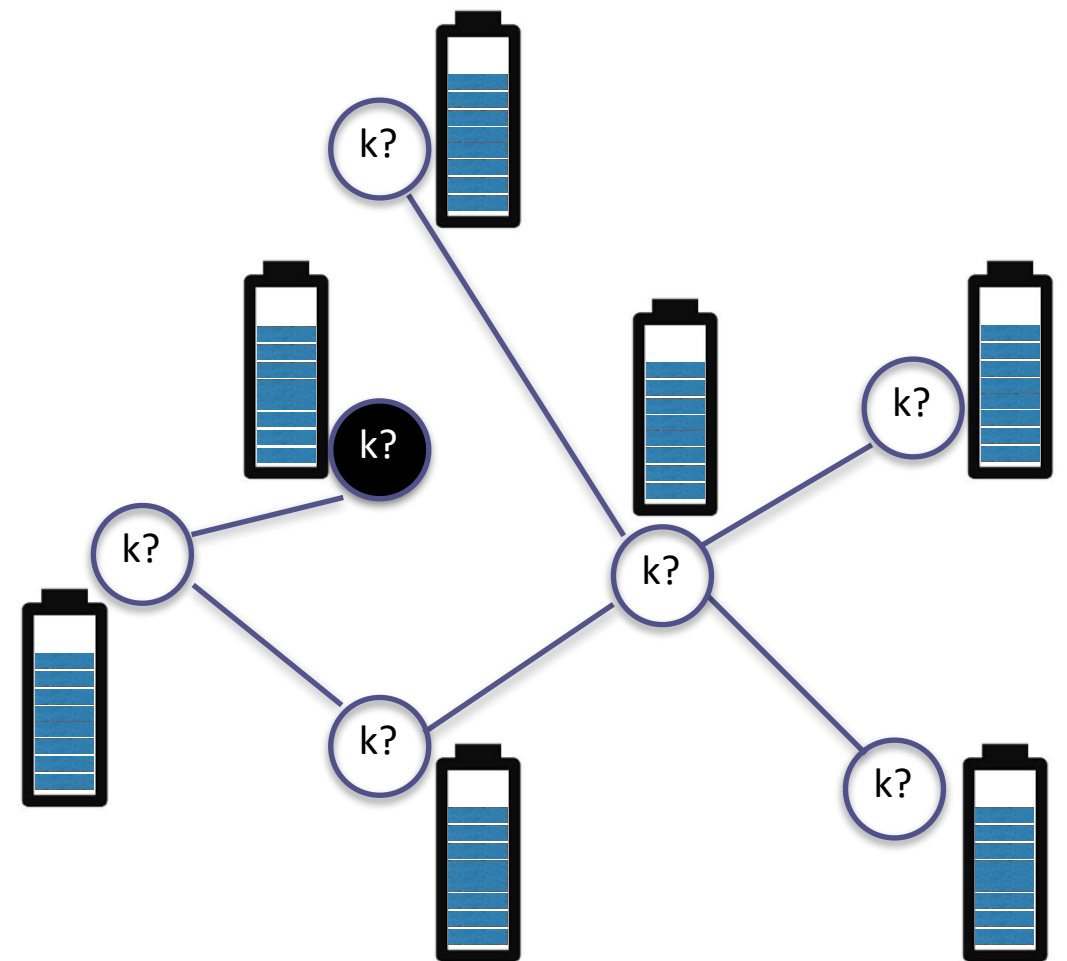
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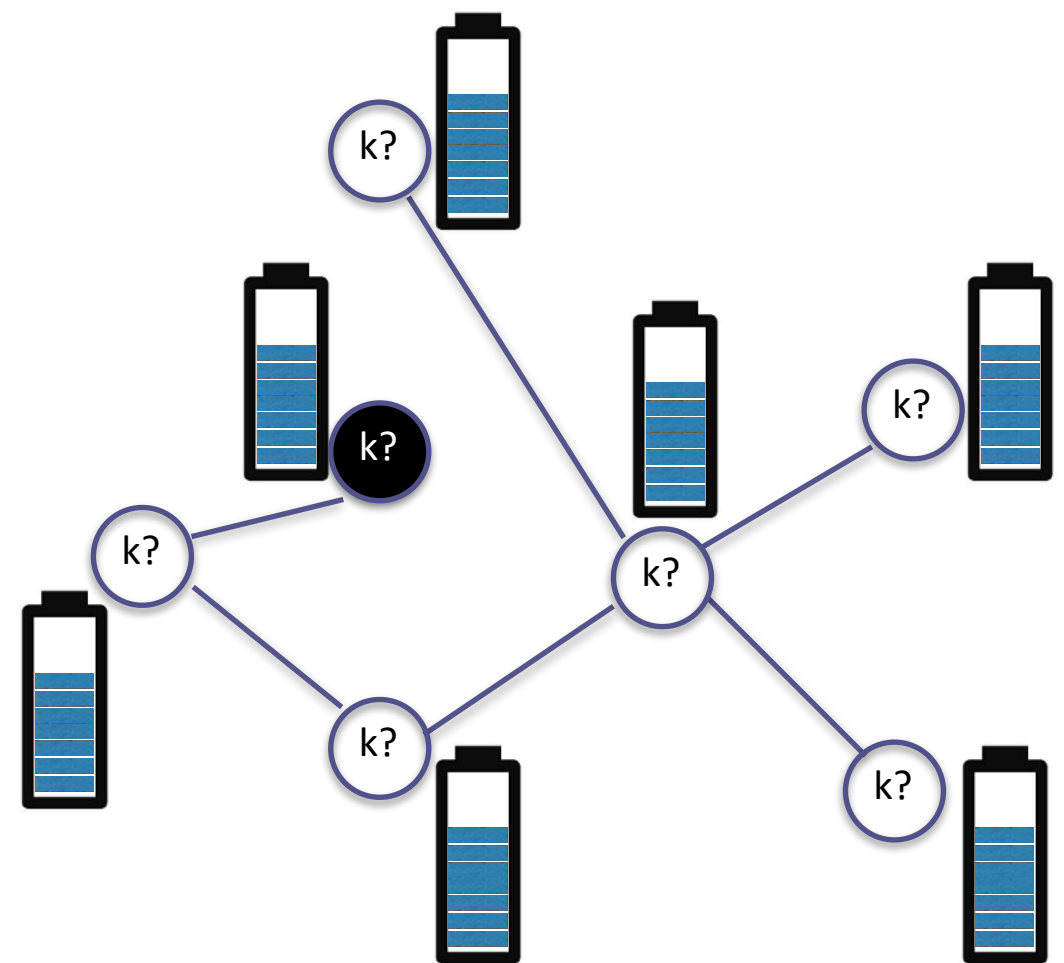
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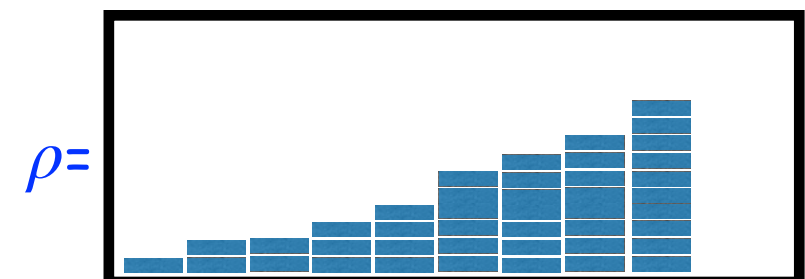
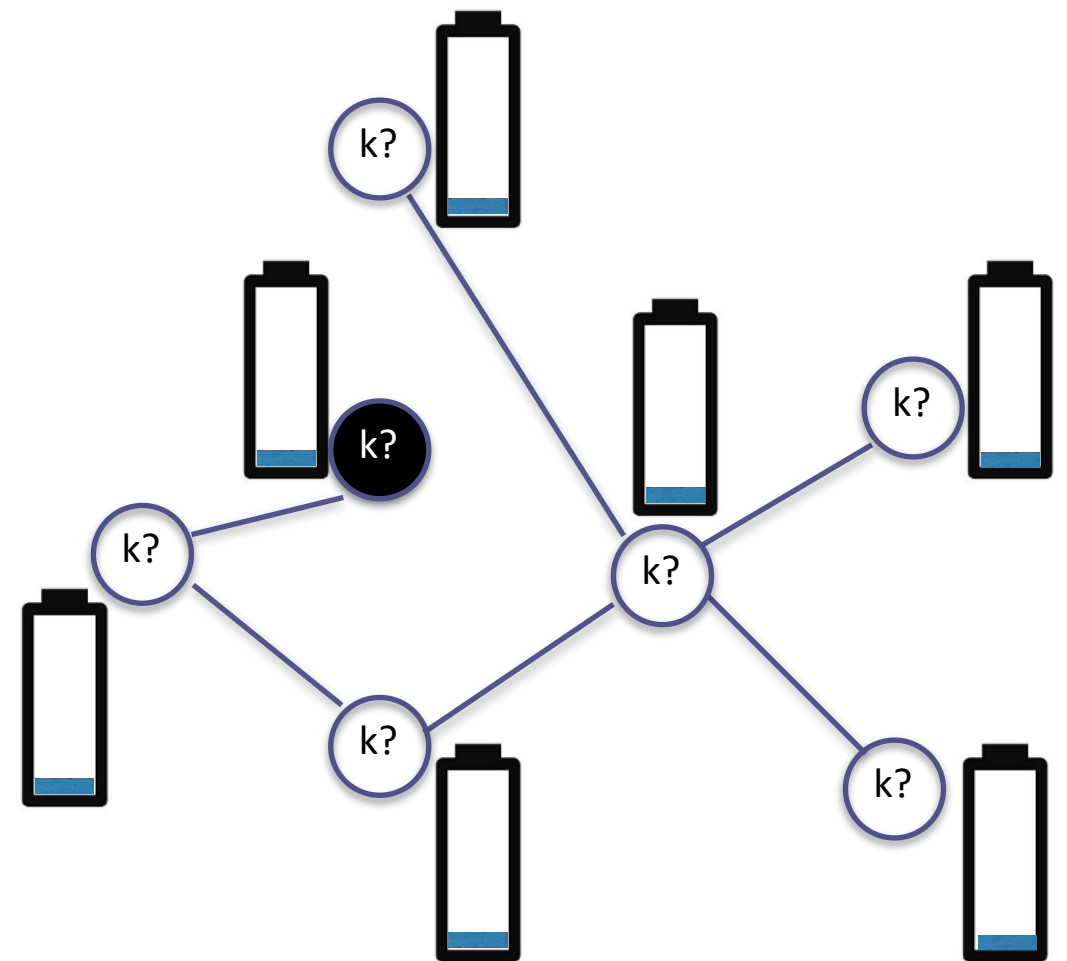
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truncated to $O(\log n)$ bits
- counted remove potential: $\rho = \rho + \Phi, \Phi = 0$
- counted decide according to ρ
- counted notify if $k \geq n$
- try next k if needed

After $p(k)$ phases...



RMC theory and practice

RMC is **simple** (light implementation),

works with **restricted resources**, and tolerates **disconnections**

⇒ **suitable for A-VANET computations, but!**

... the current theoretical analysis applies to adversarial dynamicity:

$$\tilde{O}\left(\frac{n^{1+2T(1+\epsilon)}}{\ell i_{\min}^2}\right) \Rightarrow \tilde{O}(n^{9+10\epsilon}), \text{ for } T = 5, i \in \Theta(n), \ell \in O(1).$$

On the other hand, lower bounds known:

$$\Omega(\log n) \quad \text{and} \quad \Omega(\mathcal{D})$$

Large polynomial gap!! ⇒ we evaluate experimentally on real traffic.

Hypothesis: large polynomial speed-ups on real traffic.

Simulation Techniques

epochs:

- one for each estimate k
- initialize potentials: $\Phi_{uncounted} = \ell, \Phi_{counted} = 0$

$p(k)$ phases:

(to let counted remove “enough” potential ρ)

change # phases function until incorrect

$r(k)$ rounds:

(to “average” the current potentials Φ)

change # rounds function until incorrect

mass distribution:

- broadcast Φ and receive neighbors' Φ_i

- $\Phi = \Phi + \sum_{i \in N} \Phi_i / d(k) - |N| \Phi / d(k)$
truncated to $O(\log n)$ bits

- counted remove potential: $\rho = \rho + \Phi, \Phi = 0$

- counted decide according to ρ

change threshold functions until incorrect

- counted notify if $k \geq n$

- try next k if needed

Simulations

Input topologies:

- Extracted from traces of taxi trips in NYC with proximity defined by street location. (Massive database, technical challenge.)
- “Enhanced” path with proximity defined by short reachability. (Simulates highways.)

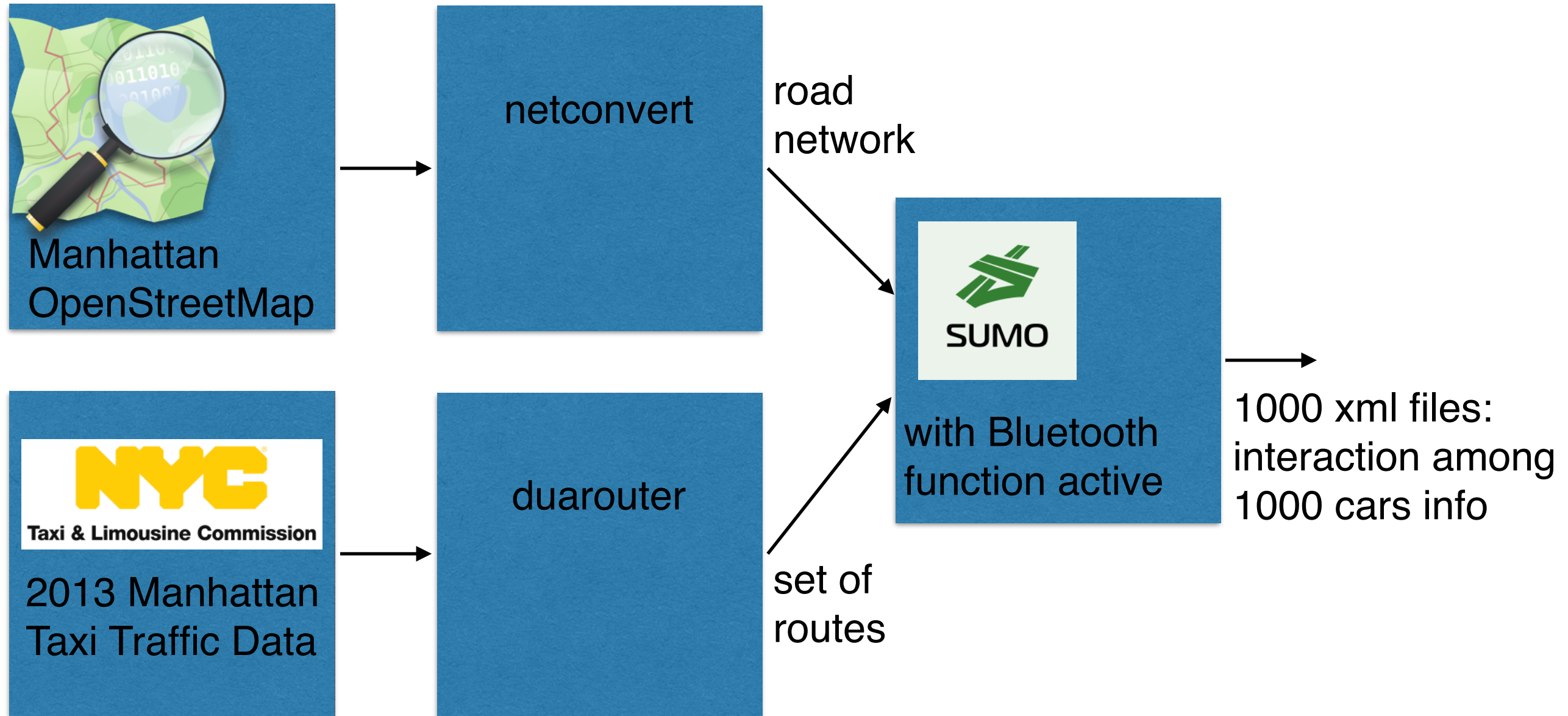
Parameters:

- $n = \{128, 256, 500, 600, \dots, 1000\}$ network size
- $\ell = \{30, n/4 - 1\}$ counted nodes
- $T = \{1, 2.5, 5, 11\}$ connectivity
- Other algorithm parameters and asymptotic notation constants fixed to small values.

Evaluated against:

- RMC theoretical running time.

NYC Taxi Traffic Data Extraction



Results

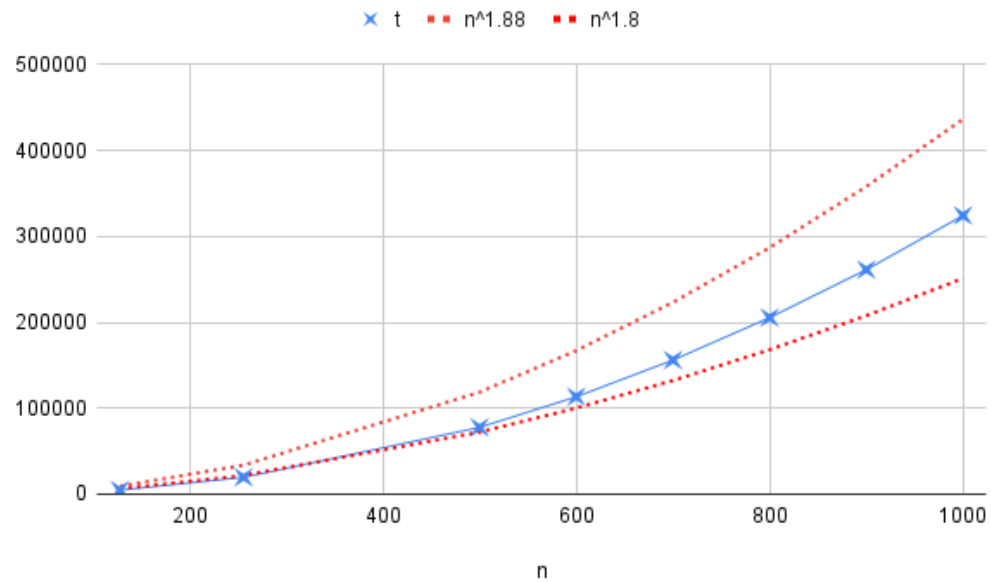


Fig. 1: NYC taxi traffic input for $\ell = 30$ and $\mathcal{T} = 5$.
Dotted lines correspond to functions of n bounding the running time.

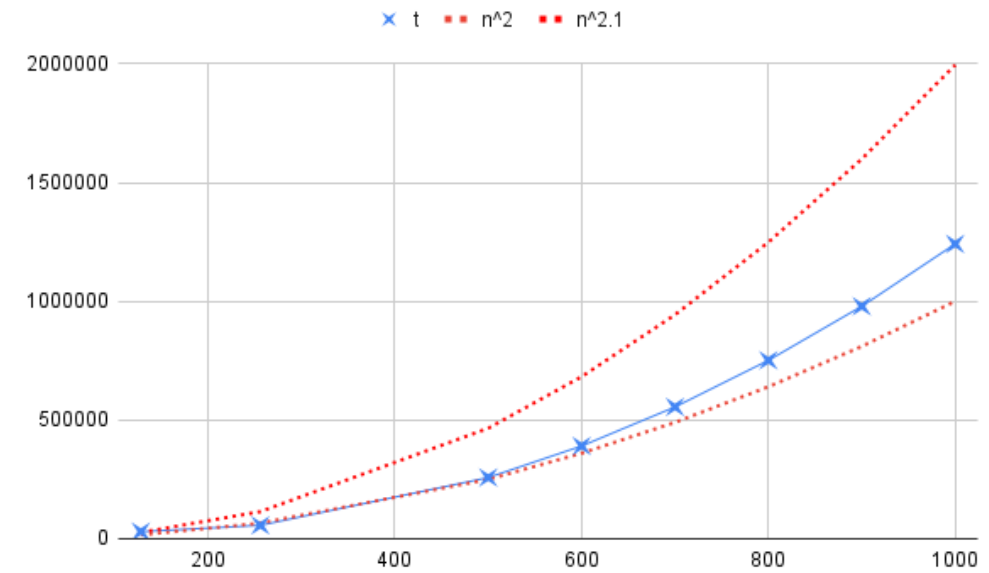


Fig. 2: Highway input for $\ell = 30$ and $\mathcal{T} = 11$.
Dotted lines correspond to functions of n bounding the running time.

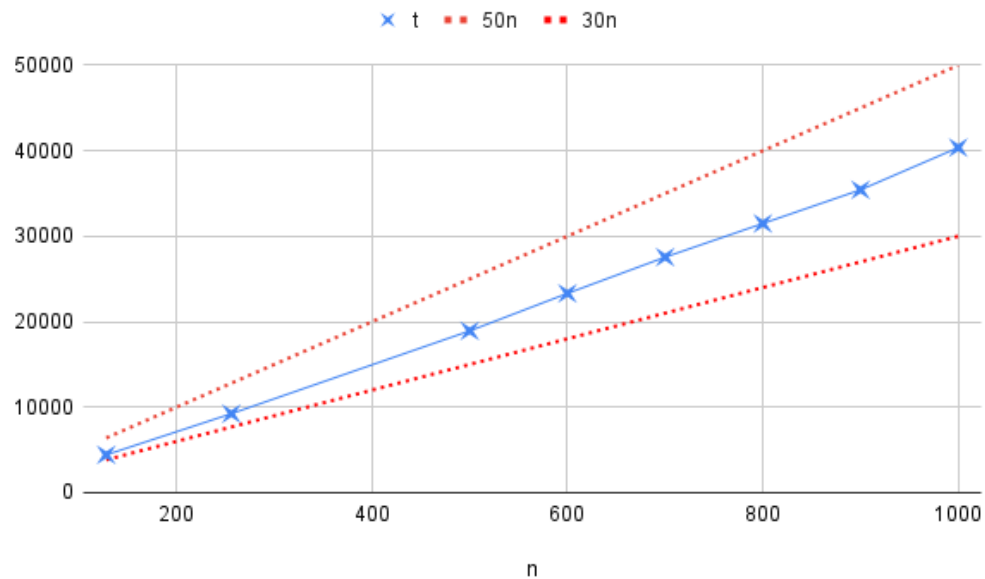


Fig. 3: NYC taxi traffic input for $\ell = n/4 - 1$ and $\mathcal{T} = 5$.
Dotted lines correspond to functions of n bounding the running time.

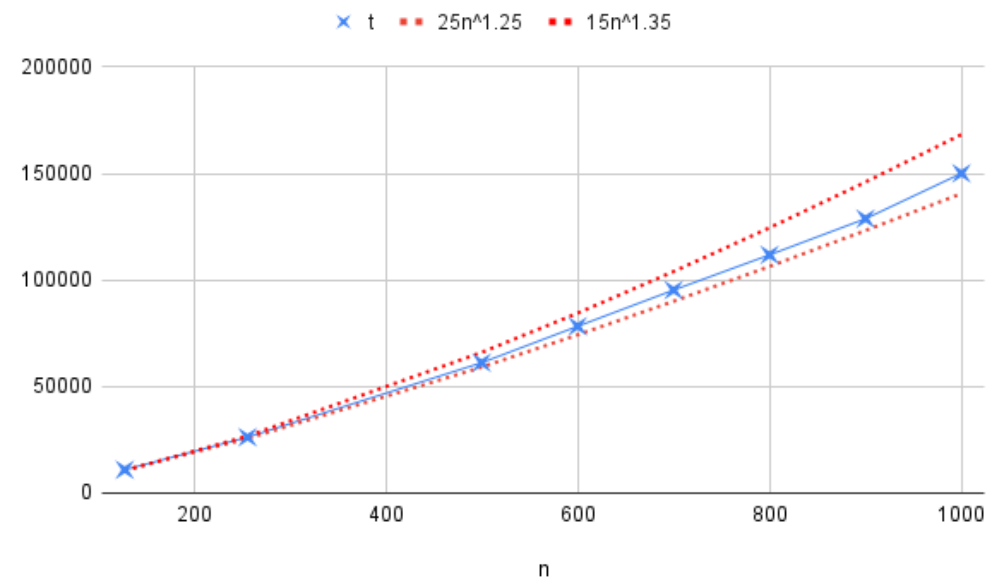


Fig. 4: Highway input for $\ell = n/4 - 1$ and $\mathcal{T} = 11$.
Dotted lines correspond to functions of n bounding the running time.

In the topologies tested, RMC is $\approx n^7$ times faster than the worst-case theoretical running time, confirming our hypothesis.

Thank you!

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