Computing Aggregate Functions in Sensor Networks

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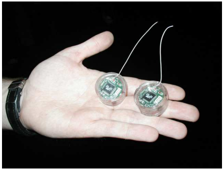
PRDC 2009

A Sensor Network



Intel Berkeley Research Lab

Capabilities
- processing
- sensing
- communication

Limitations
- range
- memory
- life cycle

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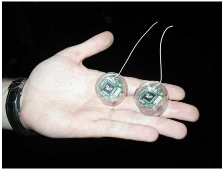

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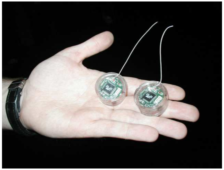

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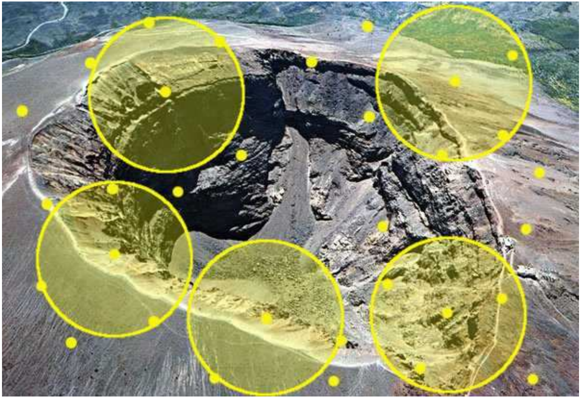

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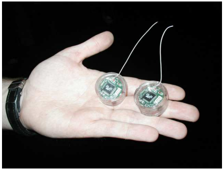

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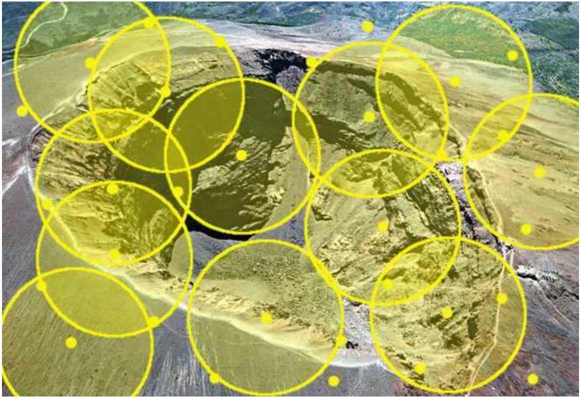

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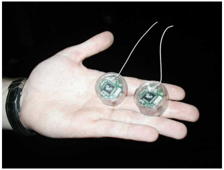

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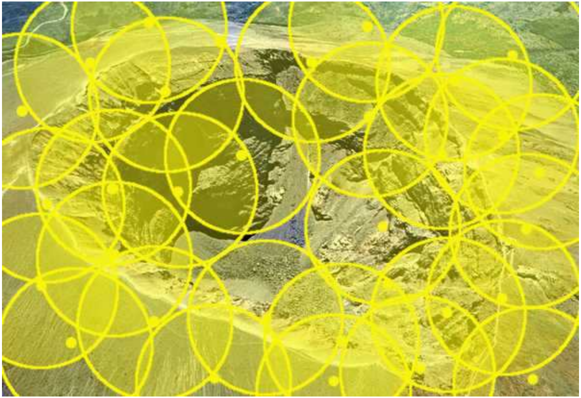

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The Problem

Node gets *input-value* (sensed, measured, etc.)

- unreliability ⇒ can not rely on individual sensors data ⇒ aggregate!

Algebraic aggregate functions:

- average
- maximum, count, sum, quantiles, etc. (easy from average [KDG03])

What average?

- lack of position information ⇒ aggregate all.
- sink nodes must receive ⇒ result to all nodes.
- input-values change over time ⇒ need global synch.
- multi-hop ⇒ impossible to aggregate in one step.
- under arbitrarily failures ⇒ aggregation is intractable! [BGMGM03]

Problem

Compute the average among all-nodes input-value at a given time step and distribute the result to all nodes under bounded failures.

Connectivity

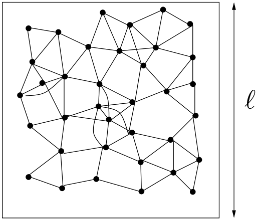
Node Deployment in Sensor Networks

- Hostile or remote environment

  $\Rightarrow$ deterministic deployment not feasible

  $\Rightarrow$ *controlled* random deployment.

- Arbitrary Density: the Geometric Graph $\mathcal{G}_{n,r}$.



- $[0,1]^2$
- Structural properties depend on relation among r and n.
- Connectivity/coverage guarantee.

Node Constraints

- Constant memory size.
- Limited life cycle.
- Short transmission range.
- Low-info channel contention:
  - Radio tx on a unique shared channel.
  - No collision detection.
  - Non-simultaneous rx and tx.
- Local synchronism.
- Discrete tx power range.
- No position information.
- Unreliability.
- Adversarial wake-up schedule.
- No global controller.
- No initial infrastructure.

tx = transmission.
rx = reception.

Other

- Failures: $\leq f$ failures separated $\geq T$ steps.
- Input values distribution: adversarial.
- Topology knowledge: unknown except for n.
- Failure-free *sink* node, knows D and Δ.
- ID: unique of $O(\log n)$ bits.
- Metrics:
  - time $\rightarrow$ slots.
  - energy $\rightarrow$ transmissions.

Previous Work

- Hierarchical Aggregation: tree convergecast.
  - Gupta et al. 01: $O(\log^2 n)$ *rounds*, no contention resolution.
  - Kollios et al. 05: $\omega(\log n)$ memory.
  - Madden et al. 02
- Non-hierarchical Aggregation: mass distribution.
  - Boyd et al. 06: prob $O(\log n + \log(n/\epsilon)/(1-\lambda_{max}((\mathbf{I}+\mathbf{P})1/2)))$ *rounds*.
  - Kempe et al. 03: similar bounds, one hop.
  - Chen et al. 06: prob $O(\Delta^3 \log(\sum_i (v_i - \overline{v})^2/\epsilon^2)/a(G))$ *rounds*.
  - ALL: $\omega(\log n)$ memory, no contention resolution, synch start.
- Geographic.
  - Dimakis et al. 08: needs position information.

Previous Work

- Hierarchical Aggregation:

  pros fast.

  cons failures → network partitioned.
  limited memory → can not be implemented.

- Non-hierarchical Aggregation:

  pros more resilient to failures.

  cons higher energy consumption.

- Our protocol:

  interleave both! with limited memory and low energy consumption.

Protocol

- **Preprocessing**
  - Partition nodes in *delegates* and *slugs*.
  - Reserve blocks of time steps for local use.
- **Aggregate Computation Scheme**
  - **Trigger:** sink broadcast (τ_1, Δ, D).
  - **Collection:** delegates aggregate slugs input value.
  - **Computation:** delegates compute aggregate function.
  - **Dissemination:** delegates distribute the result.

Protocol

1. **Preprocessing:**
   - Partition nodes in *delegates* and *slugs*.
     - every slug is at $d \leq \alpha r$ from some delegate $(0 < \alpha \leq 1/4)$
     - every pair of delegates are at $d > \alpha r$

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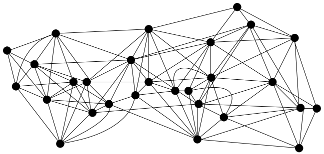
MIS(αr)



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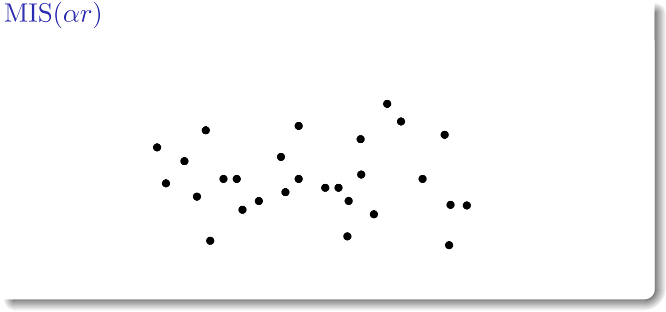

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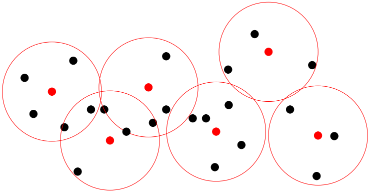

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   - Every delegate reserves blocks of time steps for local use.

     s.t. delegate and slugs can communicate without collisions.

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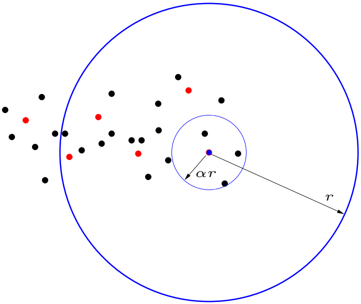

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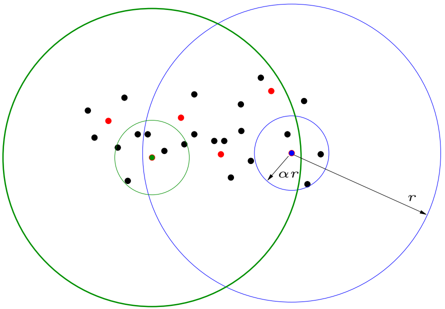

$$2\alpha \leq \beta \leq r/2$$

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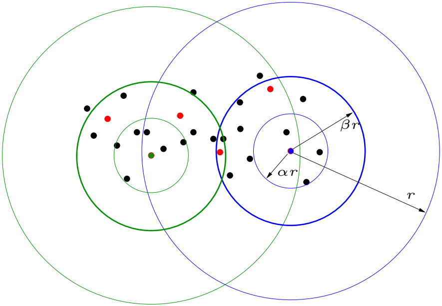

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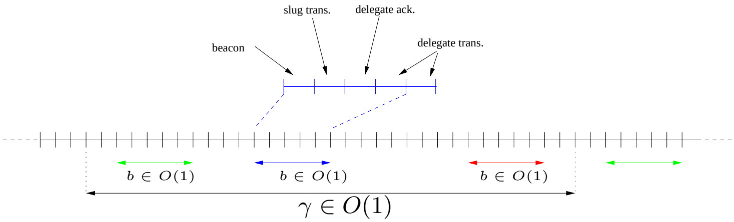

From now on, delegates use βr and slugs αr, in reserved slots.

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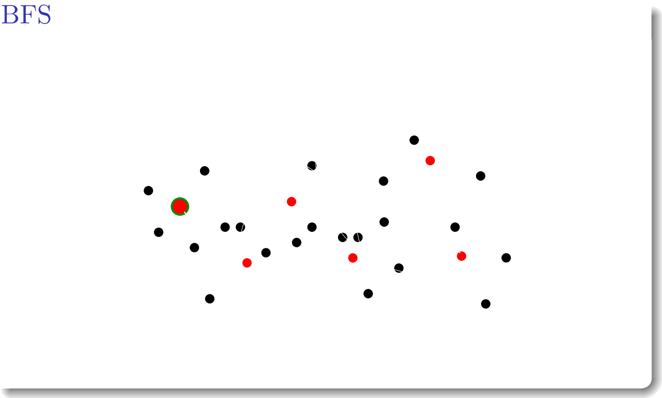

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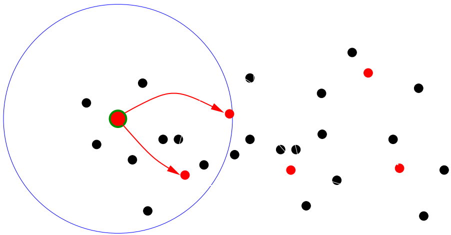
BFS



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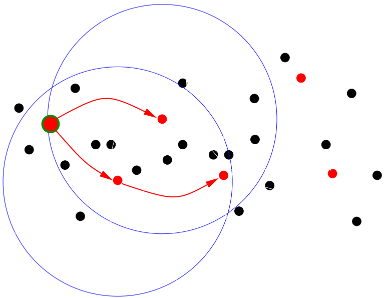

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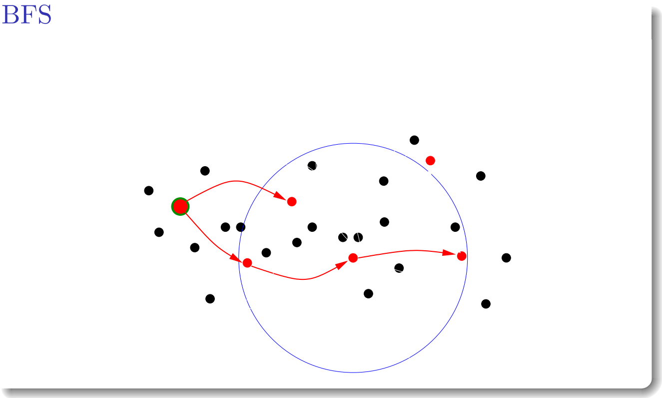

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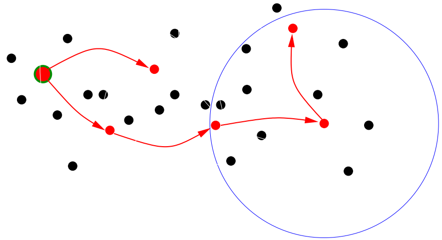

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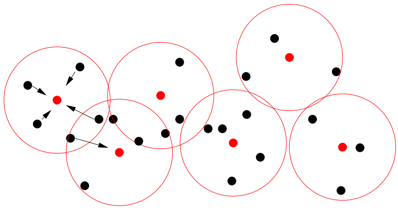
Windowed protocol



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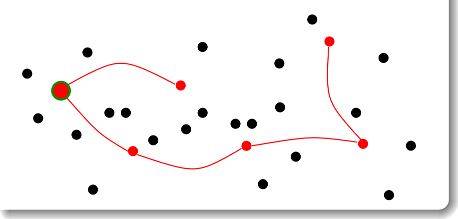
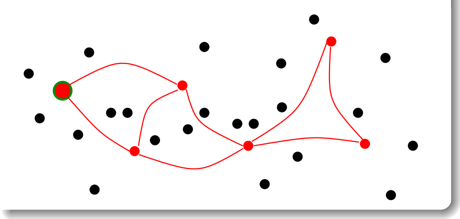
4. **Computation & Dissemination:** tree-based AND mass-distribution.

Tree-based



Aggregate at sink.

Mass distribution



Iteratively share a fraction.

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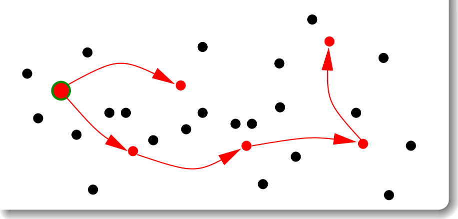

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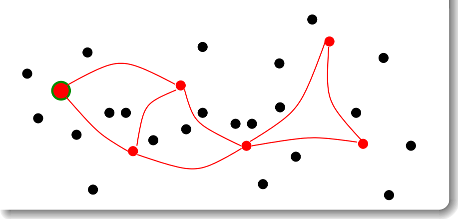

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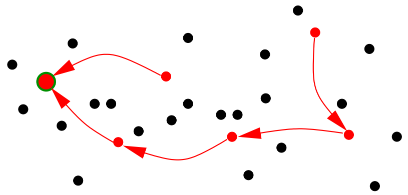

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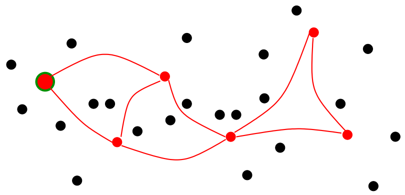

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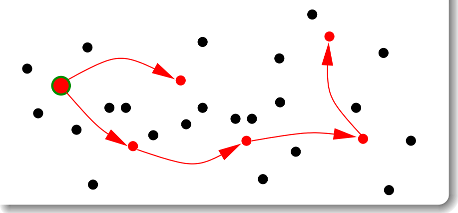

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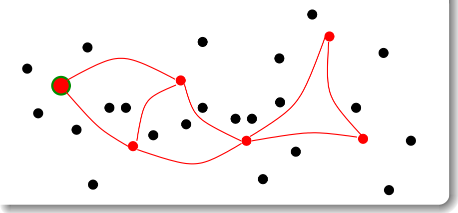

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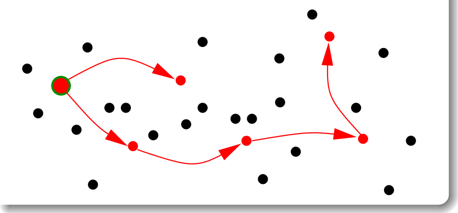

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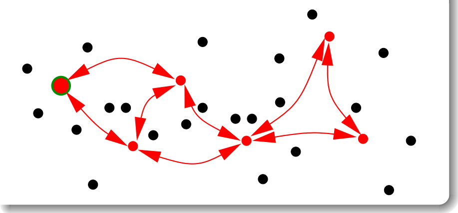

Iteratively share a fraction.

Bounds

1. **Preprocessing:**
   - MIS(αr). W.h.p. node i is in the partition within

$$O(\log^2 n) \text{ steps [MW'05].}$$

   - Coloring(r). W.h.p. delegate i reserves a block within

$$O(\log n) \text{ steps [FCM'07].}$$

2. **Trigger:** BFS(sink). Node i receives τ_1 within

$$O(D) \text{ steps.}$$

3. **Collection:** W.h.p. delegate i receives all slug values within

$$O(\Delta + \log^2 n) \text{ steps.}$$

Up to here, $O(D + \Delta)$ steps w.h.p.

Bounds

4. **Computation & Dissemination:**
   - tree-based: w.h.p. node i holds final value in
     $$O(D + f \log^2 n) \text{ steps.}$$
   - mass-distribution: w.h.p. node i holds final value in
     $$O\left(\frac{f - \log \varepsilon + \log(\nu_{max}/\nu_{min})}{\Phi_{min}^2}\right) \text{ steps.}$$

   $\Phi_{min} = \min_{k \in \{0,1,\ldots,f\}} \Phi_k$.
   Φ_k: conductance of underlying graph after kth failure.
   ε: relative error

   Adding $O(D + \Delta)$ to these bounds...

Bounds

Overall time efficiency

Theorem

$\exists \kappa_1, \kappa_2 > 0$ such that, if $T \geq \kappa_2 \log^2 n$, w.h.p., within

$$O(\Delta + D + f \log^2 n) \text{ time steps after } \tau_1 - \kappa_1(D + \log^2 n),$$

all nodes hold the same value in the range

$$\left[\frac{\overline{\nu}|V'| - f\nu_{min}}{|V'| - f}, \frac{\overline{\nu}|V'| - f\nu_{max}}{|V'| - f}\right].$$

Optimal if $f \in o(n^c)$ for any constant c.

Bounds

Overall time efficiency

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$\exists \kappa_1, \kappa_2 > 0$ such that, if $T < \kappa_2 \log^2 n$, w.h.p., within

$$O\left(\Delta + D + \frac{f - \log \varepsilon + \log \frac{\nu_{max}}{\nu_{min}}}{\Phi_{min}^2}\right) \text{ time steps after } \tau_1 - \kappa_1(D + \log^2 n),$$

all nodes have converged to a value in the range

$$[\nu_{max}, \nu_{min}]$$

with relative error $0 < \varepsilon < 1$.

Conclusions

- Combined algorithm is early stopping.
  - Non-frequent failures
    $\longrightarrow$ tree-based returns result fast and aborts mass-distribution.
  - Frequent failures
    $\longrightarrow$ mass-distribution returns at least an approximation later.
- All analyses include all communication costs.
- First optimal early-stopping for aggregation.

Open problems

- Only one radius.
- Geographic average.
- Other hierarchical topologies.
- Relax some restrictions.
- Mobile.

Thank you