Initializing Sensor Networks of Non-uniform Density in the Weak Sensor Model

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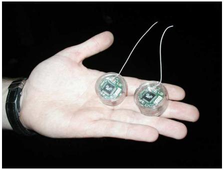
Universidad Rey Juan Carlos

WADS 2007

A Sensor Network



Intel Berkeley Research Lab

Capabilities

- processing
- sensing
- communication

Limitations

- range
- memory
- life cycle

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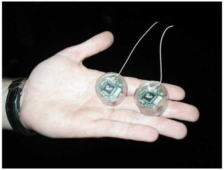

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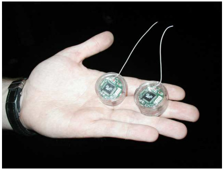

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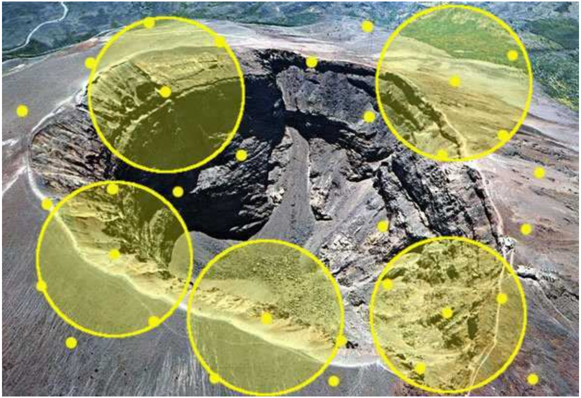

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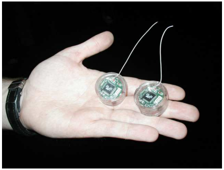

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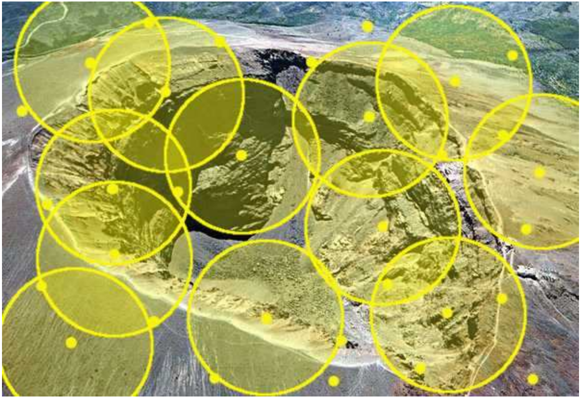

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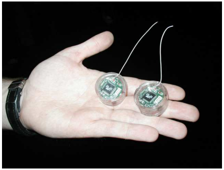

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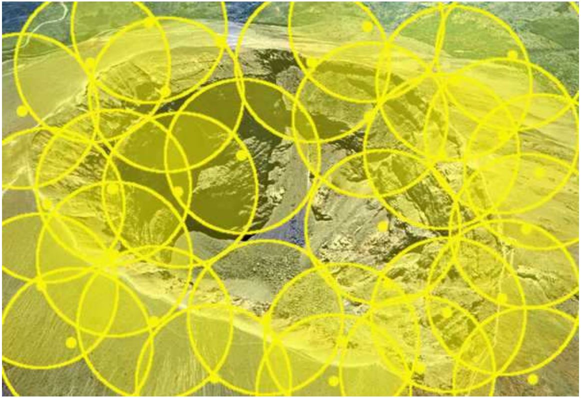

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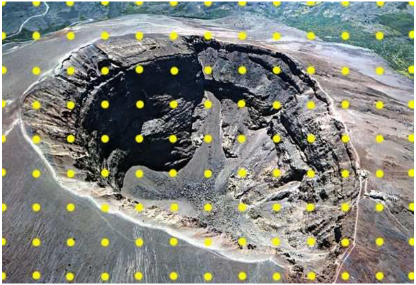
Node Layout

Radio Networks: arbitrary



Sensor Networks: uniform

Example of a feasible model:

a multiple bivariate normal distribution.

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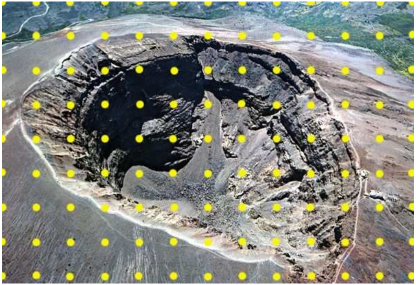

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More generally, we assume a Smooth Distribution:

1. In any disc of radius $r/2$:

   the number of nodes is at most some $\Delta \leq n$.
2. For any const. $\alpha > 0$, in any disc of radius αr, $\exists$ const. $\beta > 0$ such that:

   the number of nodes is at least $\beta \log n$.

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Sensor Network Bootstrapping

Communication is through radio broadcast.

Still, explicit links are necessary.

Lack of collision detection and unknown topology $\Rightarrow$

establishing them from scratch is not trivial!

- How do we understand limitations?
  The Weak Sensor Model.
- How are sensors distributed?
  Geometric Graph.
- What kind of network do we want?
  Hop-optimal, $O(1)$ degree (concludes from WSM).[FCFM05]
- Under smooth distributions:
  Still any connected GG has a hop-optimal subgraph, $O(1)$ degree.

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Previous Work
Upper Bounds

Sensor Network formation:

- Sohrabi et al.,00: Flat topology.

  Number of channels function of density.
- Blough et al., 03: k-neighbors protocol.

  Distance estimation.
- Song et al., 04: OrdYaoGG structure power spanner.

  Distance estimation, directional antenna.

  All: memory size function of density and no contention resolution in the analysis.
- FCFM, 05: $O(\log^2 n)$, whp, Sensor Network bootstrapping (RGG).

Previous Work
Lower Bounds

Clear Transmissions:

- KM-98: $\Omega(\log n)$, expected.
- JS-02: $\Omega\left(\frac{\log n \log(1/\epsilon)}{\log\log n + \log\log(1/\epsilon)}\right)$, wp$(1-\epsilon)$, uniform, one-hop.
- FCFM-06: $\Omega(\log n \log(1/\epsilon))$ wp $1-\epsilon$, uniform, one-hop.
  $\Omega(\log\log n \log(1/\epsilon))$ wp $1-\epsilon$, uniform, RGG.

Our Results
Lower Bounds

- Clear transmission:
  a node produces a clear transmission at time t,
  if every two-hop neighbor does not transmit in t.
- Clear Transmission problem:
  every node has to either receive or produce a clear transmission.
- Group Therapy problem:
  every node must be *heard*.
- Regardless of randomization:
  $\Omega(\Delta)$ for group therapy.
- Uniform protocols:
  $\Omega(\Delta + \log \Delta \log(1/\epsilon))$, w.p. $1-\epsilon$, using previous clear transmission bound.
- Fair protocols:
  $\Omega(\Delta(\log \Delta + \log(1/\epsilon)))$, w.p. $1-\epsilon$, for group therapy.
- Fair protocols with uniform distributions:
  $\Omega(\log^2 n)$, expected for clear transmission.

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Our Results
Upper Bounds

- Under smooth distributions and the WSM:
  distributed protocol builds $O(1)$-degree hop-optimal network,
  each node joins the network w.h.p. within $O(\Delta \log n)$ steps.

- Includes $O(\Delta \log n)$-fair protocol where
  each node produces a Clear Transmission.

  If every node produces a Clear Transmission
  $\Rightarrow$ Group Therapy problem is solved
  $\Rightarrow$ this protocol matches the lower bound.

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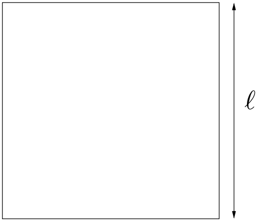

The geometric graph model $\mathcal{G}_{n,r,\ell}$.

- $[0,\ell]^2$
- Structural properties depend on relation among r, n and ℓ.

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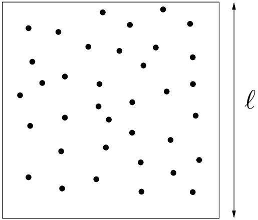

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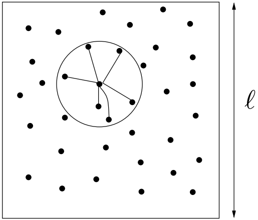

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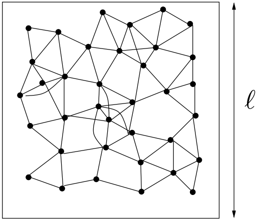

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Node Constraints

The Weak Sensor Model[FCFM05]

- Constant memory size.
- Limited life cycle.
- Short transmission range.
- Low-info channel contention:
  - Radio tx on a shared channel.
  - No collision detection.
  - Non-simultaneous rx and tx.
- Discrete tx power range.
- Local synchronism.
- One channel of communication.
- No position information.
- Unreliability.
- Adversarial wake-up schedule.

$$tx = transmission.$$
$$rx = reception.$$

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tx = transmission.
rx = reception.

A reduced model for a stronger lower bound

- Shared channel.
- No collision detection.
- Non simultaneous transmission and reception.
- Local synchronization.
- Adversarial wake up.

Known in the literature as: Radio Network.

Definitions

Fair protocol: sequence $p_1, p_2, \ldots$.

- every active node transmits with probability p_ℓ at time-slot t_ℓ.
- $p_\ell \in \{2^{-j} | 1 \leq j \leq \log n\}$.

Adversary:

- Wakes up 2^i *active* nodes at time t_1 in a neighborhood of density Δ, $i \in [1, \log \Delta]$.
- Active nodes: stay active and run the protocol.
- Non-active nodes: do not participate in the protocol.

LP formulation

Let

- $p_{ij} \triangleq$ probability that a node fails to achieve a non-colliding transmission when 2^i active nodes transmit with probability 2^{-j}.
- $t_j \triangleq$ number of time slots where nodes transmit with probability 2^{-j}.

Then, for each $i \in [1, \log \Delta]$, we want

$$2^i \prod_j p_{ij}^{t_j} \leq \epsilon$$

$$\sum_j t_j \ln(p_{ij}) \leq \ln(\epsilon) - \ln 2^i.$$

We can obtain a lower bound minimizing the total number of time slots under these constraints $\rightarrow$ LP?

LP formulation

$$\sum_j t_j \ln(p_{ij}) \leq \ln(\epsilon) - \ln 2^i$$

primal

Minimize $\mathbf{1}^T \mathbf{t}$,
subject to:
$$\mathbf{P}\mathbf{t} \geq \boldsymbol{\epsilon}$$
$$\mathbf{t} \geq \mathbf{0}$$

dual

Maximize $\boldsymbol{\epsilon}^T \mathbf{u}$,
subject to:
$$\mathbf{P}^T \mathbf{u} \leq \mathbf{1}$$
$$\mathbf{u} \geq \mathbf{0}$$

Where:
$$\mathbf{t} \triangleq [t_j]$$
$$\boldsymbol{\epsilon} \triangleq [-\ln(\epsilon) + \ln 2^i]$$
$$\mathbf{P} \triangleq [-\ln(p_{ij})]$$

Primal LP has a finite solution $\Rightarrow$ dual LP has a finite solution
$\Rightarrow$ any feasible objective function value for the dual
is a lower bound on the value of the primal!
(Weak LP Duality Theorem)

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Slack variables $u_i = 2^i \left(1 - \frac{1}{\sqrt{e}}\right)^2$ verify these contraints, then...

LP formulation

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Theorem

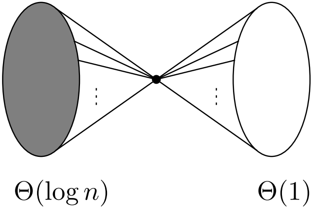
$$\mathbf{1}^T \mathbf{t} \in \Omega(\Delta(\log \Delta + \log(1/\epsilon)))$$

Clear Transmissions

Fair protocols under uniform distributions

Adversary:

- Wakes up $\Theta(n/\log n)$ disjoint *clique-pairs* at time t_1.
- Active nodes: stay active and run the protocol.
- Non-active nodes: do not participate in the protocol.



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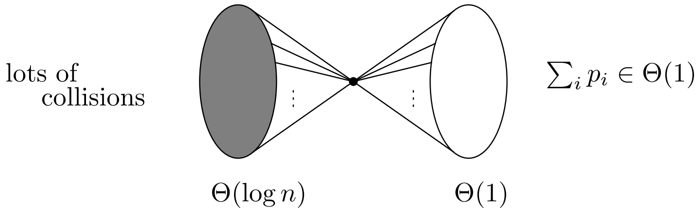

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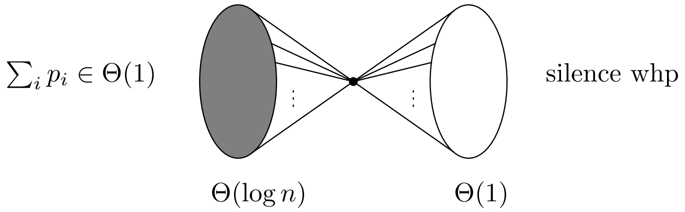

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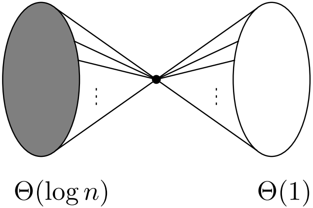

Minimizing the probability of failing to achieve a Clear Transmission in a low density clique...

Clear Transmissions

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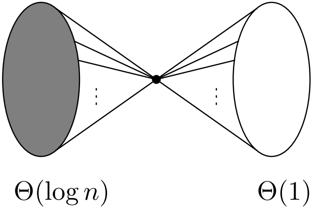
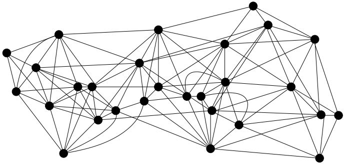


Theorem

$\Omega(\log^2 n)$ *expected time to solve the Clear Transmission problem.*

Disk-cover Algorithm
[FCFM-05]

Given an RGG, find a CHSG as follows:



- Add all nodes.
- Lay down disks of radius $ar/2, 0 < a < 1$ centered on uncovered nodes.
- Add all edges that connect these *bridges*.
- Expand the disks to a radius of $br/2, a < b < 1$.
- Add edges to form a constant-degree *disk-spanner*.

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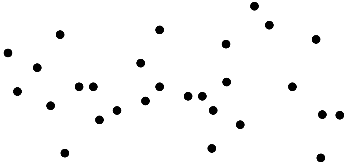

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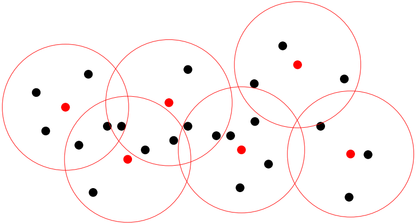

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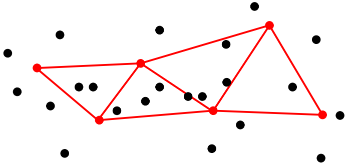

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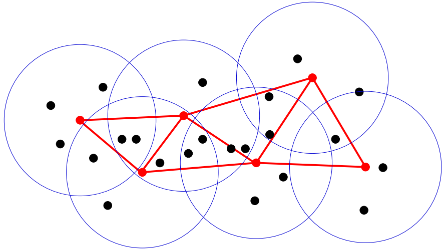

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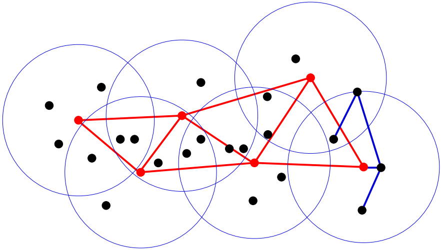
Disk-cover Algorithm
[FCFM-05]

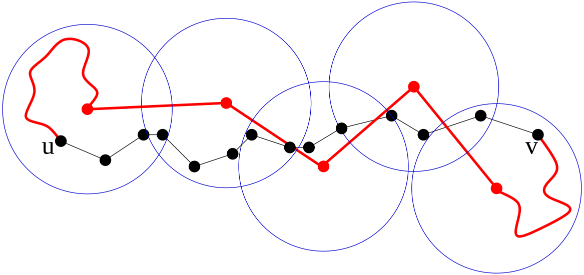
Given an RGG, find a CHSG as follows:



- Add all nodes.
- Lay down disks of radius $ar/2, 0 < a < 1$ centered on uncovered nodes.
- Add all edges that connect these *bridges*.
- Expand the disks to a radius of $br/2, a < b < 1$.
- Add edges to form a constant-degree *disk-spanner*.

Hop-optimality

What is the optimal path between u and v?



Theorem (FCFM-05)

$d(u, v) \in O(D(u, v)/r + \log n)$ *is asymptotically optimal.*

We need a logarithmic diameter...

Spanner Construction

1. **Predecessor-identification phase**: every node broadcasts its ID for $\gamma_1 \Delta \log n$ steps with probability $1/\Delta$, $\gamma_1 > 0$ some constant.
2. **Self-enumeration phase**: upon receiving the rank i of its predecessor, a node defines its rank as $i + 1$ and broadcasts it with constant probability $p < 1$ for $\gamma_2 \log n$ steps, $\gamma_2 > 0$ some constant.
3. **Link-definition phase**: Each node broadcasts its ID and rank for $\gamma_1 \Delta \log n$ steps with probability $1/\Delta$.

Lemma

If every node repeatedly transmits with probability $1/\Delta$ every node achieves a Clear Transmission within $O(\Delta \log n)$ time steps w.h.p.

Theorem

Any node running the spanner algorithm joins the spanner within $O(\Delta \log n)$ time steps w.h.p.

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Thank you