CS837 2018 Quantum Computing Final and Qualifying Examination (Partial Credits Possible).

Problem 1 (26 points) – Matching

Fill in the match numbers. *Some have more than 1 match*.

#	Question	Answers	Match #
1	|0> & |1>	CNOT	
2	<0|	Pauli operator X	
3	$(b\|1\rangle a\langle 1\|)$	Pauli operator Y	
4	$a\|0\rangle + b\|1\rangle$	Pauli operator Z	
5	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	Inner Product	
6	(|00>+|11>)/sqrt(2)	[10]	
7	$\|0\rangle^{\dagger}$	tr()=0	
8	BraKet	Hadamard operator H	
9	|0><0|+|1><1|	Bell state	
10	|0><0|-|1><1|	[1000]	
11	|00>	Entangled state	
12	<00|	|0> tensor product |0>	
13	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Computational basis state	
14	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$	-I	
15	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	I	
16	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	Rotation Matrix	
17	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	iY	
18	$\begin{bmatrix} e^{i\pi} & 0 \\ 0 & e^{i\pi} \end{bmatrix}$	A superposition state	
19	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	tr()=ab	
20	|1><0|+|0><1|	2 probability amplitudes	

Problem 2 (10 points) – Eigen Decomposition of the Pauli Matrix X

The evolution of a closed quantum system is described by unitary transformations (operators) acting on states to form new states. We are concerned with eigenvalues and eigenvectors because an operator's action is totally determined by its eigenvalues and eigenvectors through the spectral decomposition, where an operator is the sum of its eigenvalues times the projectors on its eigenvectors. For the Pauli "not" matrix X find the eigenvalues, the eigenvectors, the diagonalized matrix D from the eigenvalues, the projection matrix P from the eigenvectors, and show that PDP^{-1} yields the X matrix.

Problem 3 (15 points) – Implementing a controlled-*U* operation for arbitrary single qubit *U*

It is possible to implement a controlled-*U* operation for arbitrary single qubit *U* using only single qubit operations. The procedure applies a phase shift $e^{i\alpha}$ on the target qubit controlled by the control qubit, that is:

$$|00\rangle \to |00\rangle; \quad |01\rangle \to |01\rangle; \quad |10\rangle \to e^{i\alpha}|10\rangle; \quad |11\rangle \to e^{i\alpha}|11\rangle.$$

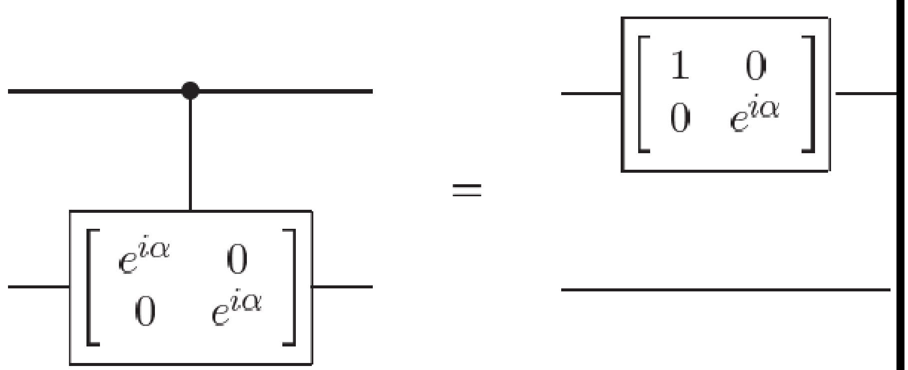


Figure 4.5. Controlled phase shift gate and an equivalent circuit for two qubits.

Verify that the right-hand side of the figure is equivalent to the left-hand side.

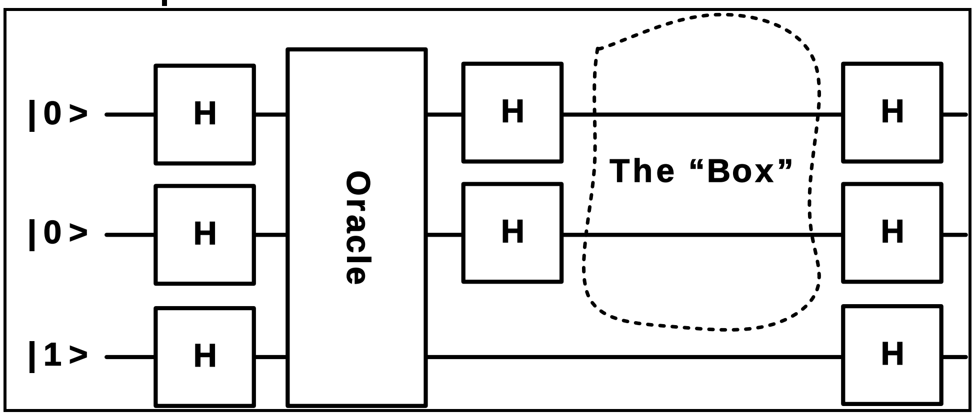
Problem 4 (14 points) – Quantum Circuit Identities

In the construction of quantum circuits, it is useful to be able to simplify circuits by inspection, using well-known identities. Prove the following three identities:

1. HXH=Z
2. HYH=-Y
3. HZH=X

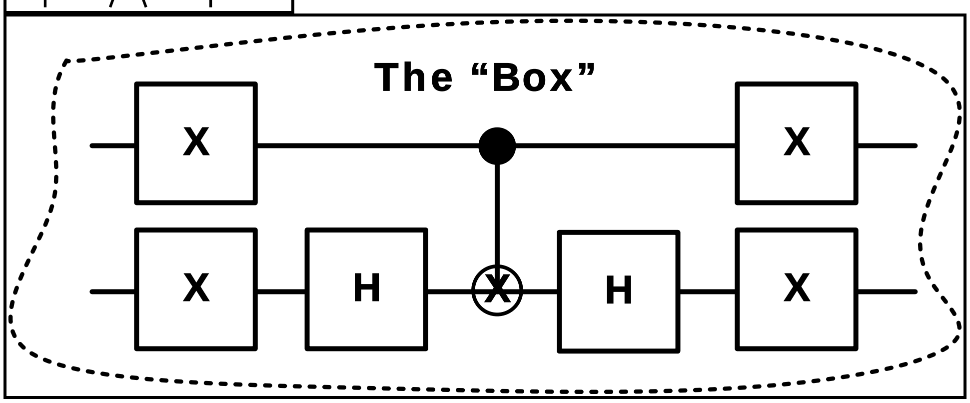
Problem 5 (20 points) – Quantum Search Algorithm

The quantum search algorithm example from the textbook has a search space of N = 4. The circuit that performs a single Grover iteration, shown below, involves the top two qubits initialized to |0> and the third to |1>.



Verify that the gates in the dotted box in the figure perform the following conditional phase shift operation up to an unimportant global phase factor:

$$2\left|00\right\rangle\left\langle 00\right| - I$$



Problem 6 (15 points) – Analysis of a Quantum Computing Circuit

For the following circuit consisting of a Hadamard gate and a CNOT gate, compute the operator matrix of the full circuit and the input vector to the circuit. Then compute the output of the circuit in vector form and convert it to the Dirac (BraKet) notation. Describe the nature of the output state and explain why this is a useful circuit.