Proceedings of

Graph Theory Day 70

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The Metropolitan New York Section of The Mathematical Association of America
Hosted by
Seidenberg School of Computer Science and Information Systems, Pace University
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Edited by Sung-Hyuk Cha
Edgar G. DuCasse
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Preface

We are very pleased to have the opportunity to organize Graph Theory Day 70. This conference is sponsored by the Metropolitan New York Section of The Mathematical Association of America and hosted by the Seidenberg School of Computer Science and Information Systems at Pace University. Graph Theory Day is a biannual New York based conference, in its 35th year. It occupies a unique place among conferences, presenting both new research and exceptional student papers, providing opportunities for both faculty and student participation. The purpose of the Graph Theory Day is to provide a learning and sharing experience on recent developments in Graph Theory. The conference is welcoming to a range of participants, open to both researchers in the field and students. While experts give talks, they are targeted at audiences in general discrete mathematics and computer science with an eye dedicated towards students.

Two eminent invited speakers, Professor Christina Zamfirescu of Hunter College and the Graduate Center, CUNY and Ms. Liana Brancati of the Montefiore Medical Center, have contributed to the conference. We are grateful to them.

For the first time, the GTD committee has compiled submitted abstracts into the Proceedings of Graph Theory Day 70. We received 14 abstracts and had 30 participants by the preregistration deadline. We have strived to publish well-written abtracts that present important original research results and/or open problems relevant to Graph Theory. We would like to express our gratitude to all the contributors and participants. Finally, we hope that you will benefit from this conference and its proceedings.

S.-H. Cha, E. G. DuCasse, and L.V. Quintas

Digraphs: Intersection, Transformations and Applications to Questionnaire Design

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A great deal of research has been done in the area of transformations on graphs and digraphs, found in connection with work done in groups on graphs. We will deal with 4 transformations of a digraph, namely the line digraph, total digraph, middle digraph, and subdivision digraph. On the other hand, using intersections of sets belonging to a family of sets, in order to define the edge connections in a graph is so natural that it arose independently in a number of areas in connection with both pure and applied mathematics, and has been studied for over 7 decades.

While a lot of research has been done on various types of intersection graphs, the study of similar concepts for digraphs has just started. Beineke and Zamfirescu [1] in 1982 and Sen, Das, Roy, West in 1989 introduced and studied independently a natural analogue of the intersection graph model for digraphs. Beineke and Zamfirescu [1] made for the first time a connection between these new intersection digraphs and transformations on digraphs.

The intersection number of a digraph D is the minimum size of a set U, such that D is the intersection digraph of ordered pairs of subsets of U. The paper describes much of the work done in the area of intersection graphs and digraphs [2], and proves two main results:
Theorem 1 The intersection number of the line digraph of D equals the number of vertices of D that are neither sources nor sinks.
Theorem 2 If D contains no loops, the intersection numbers of total digraph, middle digraph and subdivision digraph of D are all equal to the number of vertices of D that are not sources, added to the number of vertices of D that are not sinks.

In the second part of this paper we introduce as in [3] a special type of graph, the survey chart, a directed acyclic graph with additional properties, which we use as a tool for designing and improving survey questionnaires, in order to turn a complex questionnaire into a questionnaire that is easier to visualize, test and analyze. We define and perform a series of transformations which bring an initial structure closer to a structure we prefer, for it is more more amenable to analysis, verification of the coverage of questions, and readability of the flows of questions.

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Graphs whose vertices are the positive divisors of a given integer

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This presentation seeks to give an overview of the properties of the graph $Div(N)$ whose vertex set is the set of all positive divisors of a given integer N, and whose edges connect two divisors a, b with $b < a$ if and only if $a = pb$ for some prime p. The structure of this graph depends entirely on the exponents in the prime factorization of $N = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ where $0 < e_i$. We note that:

$Div(N)$ has order $\prod_1^k (e_i + 1)$, size $\prod_1^k (e_i + 1) \sum_1^k \frac{e_i}{e_i+1}$, and is the Cartesian product of paths. Specifically, $Div(N)$ is a multi-dimensional-grid.

By introducing a relation on the vertices of $Div(N)$, the graph is put in the context of posets. Here $Div(N)$ is isomorphic to the Hasse diagram of the divisors of N.

Depending on the factorization of N, $Div(N)$ is a Boolean algebra, a Post algebra, or in general a P-algebra. Since $Div(N)$ is a grid there are a number of known properties concerning its distance parameters, traceability, and Hamiltoncity.

$Div(N)$ is planar when $k \leq 2$ or when $k = 3$ and at most one of the e_1, e_2, e_3 is greater than 1. $Div(N)$ is non-planar in all other cases.

There are many properties of $Div(N)$ that remain to be investigated [1].

Current research involves various algorithms concerning computation of graph parameters of $Div(N)$. Progress is also being made on extensions of $Div(N)$ to digraphs with various transition probabilities.

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Addressing a Graph

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The hypercube,Q_n, has the vertex set $\{(x_1, x_2, \cdots, x_n)|x_i = 1 \text{ or } 0\}$. Furthermore, the (Hamming) distance between vertices $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$ is given by $\sum_{i=1}^{n} |x_i - y_i|$. In particular, x and y are adjacent when $\sum_{i=1}^{n} |x_i - y_i| = 1$, that is, when their addresses differ in exactly one place [1].

Graham and Pollack [2], tried to address graphs other than hypercubes using binary labels. This did not work for non-bipartite graphs. So they modified the addresses to include a third element, *, which permitted agreement with 0 or 1. As an example, the vertices of K_3 can be labeled 00, 10, and *1.

Given a graph G, let $f(G)$ be the minimum number of places in a valid addressing of G. Thus, $f(K_3) = 2$. In citeWinkler1983, Winkler shows that if G is a connected graph on n vertices, then $f(G) \leq n1$. The inequality becomes equality if G is a tree or K_n.

Various properties are shown and several problems are presented. See [4] for more information.

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The Unreliability of Paths and Related Graphs in the Neighbor Component Order Edge Connectivity Network Model

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Let G be a finite simple graph. Consider a model in which edges of G fail independently, and when an edge fails we remove it from G along with the incident vertices. We say that a set of edges F is a *failure set* of G if after all edges of F fail, the components of the induced subgraph all contain at most $k-1$ vertices, for some prescribed $k>0$. If the edges fail with probability ρ, then the *unreliability* of G, denoted $U_k(G,\rho)$, is the probability that a randomly selected set of edges is a failure set. Let P_n be the path on n vertices. We will prove a general recursive formula on n for $U_k(P_n,\rho)$ that holds for any fixed k and ρ. We then solve this recursion when $k=1$ to get a closed form expression for $U_1(P_n,\rho)$. Finally, we use this result to derive closed forms for $U_1(C_n,\rho)$ and $U_1(M_{n,m},\rho)$, where C_n is a cycle on n vertices and $M_{n,m}$ is a unicycle that consists of a cycle on m vertices with a path on $n-m+1$ vertices extending from one of the cycle vertices (see figure below).

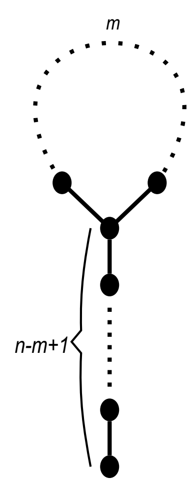

Figure 1: The graph $M_{n,m}$

On Extended Vertex Cover, Clique, and Independent Set Problems

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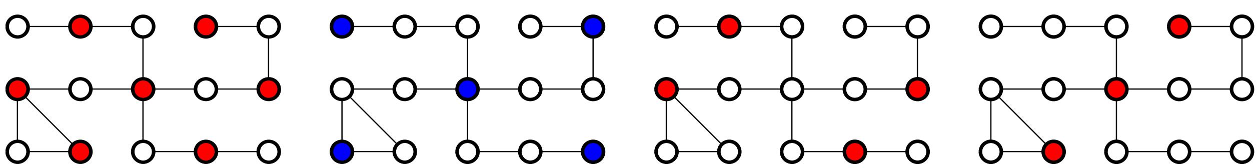
A *vertex cover* of a graph is a subset of vertices such that each edge of the graph is incident to at least one vertex of the set. Finding a minimum vertex cover is one of the classic *NP-hard* problems [1]. Here the vertex cover is extended from direct incident to path length, d and the problem of $VC_d(G)$ is formally defined as follows.

Input: $G = (V, E)$, a positive integer d, and an integer $0 < k < |V|$
Output: $V_c = \{v_x \in V | \forall (a, b) \in E, \exists v_x \in V_c(sp(v_x, a) < d \cup sp(v_x, b) < d)\}$ and $|V_c| = k$
where sp(x,y) is the shortest path length between x and y

When $d = 1$, $VC_1(G)$ is the same as the original vertex cover problem and $|VC_1(G) = 7|$ in the example of Fig. 1 (a). Optimal solutions for $|VC_2(G)|$ and $|VC_3(G)|$ are 4$|$ and 3 as shown in Fig. 1 (c) and (d), respectively. A simple *greedy algorithm* which takes the vertex with the highest



(a) $|VC_1(G)| = 7$ (b) $|VC_2(G)| = 5$ (c) $|VC_2(G)| = 4$ (d) $|VC_3(G)| = 3$

Figure 1: Deletable prime graph in Octal representation.

degree into the vertex cover finds the optimal solution in the particular case in Fig 1 (a), however, $|VC_2(G)| = 5$ is found when $d = 2$ because $deg_2(c) = 8$ for the center vertex c as shown in Fig 1 (b). Note that $deg_d(v)$ is the number of edges that are reachable within length d as defined in eqn (1).

$$deg_d(v) = |E_d(v)| \text{ where } E_d(v) = \{(a, b) \in E | sp(v, a) < d \cup sp(v, b) < d)\} \tag{1}$$

This work shows the proof that $VC_d(G)$ problem is *NP hard*. This extended path concept is applied to define *Clique*$_d$ and *Independent Set*$_d$ problems which are also proven to be *NP hard*. Several open problems regarding these problems are addressed.

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Graph meets Ontology at Morgan Stanley

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Effective control of a large enterprise infrastructure requires accurate inventories as well as an understanding of the relationships between hard, soft, and conceptual resources.

The structure and extent of the IT infrastructure can relatively easily be described in a graph database. However, in order to make effective use of the data, we require a semantic overlay that supports effective queries and analytics over the graph.

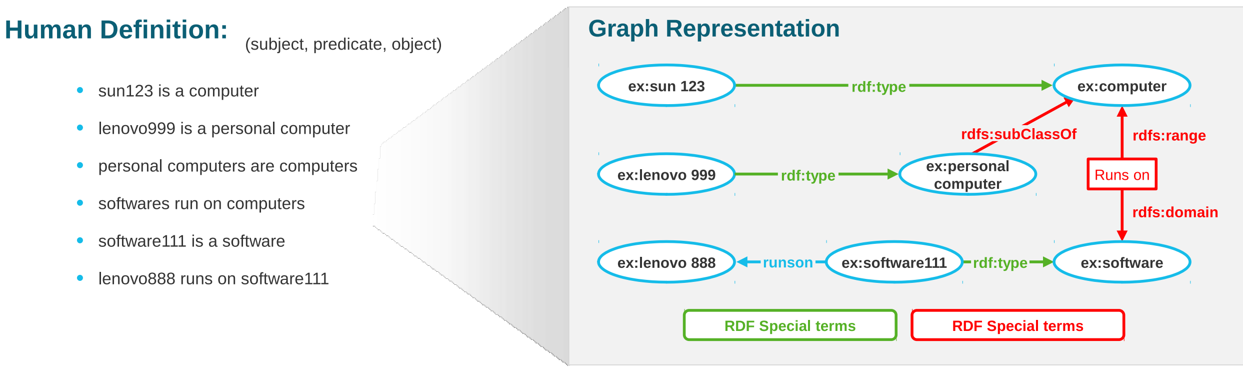

Figure 1: A small example of inventory and relationships

We discuss the approach being taken at Morgan Stanley. We touch on the use of semantic web standards to build the solution and discuss its merits by outlining use-cases and examples.

Further reading:

- `http://developer.marklogic.com/learn/semantics-exercises`
- `http://www.linkeddatatoiols.com/introducing-rdf`
- `http://www.cambridgesemantics.com/semantic-university/sparql-by-example`
- `http://www.w3.org/TR/rdf-sparql-query`

An Infinite Class of k-long Graphs

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Let n and k be nonnegative integers such that $n \geq 1$. The n-th k-long number is $n(n+k)$ and is denoted O_n^k [1]. Thus $O_2^4 = 2(2+4) = 12$, for example. Many interesting properties of k-long numbers are presented in [1].

A graph, G, is k-long if (1) its vertices are labeled with distinct k-long numbers, (2) the weight of each edge is the product of the labels of its endvertices, and (3) the edge weights are distinct k-long numbers [2, 3]. Infinite classes of k-long graphs are presented and their properties are examined. It is shown that the complete graph K_3 is k-long for every $k \geq 0$.

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Probability on a Digraph whose Nodes are the Positive Divisors of a Given Integer

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The transition digraph $\overrightarrow{Div}(n)$ defined below is studied with respect to its probabilistic properties and for its potential as a graph game.

Definition: $Div(n)$ with $n = p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$ $0 < n_i$ for $i = 1, 2, \cdots, k$, is the graph whose vertices are the positive divisors of n and whose edges connect two divisors a and b of n, $b < a$ if and only if $a = pb$ for some prime p [1].

Definition: $\overrightarrow{Div}(n)$ is the digraph with transition probabilities obtained as follows.
(A) Replace each edge of $Div(n)$ with a directed 2-cycle.
(B) Let $v = p_1^{v_1} p_2^{v_2} \cdots p_k^{v_k}$ $0 \le v_i \le n_i$.
For each i such that $1 \le i \le k$:
If s is adjacent to v with $s > v$, then assign $\frac{n_i - v_i}{2|S|}$ to the arc $v \to s = p_i v$, where
S is the multiset consisting of $(n_i - v_i)$ copies of p_i and
If t is adjacent to v and $v > t$, then assign $\frac{v_i}{2|T|}$ to the arc $v \to t = \frac{v}{p_i}$, where
T is the multiset consisting of v_i copies of p_i.
This assignment defines the transition probabilities on the out-going arcs at each node v in $\overrightarrow{Div}(n)$.

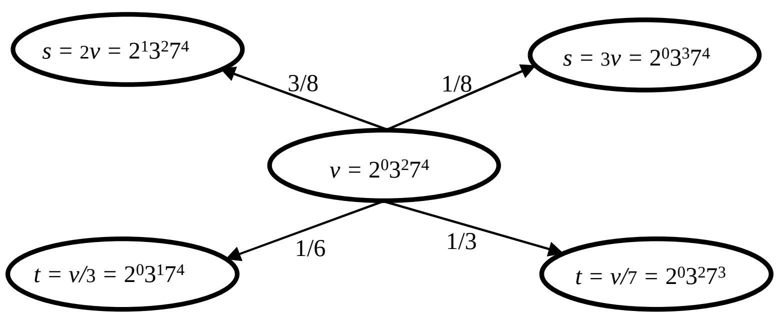

Figure 1: Drawing of node $v = 2^0 3^2 7^4$ with its out-going arcs in $\overrightarrow{Div}(2^3 3^3 7^4)$.

Open Problems: Determine probabilistic properties of $\overrightarrow{Div}(n)$ and create games on $\overrightarrow{Div}(n)$.

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Car Service Transportation Network Scheduling Optimization Algorithm

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The Uber Scheduling Optimization Algorithm takes a pick up request made from a mobile device and finds a list cars that will be the most efficient candidates for servicing the pick up request from an array of all the possible vehicles, with and without passengers (taking into consideration the route of the car with the passenger) optimizing not only the shortest wait time, increasing customer satisfaction and driver profits, but also reducing the CO2 emissions factor associated with vehicle services.

Here we review various approaches to the optimal algorithm such as in [1, 2]. While this is a classic bipartite graph matching problem and numerous algorithms have been suggested, we found that using merge sort and shortest path with memorization algorithms to be the most efficient in solving the complexity of the problem with most optimal complexity from $O(n^3)$ to $O(n \log n)$ in computational time complexity with the shortest path algorithm. We also built an application to illustrate the results of the algorithm.

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k-Rooted Deletable Prime Graphs of Depth j

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A k-rooted deletable prime graph of depth j is defined as follows. Let k be a single digit prime number, that is, let $k = 2, 3, 5$, or 7, and let it be the label of a vertex. Now find all two digit primes formed by putting a single digit either before or after k and let these be the labels of vertices adjacent to vertex k. These vertices form the distance set D_1, as they are adjacent to k. If $k = 2$, for example, then D_1 consists of two vertices labeled 23 and 29.

For each vertex, x, in D_1, add vertices whose labels are three digit primes that result by adding putting one digit into the (prime) label of x, in any position. These new vertices form D_2. If 23 and 29 are in D_2, as in our previous example, then 223 and 239 are among the vertices in D_3 because they are adjacent to 23. Note also, that 239 is also adjacent to 29, resulting in a 4-cycle. This illustrates that a k-rooted deletable prime graph of depth j need not be a tree.

Continue this process until we obtain D_j and we are done.

The labels of the endvertices of the graph will be so-called *deletable primes* [1], that is, primes that remain prime when the digits are removed, one at a time, in any chosen order. See [2] for an example and a discussion of the special cases, left and right-truncatable primes. We obtain various properties of k-rooted deletable prime graphs of depth j.

References

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[2] D. Wells, *Prime Numbers*, John Wiley & Sons, 2005.

Deletable Prime Graphs in Different Radix Number Systems

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Deletable primes are primes such that removing some digit leaves either empty or another deletable prime. It was conjected in [1] that there are infinitely many of deletable primes with implilcit definition of deletable primes. Here this conjecture is generalized with different radix number systems not just in decimal number system. *Deletable primes in radix* r are primes represented by the radix r such that deleting some digit leaves another deletable primes in radix r recursively with base single digit prime numbers as defined in eqn (1) with relatively large $r \geq 8$.

$$\mathrm{DP}(p_r) = \begin{cases} \exists p'_r \in \mathrm{DS}(p_r)(\mathrm{DP}(p'_r)) \wedge p_r \in P & \text{if } p_r \geq r \\ p_r \in P & \text{if } p_r < r \end{cases} \tag{1}$$

$$\text{where } \mathrm{DS}(p_r) = \{p'_r | \forall k \in Z(0 \leq k \leq \lfloor \log_r(p_r) \rfloor \wedge ((p'_r = div(p_r, r^{k+1}) + mod(p_r, r^k)) \in P))\} \tag{2}$$

For example, 1101_2 in binary is a deletable prime since 101_2 and 111_2 are also primes where 11_2 and 10_2 are base primes in binary. Are there infinately many binary,*octal* and *hexadecimal* deletable primes?

Let $G(p_r)$ be a deletable prime graph of a prime p_r of n digit long represented in radix r. The root vertex is p_r and other vertices are other deletable prime numbers $< p_r$ that can be edited from p_r by deleting certain digits from p_r. Terminal nodes are either single digit primges or base prime numbers as defined depending on the radix. Each arc represents a signle deletion opertation from one prime number to a smaller prime number. The deletion relationships in $(203_8, 3_8)$ and $(605_8, 5_8)$ as shown in Fig. 1 were unclear or implicit in [1] while the recursive explicit definition in (2) clearly includes them in the arc set. 225_8 is a prime but not a deletable prime in Octal.

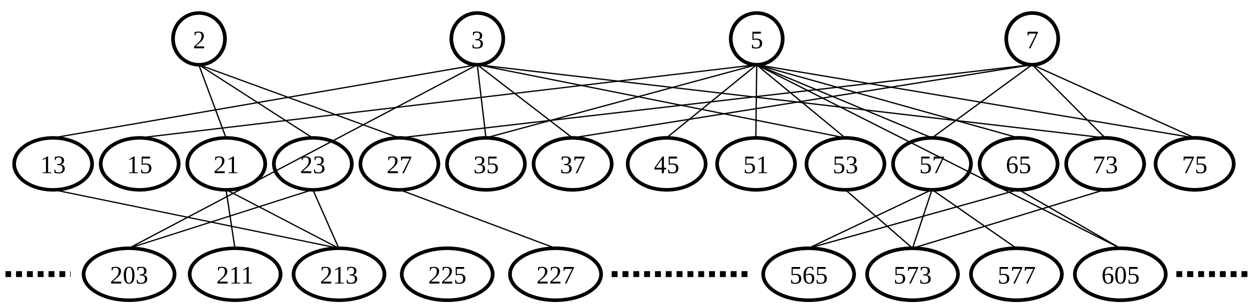

Figure 1: Deletable prime graph in Octal representation.

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Merging Domain Ontologies using Graph Algorithms

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A *Domain Ontology* is a directed graph that represents a certain domain specific knowledge where concepts (terms) and their relationships are represented as vertices and arcs, respectively. Successful implementation of a domain ontology enables to retrieve information with better automatic or semiautomatic reasoning [1]. While the term, *'edge'* in Graph Theory domain means a link between two vertices, its meaning varies depending on domains such as computer vision, architecture, etc. For example, the term *'edge'* in *'water's edge election'* means completely different in corporatae tax domain. Also, the term, *'graph'* means usually either bar or line chart in many domains of the world whereas it is specifically a set of vertices and a set of their relations in Graph Theory domain. Fig. 1 shows components in a Tax domain ontology.

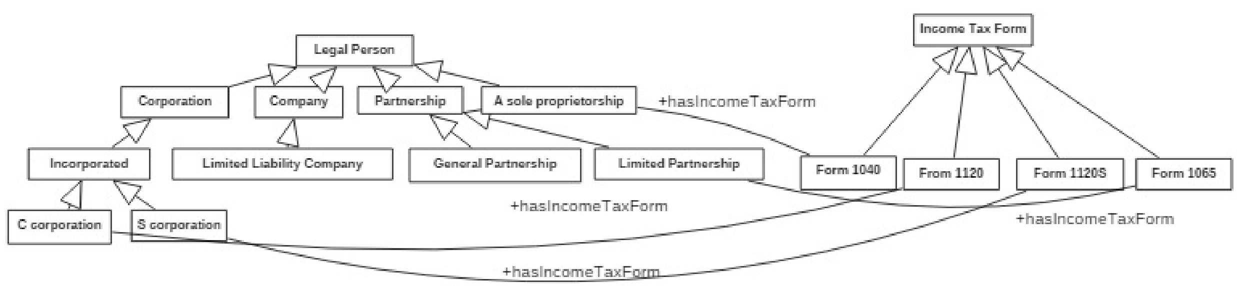

Figure 1: An Example of Components in a Tax Domain Ontology

Merging domain ontologies into a more general ontology is a challenging problem. While time consuming and subjective manual process to merge domain ontolgies have been practiced, an objective automatic algorithm is of great interest. Here the domain ontology merging problem is viewed as graph theory problems such as finding mismatches betwen graphs and similarity between graphs.

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Partially Ordered Bounded Integer Partition and Computing its Properties with Memoization Technique

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Partitioning a positive integer n into k parts is an important combinatorial problem, i.e., distributing n unlabeled balls into k unlabeled urns [1]. The bounded integer partition problem is counting the number of partitions where the capacity of each urn is limited to b. The *partially ordered relation* on integer partition is a level graph where kth level has $p_b(n,k)$ number of nodes. Edges from k to $k+1$ level indicate splitting an urn to two and those from $k+1$ to k level mean merging two urns into one as indicated in Fig. 1.

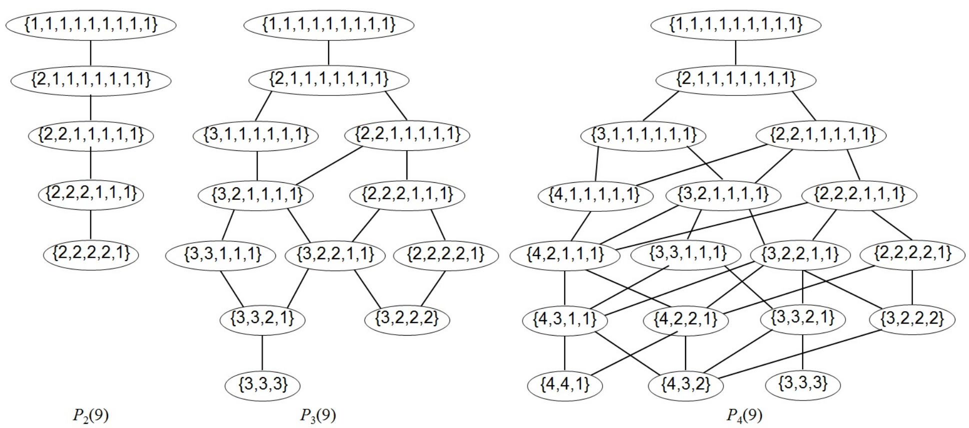

Figure 1: Partially ordered upper bounded Integer Partiion relations

Let $P_{l,u}(n,k)$ be the partition of an integer n into k smaller integer part where each part has lower and upper bounds. Let $p_{l,u}(n,k)$ be the number of nodes in k level in the partially ordered relation and its formula is given in (1).

$$p_{a,b}(n,k) = \begin{cases} 0 & \text{if } u > l, k < 1, k > n, n < lk, \text{ or } n > uk \\ 1 & \text{if } n = lk \text{ or } n = uk \\ p_{a,b}(n-1,k) + p_{l,u-1}(n-k,k) & \text{if } 1 < k < n \text{ and } lk < n < uk \end{cases} \tag{1}$$

This bound does not in essence change in the computational time complexity of most naïve algorithms. We found that a *memoization* technique makes the computation faster with the addition of this extra bound, i.e., memoization pulls even farther ahead of dynamic programming. This extra bound causes more nodes to terminate meaning memoization has even less work to do to complete the problem whereas dynamic programming would still approach the problem in the same way, not saving much time at all [1].

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Graphs Whose Vertices are Forests with Doubly Bounded Degree: Fundamentals

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A graph G is said to be an f-graph if G has no vertex of degree greater than f and its properties had been studied in [1, 2, 3, 4]. A vertex in a graph of degree greater than 1 is called *internal*. An f-tree with each internal vertex having degree at least L is called an (L, f)*-tree*. Define $F(n, L, f)$ to be the graph with the vertex set the unlabeled (L, f)-forests of order n and two vertices in $F(n, L, f)$ adjacent if and only if they differ by exactly one edge. Note that if v and u are adjacent vertices, then either v is a one-edge deleted subforest of u or v is a one-edge extended super f-forest of u.

Current research on this class of graphs involves investigation of fundamental properties of $F(n, L, f)$. This presentation will serve as a brief examination into basic properties of $F(n, L, f)$, including the construction of these graphs and basic invariants. In addition, a special case of these graphs where $L = f = k$ to form $F(n, k, k)$ will be presented.

References

[1] E.G. DuCasse, L.V. Quintas, A. DePhilips, and A. Michel, Graphs whose vertices are forests with bounded degree: Order and Size. *Congressus Numerantium* 201 (2010) 65-74.

[2] E.G. DuCasse, L.V. Quintas, and M. Zimmler, Graphs whose vertices are forests with bounded degree: Traceability. *Bulletin of the Institute of Combinatorics and its Applications* 66 (2012) 65-71.

[3] R. Neville, Graphs whose vertices are forests with bounded degree, *Graph Theory Notes of New York, New York Academy of Sciences*, LIV (2008) 12-21.

[4] L.V. Quintas and J. Szymaski, Graphs whose vertices are forests with bounded degree: Distance problems. *Bulletin of the Institute of Combinatorics and its Applications* 28 (2000) 25-35.

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