Proceedings of

Graph Theory Day 72

November 5 2016

Sponsored by
The Metropolitan New York Section of The Mathematical Association of America
Hosted by
Seidenberg School of Computer Science and Information Systems, Pace University
New York, New York

Edited by Sung-Hyuk Cha
Edgar G. DuCasse
Louis V. Quintas

Proceedings of Graph Theory Day 72

Editors:

Sung-Hyuk Cha, Pace University
Edgar DuCasse, Pace University
Louis V. Quintas, Pace University

Program Committee:

Sponsored by

The Metropolitan New York Section of The Mathematica Association of America

Hosted by

The Seidenberg School of Computer Science and Information Systems
Pace University

Published by Pace University Press

Table of Contents

Preface

We are very pleased to have the opportunity to organize Graph Theory Day 72. This conference is sponsored by the Metropolitan New York Section of The Mathematical Association of America and hosted by the Seidenberg School of Computer Science and Information Systems at Pace University. Graph Theory Day is a biannual New York based conference, in its 36th year. It occupies a unique place among conferences, presenting both new research and exceptional student papers, providing opportunities for both faculty and student participation. The purpose of the Graph Theory Day is to provide a learning and sharing experience on recent developments in Graph Theory. The conference is welcoming to a range of participants, open to both researchers in the field and students. While experts give talks, they are targeted at audiences in general discrete mathematics and computer science with an eye dedicated towards students.

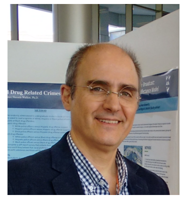
Two eminent invited speakers, ProfessorMiguel A. Mosteiro from Pace University and Dr. Matthias Autrata from Morgan Stanley, have contributed to the conference. We are grateful to them. We would like to express our gratitude to all the contributors and participants. Finally, we hope that you will benefit from this conference and its proceedings.

S.-H. Cha, E. G. DuCasse, and L.V. Quintas

Ad-hoc Affectance-selective Families for Information Dissemination in Bipartite Graphs

Miguel A. Mosteiro
Computer Science Department, Pace University, New York, NY, US
mmosteir@pace.edu

The connectivity of Computer Networks is usually modeled as a graph. Each station is represented by a node and if two nodes are able to communicate they are connected by a link. Wireless Networks protocols are designed to achieve such communication in spite of interference. When designing a multi-source multi-message dissemination algorithm, it is often easier to first aggregate messages in one node. This reduces the dissemination problem to a single-source multi-message broadcast problem. To further simplify the task, most solutions define a BFS-type tree rooted at the source on top of the connectivity graph, and try to disseminate information from layer to layer in this tree. In fact, if one has a fast way of transmitting messages from one layer to the next, repeating this and using pipelining would yield a fast broadcast algorithm. Thus, the crux of the dissemination problem lies in how fast can this task be solved in bipartite graphs. In this talk, I will describe our ongoing work on message dissemination in bipartite graphs, under a novel model of interference called affectance. Our protocol is based on a combinatorial structure called selective families, which we recreate to cope with affectance. The construction of affectance-selective families designed ad-hoc for a given topology is of independent interest.



Miguel A. Mosteiro is an Assistant Professor in the Computer Science Department at Pace University. Before, he was Assistant Professor at Kean University, Research Professor at Rutgers University, and Research Fellow at the University of Liverpool (UK) and the Universidad Rey Juan Carlos (Spain). He obtained his PhD in Computer Science from Rutgers University, and his BE in Electronics from Universidad Tecnologica Nacional (Argentina). His research interests span various topics in the broad areas of algorithms and distributed computing. Currently, he focuses on algorithms for IoT systems and 5G networks, the application of game theory to Internet computing, approximation algorithms for cloud computing, and computational geometry. Prof. Mosteiro has 50+ peer-reviewed publications and multiple international collaborations (UK, France, Poland, Spain, Israel, etc.). He is Assistant Editor of the ACM Transactions on Algorithms, has been TPC member of multiple conferences, and will be TPC chair of Latin 2018. Prof. Mosteiro received the 2016 Kean University Undergraduate Research Mentor of the Year Award, and his student Maitri Chakraborty was selected for the 2016 Undergraduate Student Researcher of the Year Award.

Semantic Graph Databases at Morgan Stanley

Matthias Autrata
Enterprise Infrastructure, Morgan Stanley, New York City, NY, USA
matthias.autrata@morganstanley.com

The operation and management of large IT infrastructures requires resource information from many sources to be aggregated and integrated. It is important to understand the relationships between different resource types in order to address problems such as incident management, threat analysis, or capacity planning.

This data is modelled using a set of linked ontologies and represented in a graph with a semantic inference model. This supports a number of different applications and analytics over the graph.

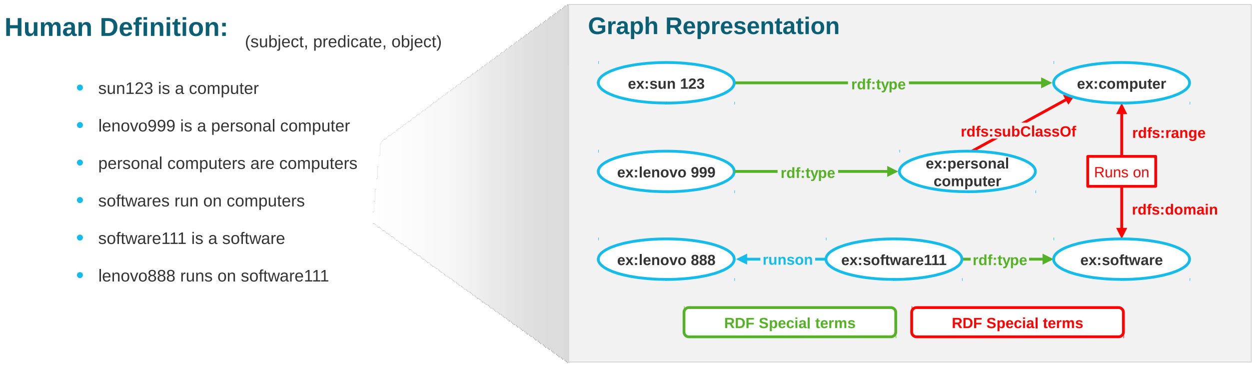


Figure 1: A small example of inventory and relationships

We discuss the approach being taken at Morgan Stanley. We touch on the use of semantic web standards to build the solution and discuss its merits by outlining use-cases and examples.

Further reading:

- http://developer.marklogic.com/learn/semantics-exercises
- http://www.linkeddatatools.com/introducing-rdf
- http://www.cambridgesemantics.com/semantic-university/sparql-by-example
- http://www.w3.org/TR/rdf-sparql-query

Dr. Matthias Autrata received his Ph.D. in Mathematics from University of Ulm, Germany and earned an MS in Mathematics from University of Southern California. Dr. Autrata is an executive director at Morgan Stanley where he is responsible for asset and configuration data management globally. He is also a member of the corporate architecture team responsible for setting the Firm's IT strategy. Previously, Matthias worked at a number of international banks, including UBS, Deutsche Bank, and ABN AMRO. His professional interests include data-management, real-time, event-driven architecture, and applications of semantic web and linked data technologies.

Triangular Graphs

A. Delgado[1], M. Lewinter[2], and K. Phillips[3]
[1]Columbia University, NY, USA
[2]ADU Services, White Plains, NY, USA
[3]Greeley HS, Chappaqua, NY
anthony.delgado12@gmail.com, marty1729@hotmail.com, kmecpp@gmail.com

The n-th triangular number, $tn = 1 + 2 + 3 + \ + n = n(n + 1)/2$. A graph is *triangular* if

1. Each vertex has a label consisting of a distinct triangular number.
2. Each edge has a distinct weight equal to the product of the labels of its endvertices.
3. The edge weights are distinct triangular numbers.

We show that all trees are triangular and present several open questions. This paper is in the spirit of two previous papers (see [1] and [2]) which involved k-long numbers, that is, numbers of the form $n(n + k)$. These numbers constitute a generalization of oblong numbers, that is, numbers of the form $n(n + 1)$.

References

[1] A. Delgado M. Lewinter L.V. Quintas, k-long graphs, *Cong.Num.*, 196, (95-106) 2009

[2] A. Delgado M. Lewinter B. Phillips, k-long graphs II, *GTN of NY*, LXVI (29-34) 2014

RANDOM ONE-EDGE TRANSFORMATIONS OF GRAPHS

E.G. DuCasse, L.V. Quintas*
Pace University, New York, USA
K.T. Zwierzyński
Poznań University of Technology, POLAND

Abstract. A brief description is given of random graphs leading from the randon graph process to the random *f*-graph process (involving graphs with bounded degree) to variations of the latter.

Given an unlabeled graph *G* a ***one-edge transformation*** *of G* is either an unlabeled one-edge extension of *G* or an unlabeled one-edge deletion of *G*.

Example. The 6-cycle, C_6, has two one-edge extensions and one one-edge deletion. (Figure 1)

Next, starting with *n* isolated vertices, that is, K_n^c the complement of the complete graph K_n, then randomly add one edge at a time until the unique terminal graph K_n is reached. This is called the ***random graph process*** **RGP(*n*) of order *n***. Let **R(*n*)** denote the ***Transition Digraph*** for this process. Here the transformed graphs are arranged in ***levels*** according to the size of the graphs.

Example. The transition digraph R(4) has order 11 and size 15 (with a loop at K_4**)**. (Figure 2)

A graph with no vertex of degree *f* is called an ***f*-graph**.

Graphs that are the union of cycles, paths, and isolated vertices are **2-*graphs***. In particular, cycles are 2-graphs.

The process analogous to the RGP(*n*) in which the graphs involved are restricted to be *f*-graphs is called the ***random f-graph process*** **R*f*GP(*n*)** ***of order n*** with transition digraph **R*f*(*n*).**

Example. The transition digraph R2(4) has order 7 and size (with loops at C_4 and $C_3 \cup K_1$). (Figure 3)

The significant difference between the RGP(*n*) and the R*f*GP(*n*) is
(a) At each step of evolution in the R*f*GP(*n*) the number of **admissible** edges from which an edge can be randomly chosen to be added to a given graph on level *t* to obtain a graph on level *t* + 1 is not a simple formula of *n* and *t* as is the case for RGP(*n*), this being, $\binom{n}{2} - t$. In the R*f*GP(*n*) any edge that might be added that would introduce a vertex of degree greater than *f* would not be admissible. This can be seen by comparing R(4) with R2(4).

(b) The number of terminal graphs in the RGP(*n*) is exactly one, whereas the number of terminal graphs in the R*f*GP(*n*) for $f < n - 1$ is greater than one, as illustrated by R(4) and R2(4).

A treasure chest of interesting problems from the elementary to those suitable for advanced research is available in the classic RGP(*n*) and R*f*GP(*n*) context and in the variations that have been introduced.

Problems
1. Study the graphs on a given level *t* of the R*f*(*n*) with respect to enumeration, probability distribution, and extremal structures.
2. Allow for edge deletion and thereby define the ***reversable random f-graph process*** **RR*f*GP(*n*)** ***of order n***. Study the graphs obtained after *s* steps of insertions and deletions with respect to probability distributions and stuctures.
3. Restrct the graphs being studied for example to *f*-forests and thereby define the ***random f-forest process*** **R*f*FP(*n*)** ***of order n***.
4. Study the structure of the associated transition digraphs of the random processes discussed above.
For elementary problems restrict *n* to small fixed values. For advanced problems study these random processes for large fixed *n* and for *n* going to infinity.

An Introduction to Directed Graphs with Prime Divisors as Nodes

Daniel Keidar, E. G. DuCasse, L.V. Quintas
Mathematics Department
Pace University, NY, NY, US
keidar.daniel@gmail.com, educasse,lquintas@pace.edu

A brief introduction and definition of graphs with positive prime divisors as nodes, will lead into given examples and further expansion of such graphs. The presenter will build off these concepts in order to introduce digraphs of the same manner before delving into their probability distributions and traits. These probability distributions may prove to provide intriguing patterns. Concluding with real-world examples and a discussion of potential applications, the presenter will draw from concepts covered in Paces mathematics courses and computer sciences courses including structure of graphs, and prime factorization.

References

[1] E.G. DuCasse, L.V. Quintas, and L. Brancati, A graph whose vertices are all the divisors of a positive integer: Fundamentalss, *Bulletin of the Institute of Combinatorics and its Applications* 73, (47-62) 2015.

Graph Theory: An Excellent Research Topic for Mathematics Students

Anthony Delgado
Columbia University, NY, USA
anthony.delgado12@gmail.com

It is not easy to find mathematics research topics for undergraduate and high school students. Many open questions entail complicated, advanced material and require years of mathematics to understand.

By contrast, graph theory is relatively new and involves readily understood basics such as trees, cycles, domination, vertex labeling, spanning trees, and hypercubes. Graph theory is applicable to a wide range of practical problems and interacts nicely with computer science.

I interviewed a Professor Emeritus of Mathematics, to understand more about the mentoring of graph theory research, including his role in mentoring five Intel Science Research semifinalists. He noted that the students enjoyed the research and worked hard to investigate the problems he provided. Many of the students eventually obtained Masters or PhDs and pursued careers in mathematics.

In a questionnaire sent to eleven students, it was found that the students were able to begin research within four to six weeks after they started to learn graph theory. All respondents rated their projects as enjoyable and interesting. The majority of respondents indicated an increase in their motivation to work hard in their mathematics and computer science courses.

Several of their research topics will be presented.

References

[1] S. Arengo M. Lewinter, How connected are product graphs?, *GTN of NY*, XVI (14-15), 1988

[2] J. Gratt M. Lewinter, The eccentricities of a center preserving spanning tree. *GTN of NY* XVII (18-19) 1989.

[3] C. Camazine M. Lewinter, Center distance sets of F graphs. *GTN of NY* XVII (10-11) 1989.

[4] C. Delaplain M. Lewinter, The euler number of a graph. *GTN of NY* XIX (36-37) 1990.

[5] M. Haag M. Lewinter, Double embeddings of a graph in the hypercube. *GTN of NY* XX (50-51) 1991.

[6] N. Highstein M. Lewinter, The euler number of a graph II. *GTN of NY* XIX (52 53) 1991.

[7] T. Bocchi M. Lewinter, Classification of L- graphs. *GTN of NY* XXIII (29 32) 1992.

[8] T. Bocchi M. Lewinter, How many 4 cycles does a spanning tree of M(m, n) span? GTN of NY XXIII (33 34) 1992.

[9] T. Bocchi M. Lewinter, Maximal planar eulerian graphs. *GTN of NY* XXIV (47-50) 1993.

[10] M. Aaron M. Lewinter, 0 deficient vertices of spanning trees. *GTN of NY* XXVII (31-32) 1994.

[11] D. Aulicino M. Lewinter, k-equidistant sets of Qn . *GTN of NY* XXXVI (17-19) 1999.

[12] D. Aulicino F.Harary M. Lewinter, Embedding k-dimensional meshes in hypercubes. *GTN of NY* XXXVII (51-53) 1999.

[13] D. Aulicino M. Lewinter, Almost all meshes are pancentral. *GTN of NY* XXXIX (38-40) 2000.

[14] T. Bocchi M. Lewinter, A.Silverman, Factoring degree sequences. *GTN of NY* XLI (24-25) 2001.

[15] D. Aulicino M. Lewinter, Pancentral graphs. *Cong. Num.* 150, (69-72) 2001.

[16] S. Karimi M. Lewinter, J.Stauffer, Double bonds in hexacyclic hydrocarbons and maximal planar graphs. *GTN of NY* XLIII (9-11) 2002.

[17] S. Karimi M. Lewinter J.Stauffer, Double bonds in fused hexacyclic systems. *J.Math.Chem.*, 34, (297-301) 2003.

[18] M. Huang M. Lewinter, The four-cycles of a directed mesh. *GTN of NY* L (26-30) 2006.

[19] S. Karimi S. Kalyanswamy M. Lewinter, On the Equitability of Multiply-connected Monolayered Cyclofusene, *J.Math.Chem.*, 41 (59-61) 2007.

[20] S. Karimi M. Lewinter S. Kalyanswamy, Bilayered cyclofusene. *J.Math.Chem.*, 43 (892-900) 2008.

[21] S. Karimi, A. Delgado, M. Lewinter, Resonance Structure Counts in Parallelogram-like Benzenoids with Holes. *J.Math.Chem.*, 38 (913-916) 2008.

[22] A. Delgado M. Gargano, M. Lewinter, L.V. Quintas, Spanning-tree-center vertices of a graph. *Cong. Num.* 189, (43-48) 2008.

[23] A. Delgado M. Lewinter, L.V. Quintas, k-long graphs. *Cong.Num.* 196, (95-106) 2009.

[24] A. Delgado M. Lewinter, L.V. Quintas, Color sets of tripartite graphs. *Cong.Num.* 199, (39-44) 2009.

[25] S. Karimi A. Delgado M. Lewinter, A combinatorial/geometric analysis of convex cyclofusene. *J.Math.Chem.*, 47 (167-173) 2010.

[26] A. Delgado M. Lewinter L.Quintas, Equipartition sets of hypercubes II, *Bulletin of the ICA*, 63, (2011), 51-59.

[27] S. Karimi A. Delgado M. Lewinter, A combinatorial/geometric analysis of parallelogramlike benzenoids. *J.Math.Chem.*, 50 (904-913) 2012.

[28] M. Lewinter B. Phillips, Spanning k-branched starlike trees. *GTN of NY*. LX (29-31) 2011.

[29] S. Karimi A. Delgado M. Kuepfert M. Lewinter, Resonance structure counts in contorted and flat hexabenzocoronenes. *J.Math.Chem.* 51 (817-825) 2013.

[30] A. Delgado M. Lewinter B. Phillips, k-long graphs II, *GTN of NY*, LXVI (29-34) 2014

Unique Educational Opportunities at Pace University

E. G. DuCasse and L.V. Quintas
Mathematics Department
Pace University, NY, NY, US
educasse,lquintas@pace.edu

Unique opportunities exist for students at Pace University. As early as their freshman year they can be matched with faculty who will not just help them in the selection of their courses, but will guide them in work on special projects beyond the scope of material covered in traditional courses. The results of this research may be presented not only at any of the numerous outlets in Paces network for student research, but also at external workshops and conferences. Some of the conferences where students have presented their papers include the Southeastern International Conference on Combinatorics, Graph Theory, and Computing (SICCGTC) in Boca Raton, Florida, meetings of the Mathematical Association of America (MAA), Discrete Mathematics Day, Graph Theory Days, etc. Students have not only given talks at national and international symposia, but have also coauthored papers with Pace faculty which were published in peer-reviewed research journals such as the International Journal of Chemical Modeling (IJCHEMO), the Bulletin of the Institute of Combinatorics and its Applications (BICA), Congressus Numerantium, Graph Theory Notes of New York (GTNNY), the International Journal of Graph Theory and its Applications (IJGTA), etc. This preparation gives students advantages not afforded by normal college programs and positions them advantageously for their desired careers following graduation, whether that be graduate, business, law, or medical school, or a professional occupation.

Because of the importance of these students work, all costs associated with attendance at conferences such as registration fees, meals, travel expenses, etc. are borne by the University and never charged to the student. In addition, the costs associated with the publication of journal articles such as page charges, costs for reprints (which become the students intellectual property), etc. are also funded by Pace. This presentation lists some of the students Professor Quintas and I have worked with over the past several years and the talks, research papers, etc. we have produced. Many of the students we have worked with as undergraduates have continued joint research, presentation, and publication activities with us after graduation, a mutual benefit.

Discrete Signal Processing (DSP) on Graphs

Ohbong Kwon
Computer Engineering Technology
New York City College of Technology, Brooklyn, NY, US
okwon@citytech.cuny.edu

Signals and datasets that arise in physical and engineering applications, as well as social, genetics, biomolecular, and many other domains, are becoming increasingly larger and more complex. In contrast to traditional time and image signals, data in these domains are supported by arbitrary graphs. Signal processing on graphs extends concepts and techniques from traditional signal processing to data indexed by generic graphs. This abstract studies the concepts of low and high frequencies on graphs, and low-, high-, and band-pass graph filters. In traditional signal processing, there concepts are easily defined because of a natural frequency ordering that has a physical interpretation. For signals residing on graphs, in general, there is no obvious frequency ordering. I introduce a definition of total variation for graph signals that naturally leads to a frequency ordering on graphs and defines low-, high-, and band-pass graph signals and filters. I study the design of graph filters with specified frequency response, and illustrate our approach with applications to sensor malfunction detection and data classification.

In addition to the frequency analysis, I also introduce a sampling theory for signals that are supported on either directed or undirected graphs. The theory follows the same paradigm as classical sampling theory. I show that the perfect recovery is possible for graph signals bandlimited under the graph Fourier transform, and the sampled signal coefficients form a new graph signal, whose corresponding graph structure is constructed from the original graph structure, preserving frequency contents. By imposing a specific structure on the graph, graph signals reduce to finite discrete-time signals and the introduced sampling theory works reduces to classical signal processing.

Use of Graph Algorithms in Vascular Network Analysis

Chulwoo Kim and Sung-Hyuk Cha
Computer Science Department, Pace University, New York, NY, USA
enjoy0124@gmail.com, scha@pace.edu

Graph algorithms have been used to analyze the complex network of the renal glomerulus by representing vessels as edges and the branch points of the network as vertices [1]. A random graph process model was used to simulate the development of a vascular network. The correlation between the invariants of this vascular network modeled as a graph and the mechanisms of the growth of the network were studied. It was shown that the relative frequencies of sprouting and splitting during the growth of a given renal glomerulus can be estimated by the primitive invariants such as *root distance*, *radius*, and *diameter* of the graph representing the renal glomerulus network. Here the number of paths with length k is considered to compare the similarity between two vascular networks. An algorithm is devised using the dynamic programming paradigm based on the following recurrence relation in eqn 1.

$$npl_k(v_s, x) = \begin{cases} \sum\limits_{(y,x)\in E} npl_{k-1}(v_s, y) & \text{if } k > 1 \\ 1 & \text{if } k = 1 \text{ and } (v_s, x) \in E \\ 0 & \text{if } k = 1 \text{ and } (v_s, x) \notin E \end{cases} \quad (1)$$

The computational time complexity is $\Theta(kn^2)$.

References

[1] S.-H. Cha , M. Gargano, S. Chang, L. Quintas, and E. Wahl, "Estimating a Vascular Network Growth Using Random Graphs", *Machine GRAPHICS & VISION*, vol. 17, no 1/2, 2008 pp.91-103

On the Tree of Phylogenetic Trees

Yoo Jung An[1], Chulwoo Kim[2], Sung-Hyuk Cha[2]
[1]Essex County College, Newark, NJ, USA
[2] Computer Science Department, Pace University, New York, NY, USA
yan@essex.edu, enjoy0124@gmail.com, scha@pace.edu

While phylogenetic trees are widely used in bioinformatics, one of the major problems is that different dendrograms may be constructed depending on several factors such as findings of various experts and/or algorithms [1]. The problem of selecting a representative tree that is a consensus of the experts was considered in [2]. Here a phylogenetic tree of phylogenetic trees is considered. The correlation coefficient of heights of the least common ancestors between taxa is used as a proximity measure to construct a tree of phylogenetic trees as shown in Figure 1. A consensus

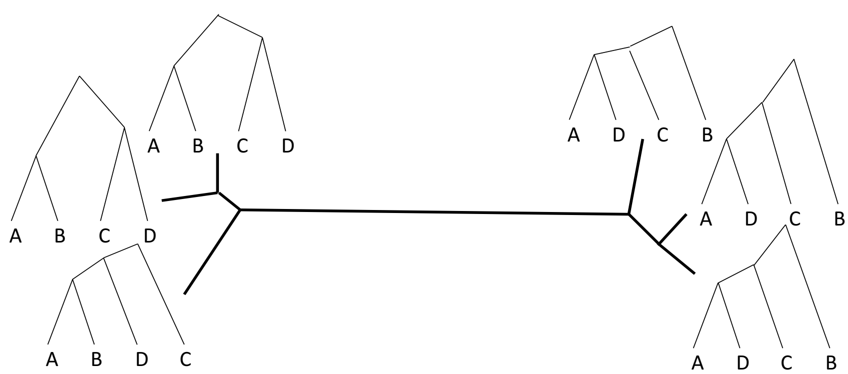


Figure 1: Tree of phylogenetic trees

tree may correspond to a centroid of distribution and becomes meaningful if the distribution is *normal Gaussian*. It is hard to see whether distribution of phylogenetic trees is normal Gaussian or multi-modal. The tree of phylogenetic trees provides insights of distribution of them as shown in Figure 1. There are two conflicting groups of phylogenetic trees and a consensus tree of all trees would not be one that all would agree on. Consensus trees of each cluster of similar phylogenetic trees would mean much more in this case. Several interesting observations based on more than 40 different phylogenetic trees and their hierarchical tree are presented.

References

[1] S.-H. Cha and Y. An, “Minimising Branch Crossings in Phylogenetic Trees”, *International Journal of Applied Pattern Recognition*, Volume 3(1), 2016, pp 22-37

[2] M. L. Garganol, “Consensus of Phylogenetic Trees Using Genetic Algorithms”, *CSIS Tech. Report*, n. 241, Pace university, Aug. 2007

Farey Sequence Sums

Jinxian Lu, Sebastian Dabrowski, Vivienne Zhu
Computer Science Department, Pace University, New York, NY, USA
{sl92982n, sd84470n, xz15682n}@pace.edu

This article considers the problem of finding the Farey sum of order N , which is a positive fraction. Our work is to explore the properties of the Farey series and the Fareys sum. We will compare and contrast the similarities and differences between Farey series and the SternBrocot Tree. Finally, we will show an application of the algorithms for evaluating the Farey sum.

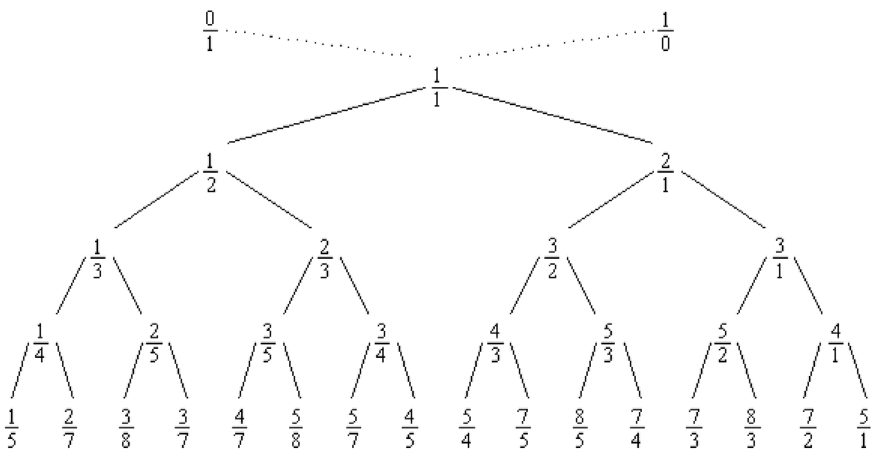


Figure 1: SternBrocot Tree

References

[1] Jonathan Ainsworth, Michael Dawson, John Pianta, and James Warwick, “The Farey Sequence” Retrieve May 2, 2016, from http://www.maths.ed.ac.uk/ aar/fareyproject.pdf