HW 1 - Solutions

Note: There are different possible solutions to some problems.

Part 1: Definition of languages

Exercise 1

Draw the tree representing $a * (b + c) * d$.

$*$ is left-associative so there is only one way to draw the tree.

$a * (b + c) * d$ is $(a * (b + c)) * d)$.

Postfix: visit the left subtree and right subtree completely before visiting the root node.

$abc + *d*$

Prefix: visit the root before visiting the left subtree and the right subtree.

$* * a + bcd$

Exercise 2

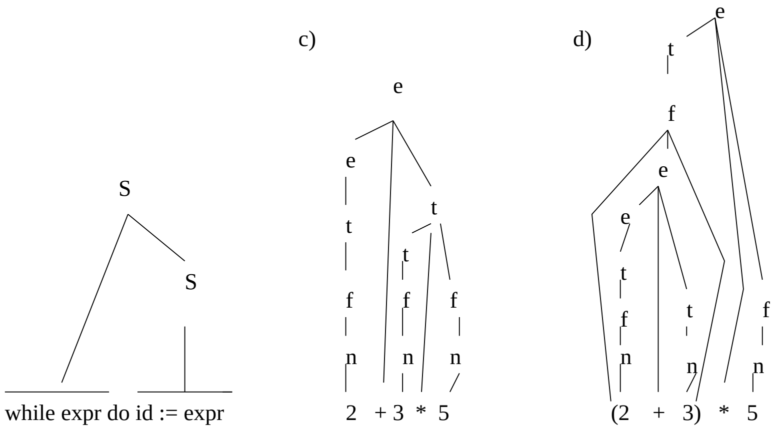
7 7 $*$ 4 2 $*$ 3 $*$ $-$

Here is the stack (in the left - top of the stack):

7
7 7
49
49 4
49 4 2
49 8
49 8 3
49 24
25

7 7 $*$ 4 2 $*$ 3 $*$ $- = 24$

Exercise 3 and 4



Exercise 5

Terminals: A and ,.

Non-terminals: $< goal >$, $< letter >$ and $< next >$.

The language is composed of A and A, A only.

Exercise 6

Us phone numbers.

```
<phone> ::= <acode> <x> <x> <x> - <x> <x> <x> <x>

<acode> ::= (<x> <x> <x>) | ^ (empty word)

<x> ::= 0 | 1 | 2 | ... | 9
```

Exercise 7

Write a BNF grammar for identifiers that consist of an arbitrarily long string of letters and digits, the first one of which must be a letter.

```
<identifier> ::= <letter><word>

<word> ::= <character><word> | ^

<character> ::= <letter> | <digit>

<letter> ::= a | b | ... | z | A | B | ... | Z
<digit> ::= 0 | 1 | ... | 9
```

Part 2: Recursion, functional language programming and SML

Exercise 1

See lecture notes.

Exercise 2

In SML with pattern matching:

```
fun pow(0) = 1
| pow(n) = 2*pow(n-1);

val pow = fn : int -> int
```

Exercise 3

In SML with pattern matching:

```
fun fib(0 ) = 1
| fib(1) = 1
| fib(n) = fib(n-1) + fib(n-2);
val fib = fn : int -> int
```

$fib(10) = 55$
$fib(40) = 102334155$

If you draw the tree of the evaluation of $fib(5)$ by the previous definition of fib you will see that the tree grows very quickly. This is an **exponential** computation so it is sure that you will not be able to compute $fib(800)$ with this definition. The question is now: How to improve the definition? (iteration, local variables...)

Exercise 4

$$f(n) = \begin{cases} n-1 & \text{if } n > 0 \\ f(f(n+2)) & \text{otherwise} \end{cases}$$

$f(10) = 10 - 1 = 9$

$f(0) = f(f(0+2)) = f(f(2)) = f(2-1) = f(1) = 1 - 1 = 0$ (inner-most evaluation)

Exercise 5

Types:

```
hd([6,1,2]);
tl [7,2,3];
```

```
hd [1];
tl [6];
explode ("abcd");
implode ["abc","def","hi"];
"f" :: ["a","c","i","l","e"];
["m","e","t"] @ ["a"," ","l","a","n","g","a","g","e"];
```

SOLUTION

hd([6,1,2]);
val it = 6 : int
tl [7,2,3];
val it = [2,3] : int list
hd [1];
val it = 1 : int
tl [6];
val it = [] : int list
explode ("abcd");
char list
implode ["abc","def","hi"];
val it = "abcdefhi" : string
"f" :: ["a","c","i","l","e"];
val it = ["f","a","c","i","l","e"] : string list
["m","e","t"] @ ["a"," ","l","a","n","g","a","g","e"];
val it = ["m","e","t","a"," ","l","a","n","g","a","g","e"] : string list

Exercise 6

```
((1,2),3);
(1,(2,3));
(1,2,3);
(1.2,("2",[4,5]));
[[3,4],[],[5]];
[(9,3,5),(1,2,1),(9,4,2)];
(["b","a"],[nil,[1,2,3]]);
```

Note : The constructor of type **list** has more priority than *****.

SOLUTION

((1,2),3);
val it = ((1,2),3) : (int * int) * int
(1,(2,3));
val it = (1,(2,3)) : int * (int * int)
(1,2,3);
val it = (1,2,3) : int * int * int

(1.2,("2",[4,5]));

val it = (1.2,("2",[4,5])) : real * (string * int list)

[[3,4],[],[5]];

val it = [[3,4],[],[5]] : int list list

[(9,3,5),(1,2,1),(9,4,2)];

val it = [(9,3,5),(1,2,1),(9,4,2)] : (int * int * int) list

(["b","a"],[nil,[1,2,3]]);

val it = (["b","a"],[[],[1,2,3]]) : string list * int list list

Exercise 7

```
int list list list
int * string list
(int * string) list
string list * (int * (real * string)) * int
((int * int) * (string list) * real) * (real * string)
```

SOLUTION

int list list list

[[[3,4],[],[4,5]],[[5,6,7,8],[1]],[[1],[2]]];

int * string list

(1,["a","b"]);

(int * string) list

[(1,"ab"),(2,"c"),(3,"fc")];

string list * (int * (real * string)) * int

(["toto","titi"], (1, (1.2,"titi"),3));

((int * int) * (string list) * real) * (real * string)

(((1,2),["toto","titi"],1.5),(1.2,"tutu"));

Exercise 8

SOLUTION

```
fun max(x,y) =
if x>y
then x
else y;
val max = fn : int * int -> int
fun max4(x,y,z,t) = max(max(x,y),max(z,t));
val max4 = fn : int * int * int * int -> int
max4(1,2,3,4);
val it = 4 : int
max4(8,2,3,4);
val it = 8 : int
```

Exercise 9

Circumference and area.

SOLUTION

First solution:

val pi = 3.14159;

fun circumference r = 2.0 * pi * r;

fun area r = pi * r * r;

Other solution:

(the 2 functions are defined at the same time)

local val pi = 3.14159 in fun circumference r = 2.0 * pi * r fun area r = pi * r * r end;

val circumference = fn : real → real

val area = fn : real → real

circumference 10.0;

val it = 62.8318 : real

circumference 10;

std_in:6.1-6.16 Error: operator and operand don't agree (tycon mismatch)

operator domain: real

operand: int

in expression:

circumference 10

Exercice 10

SOLUTION

```
Using pattern matching:
fun move([]) = []
| move (x::l) = l@[x];

val move = fn : 'a list -> 'a list
move([]); warning
move([1]);
move([1,2,3,4,5,6]);
move([1.2,4.5,6.4]);
```