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Francis Parisi
New York
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Extreme Value Theory and Standard & Poor's Ratings

Modeling the behavior of extremal events is important in many fields. These include hydrology, climatology, and engineering, as well as insurance and finance. In these fields one is often concerned about the maximum or minimum level of some value rather than the expected or average case. In the ratings process at Standard & Poor's, we are often concerned with how high (or low) some variable can go rather than the expected level of that variable. (These include foreign exchange rates, interest rates, market value declines, etc.) When answering such questions, the best tools for modeling the behavior of the extremes come from the area of extreme value theory.

STANDARD
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This paper presents the problem of extremal events in the context of Standard & Poor's ratings and demonstrates how extreme value theory (EVT) can be applied to solve the problem. In the course of pursuing this objective, we will highlight the theoretical underpinnings of EVT.

Introduction

Securities highly rated by Standard & Poor's are designed to withstand severe economic conditions. By definition, 'AAA' rated securities are remotely affected by changes in the state of the economy. The rating process tests the various components of a deal structure including default behavior of the underlying assets, interest rate changes and levels, changes in market value of the assets, prepayment assumptions, and so on. In short, highly rated securities are expected to perform even when "market" forces go to extremes. As evidence of this, we need only look at the historical corporate default study published annually by Standard & Poor's *(see references, Standard & Poor's, 2000)*. Even more convincing is the default behavior of structured transactions as reported in Hu (2000), and Kozel (2000) *(see references)*.

Given this position, it becomes clear that to accurately study the extremal behavior of financial data, one must use the appropriate statistical models. While application of extreme value theory (EVT) is not foolproof, it provides the safest method for extrapolating beyond what has been observed *(Embrechts, et al., 1997)*.

Studying extremal behavior in finance and insurance is nothing new. Insurance risk models have relied on such factors for years (the basic insurance risk model goes back to 1903 in work done by Filip Lundberg). It is key for an insurance company to set its premiums at levels high enough to cover losses from the rare but large claim. Take, for example, a property and casualty insurance company that must be able to withstand a loss proportional to the magnitude of the loss caused by Hurricane Andrew in 1992. While the occurrence of such events is infrequent, the magnitude of loss is enormous.

Extremal events manifest themselves in a variety of ways: large insurance claims, flood levels of rivers, large changes in stock prices, value-at-risk analysis of a portfolio, wind-speeds, and wave heights during a storm, to name a few. With the ever-growing securitization of risk, there is a commensurately growing need to properly model events that could shock the financial system.

The remainder of this paper develops as follows. First we present an overview of the concepts of extreme value theory and introduce the generalized extreme value distribution, the generalized Pareto distribution, and the peaks-over-threshold model. (The theoretical fundamentals of extreme value theory are presented in detail in the appendix.) Then the approach to modeling financial data is discussed. We include the distributional properties of typical financial data, and some alternative modeling techniques. Some examples of modeling financial data via extreme value methods are included.

Extreme Value Theory

Extreme value methods are applied to many areas of research, including hydrology, climatology, engineering, and finance. The basic ideas of the theory are given here so that the examples in the next section will make sense.

There are two principal distributions that are used in extreme value modeling: the generalized extreme value (GEV) distribution and the generalized Pareto distribution (GPD). An often useful method is known as the Peaks-Over-Threshold (POT) method.

Periodic (daily, monthly, yearly, etc.) maxima (or minima) follow a GEV distribution. So if we were concerned with monthly peaks in interest rates, we could fit a GEV distribution to the monthly maxima. Excesses over a given high threshold, however, follow a GPD. Suppose we were interested in the distribution of insurance claims over some high threshold, as we are in catastrophe bond ratings; those excesses would be best modeled by a GPD. Alternatively, if we were concerned about the occurrence times of the losses over some threshold and the excess distribution, we would fit a POT model. In a POT model, the number of events during a given time follows a Poisson process and the exceedances over a given high threshold follow a GPD. Since the number of events is Poisson, the interarrival times (time between events) are exponentially distributed. Therefore, by fitting a POT model we can estimate the average time between events of a given magnitude (threshold), and the distribution of the excess over the threshold. An underlying feature of a Poisson process is that the number of events in disjoint time intervals is independent. This is often not the case for financial data.

Modeling Financial Data

Standard Modeling Techniques

At Standard & Poor's, we are frequently concerned with extremal behavior in interest rates, assets class defaults, foreign exchange rates, market value declines, and many more characteristics of a transaction. The most appropriate tools to address these issues are the extreme value methods presented above and in detail in the appendix. In this section, we review some of the standard methods used to model financial data and some of the weaknesses of those methods to capture extremal behavior. The examples given are of uses of EVT in finance.

Financial time series data tend to be illbehaved in that they show jumps and fluctuations that are not modeled well by standard modeling techniques. Often many assumptions must be made, such as the data come from well-defined distributions, and the data have constant variance over time. One of the biggest problems with 'typical' models is they fail to capture extreme jumps in the data and cannot describe the extremal behavior. Of course, in the day to day realm of financial modeling, usually what is of interest is some expected behavior with some measure of dispersion or volatility. In that regard, standard modeling techniques are adequate. However, for Standard & Poor's, the extreme behavior of financial data is the concern. Extreme value methods outperform standard modeling techniques when the goal is to model extremal behavior.

While raw financial data rarely exhibit the behavior of a stationary time series, they can be made to approximate a stationary series by some straightforward transformation. A time series is called stationary if it has a finite second moment, a constant mean, and an autocovariance function that is independent of the time t. The autocovariance is strictly a function of the lag between two periods. For example, if $\{X_t\}$ is a stationary time series, then the covariance between two periods, say X_4 and X_2, is the same as that between periods X_8 and X_6; the lag is two in both cases. (See Brockwell and Davis, 1996 for an introductory level discussion and Brockwell and Davis, 1991 for an advanced level discussion about time series analysis.)

Financial data are often transformed as logarithmic differences, or as often called in the finance and economics literature, log-returns where

$$R_t = \log\left(\frac{X_t}{X_{t-1}}\right) = \log X_t - \log X_{t-1}$$

constitutes a stationary (or nearly stationary) process. Note that by Taylor's formula,

$$\log\left(\frac{X_t}{X_{t-1}}\right) = \log\left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right) \approx \frac{X_t - X_{t-1}}{X_{t-1}}$$

Therefore, $\{R_t\}$ can be considered a sequence of relative returns. Log-returns are comparable, since they represent relative change over time regardless of "unit". However, log-returns still have a complicated dependence structure.

Much of the recent research in mathematical finance has focused on modeling stochastic volatility. This allows for changes in the variance of log-returns. This concept was applied in Parisi (1998) where an autoregressive time series model is used to model changes in interest rates. At each step in the simulation, the volatility randomly transitions among three states (levels) according to a probability transition matrix. Related to this is the class of autoregressive models such as ARCH, GARCH, and ARMACH. These are AutoRegressive-Conditionally-Heteroscedastic, Generally-ARCH, and AR-Moving-Average-CH, models, respectively. In all of the ARCH type models the variance is modeled such that it varies over time and is not constant (hence heteroscedastic).

Many asset-pricing models are of the form of diffusion processes; these include the so-called Black-Scholes option-pricing model. These models are formulated as stochastic differential equations (SDE), where, as a first approximation, the fit to the data seems reasonable. However, when the goal is to model properly shocks or jumps in the data, there are problems with these models. That is, the model as a linear SDE with constant coefficients does not explain the fluctuations and jumps in real data. None of these approaches can account for the extremal behavior of financial data as well as extreme value methods. Moreover, none of these can safely predict beyond observed values.

For excellent references on stochastic calculus and diffusion models in mathematical finance, see Karatzas and Shreve (1991), and Karatzas and Shreve (1998), respectively. For recent work in stochastic volatility, see Carr and Madan (1999), and Ledoit and Santa-Clara (1999), as well as the references therein.

Applying EVT to Financial Data

In this section we consider two examples of how EVT provides the stress-testing we need to accurately rate securities at Standard & Poor's. In the first example we model the daily yields of the 91-day U.S. Treasury bill by fitting a GEV distribution. We show that this analysis, if done at the close of 1979, would have covered the record setting peaks observed in early 1980. In the second example, we fit a GEV distribution to the Korean Won/US$ foreign exchange rate using data just before the spike in late 1997.

Example 1

In this example, the question we ask is:

If we applied extreme value methods to interest rate data as of year-end 1979, would the results cover the spikes of 1980-1981?

To test the ability of extreme value theory to capture unobserved behavior this analysis uses daily yields on the 91-day US Treasury bill from 1973 to 1979 inclusive as the historical data. A GEV was fit to the data. Then using the estimated parameters from the GEV distribution, 100,000 values were simulated. (Simulation was necessary because the fitted distribution was unbounded; no finite upper limit exists.) The values were ordered and various percentiles were selected; these are the maximum, the 0.998, 0.995, 0.990, and 0.975 percentiles.

This simulation of 100,000 values was repeated 50 times to allow for randomness in each of the percentiles.The distribution of the 50 simulated values for each percentile is shown in chart 1. The alternating lines (from left to right) represent the 0.975, 0.990, 0.995, 0.998 percentiles, and the maximum, respectively.

Taking the lowest value for each percentile and counting the number of times the actual T-bill rate exceeded the percentile gives the results in table 1. The first column gives the percentile. The next column gives the lowest of the 50 interest rates (simulated from the

Table 1
Extreme Value Results Versus Actual 91-day T-Bill Yields

Percentile	Rate (%)	Exceedances in 1980-1999 (%)
Max.	37.118	0.00
0.998	19.022	0.00
0.995	16.697	0.67
0.990	14.643	4.46
0.975	12.565	8.68

GEV distribution) for each percentile. The third column shows the percent of the time the daily rate exceeded the rate in column three over the next 20 years from 1980 to 1999, inclusive.

Note that at the two highest percentiles there are no excesses; at the next level only 0.67% of the time there were excesses, and so on. The answer to the question posed in this first example is, yes, extreme value methods would have captured the 1980 experience given the observed data through 1979.

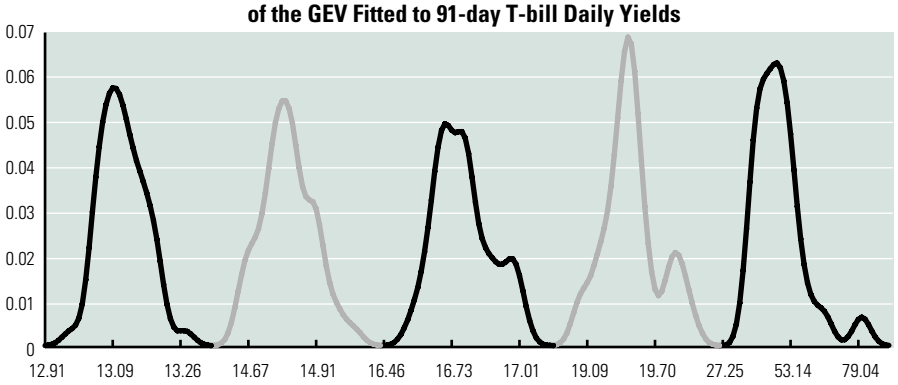
It is important to note that the results are sensitive to the data used, as is true for any data analysis. If we were to use a different period as the cut-off point, then the analysis could produce somewhat different results. Example 1 gives a sense for how high the yield on T-bills could go, but does not help in determining the timing. Of course, if we were building a dam to cover a 100-year flood, then we would build to an appropriate height. However, for ratings purposes it is onerous to assume such an extreme value for the life of a transaction. The assumption that the T-bill rate jumps immediately to 37% and stays at that level for the life of the transaction is unrealistic, if not impossible. Therefore, we need to study the extremal behavior over different time horizons.

Example 2

In this example we consider the Won/US$ exchange rate. The data include weekly forex rates from January 1980 through January 2000. A GEV distribution was fit to

Chart 1

Distribution of Select Percentiles From 50 Simulations of 100,000 Values Each, of the GEV Fitted to 91-day T-bill Daily Yields



the data up through Nov. 14, 1997; just before the sharp rise to the 1800 range *(see chart 2*). Fitting the distribution and simulating values as before, we get a value of 2568 at the 0.995 quantile versus the actual observed maximum of 1832. Hence, our GEV analysis would have captured the yet unobserved extremes of the weekly Won/US$ exchange rate.

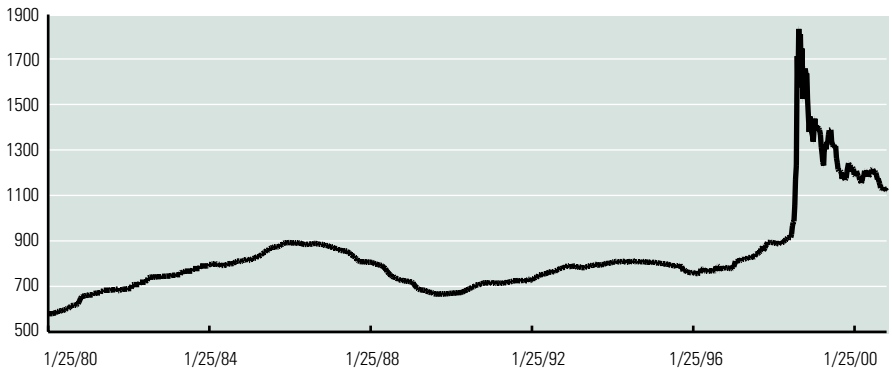
A similar analysis was done using the monthly relative change in the Won/US$ rate up to November 1997. The maximum observed in the periods that followed was about 33%. The GEV analysis gives an upper limit to the monthly change of 86.5%. The largest relative change is well within the extreme value limit.

Conclusions and Recommendation

In the rating process, Standard & Poor's is concerned with the potential stresses a security can undergo over its lifetime. The higher the rating assigned, the more the assumptions are stressed. There are a variety of statistical techniques for modeling the behavior of financial data. While many work fairly well, oftentimes assumptions must be made to make the them tractable. Unfortunately, these assumptions are often false when compared to the actual behavior of financial data. More importantly, none of the modeling techniques can adequately capture the extremal behavior of the data, especially beyond what has already been observed.

In this paper we have presented some of the theoretical framework of extreme value theory. We have given an overview of the alternative modeling techniques, and highlighted

Chart 2
Won/US$ Exchange Rate



some of their weaknesses. Finally, we have applied EVT to two examples and have demonstrated that these techniques are the most appropriate to use to model unobserved extremal behavior.

The use of extreme value distributions to determine the theoretical bounds for the behavior of financial data is justified. In order to model the behavior over time, it is necessary to apply the extreme value analysis to different time horizons.

Appendix

Most of the material in this appendix follows Embrechts, et al. (1997); the examples are from Mood et al. (1976).

Distribution of the Maxima

Classical extreme value theory has its roots in work by Fisher and Tippett, (1928). The Fisher-Tippet theorem specifies the form of the limit distribution for centered and normalized maxima. Suppose $\{X_n\}$ is a sequence of independent and identically distributed (iid) random variables and M_n is the $\max(X_1,\ldots,X_n)$. Then if there exist constants $c_n > 0$ and $d_n \in R$ (a real number), $(M_n\text{-}d_n)/c_n$ is a centered and normalized maximum. If $(M_n\text{-}d_n)/c_n \xrightarrow{d} H$ (that is, converges in distribution to H), for some non-degenerate distribution function H, then H belongs to one of the three families of extreme value distribution functions: Fréchet, Weibull, and Gumbel. These distributions have the following form:

Fréchet:

$$\Phi_{\alpha>0}(x) = \begin{cases} 0, & x \le 0 \\ \exp\{-x^{-\alpha}\}, & x > 0 \end{cases}$$

Weibull:

$$\Psi_{\alpha>0}(x) = \begin{cases} \exp\{-(-x)^{\alpha}\}, & x \le 0 \\ 1, & x > 0 \end{cases}$$

Gumbel:

$$\Lambda(x) = \exp\{-e^{-x}\}, \quad x \in R$$

Maximum Domain of Attraction

The extreme value distributions introduced above represent the limit laws for the normalized maxima of iid random variables. We need to consider the conditions on a distribution function F that imply the normalized maxima M_n converge to H. In other words, how can we choose constants $c_n > 0$ and d_n such that $(M_n - d_n)/c_n \xrightarrow{d} H$? If this condition is satisfied we say the distribution function *F* belongs to the maximum domain of attraction of *H*.

Definition 1

The random variable *X* (or the distribution function *F of X*) belongs to the maximum domain of attraction of the extreme value distribution *H* if there exist constants $c_n > 0$ and $d_n \in R$ such that $(M_n - d_n)/c_n \xrightarrow{d} H$. We write $X \in MDA(H)$ (or $F \in MDA(H)$).

Two examples will serve to illustrate the concept of the MDA and how we can find the appropriate constants.

Example 3

Consider a sample $\{X_1, X_2, \ldots, X_n\}$, from the logistic distribution $F(x) = 1/(1+e^{-x})$. First we need to find the norming constants c_n and d_n. Then we need to find the limiting distribution of $(M_n - d_n)/c_n$. We want to find "centering" constants $\{d_n\}$ close to $\mathrm{E}[M_n]$ (the expected value of M_n). Now $F(X_1), \ldots, F(X_n)$ is a random sample from the uniform distribution over (0,1); hence $F(X_n)$ is the largest of a sample size *n* from a uniform distribution over (0,1). This gives $\mathrm{E}[F(M_n)] = n/(n+1)$. Now,

$$F(\mathrm{E}[M_n]) = (1 + \exp\{-\mathrm{E}[M_n]\})^{-1} = 1 - (1 + \exp\{\mathrm{E}[M_n]\})^{-1},$$

and

$$\mathrm{E}[F(M_n)] = \frac{n}{n+1} = 1 - (n+1)^{-1}.$$

Note that $F(\mathrm{E}[M_n]) \approx \mathrm{E}[F(M_n)]$, which implies that $n \approx \exp\{\mathrm{E}[M_n]\}$ or $\mathrm{E}[M_n] \approx \log n$. A reasonable choice for the centering constants $\{d_n\}$ is the sequence $\{\log n\}$. To find the limiting distribution we proceed as follows,

$$\lim_{n\to\infty} P\left[\frac{M_n - d_n}{c_n} \le y\right] = \lim_{n\to\infty} P\left[\frac{M_n - \log n}{c_n} \le y\right]$$
$$= \lim_{n\to\infty} P\left[M_n \le c_n y + \log n\right]$$
$$= \lim_{n\to\infty} \left[F(c_n y + \log n)\right]^n$$
$$= \lim_{n\to\infty} (1 + e^{-c_n y - \log n})^{-n}$$
$$= \lim_{n\to\infty} (1 + (1/n)e^{-c_n y})^{-n}$$
$$= \exp\{-e^{-y}\} \quad \text{for } c_n \equiv 1.$$

Therefore, if we choose $\{d_n\} = \{\log n\}$ and $\{c_n\} = 1$ the limiting distribution of $(M_n - d_n)/c_n$ is $\exp\{-e^{-y}\}$, which is the Gumbel distribution.

Example 4

In this example we sample from the exponential distribution, $F(x) = 1 - e^{x/\beta}$, for $x \in [0, \infty)$. As in example 3, $\mathrm{E}[F(M_n)] = n/(n+1) = 1 - 1/(n+1)$. We have

$$F(\mathrm{E}[M_n]) = 1 - \exp\{-\mathrm{E}[M_n]/\beta\}$$
$$\approx \mathrm{E}[F(M_n)].$$

Then

$$\frac{1}{n+1} \approx \exp\{-\mathrm{E}[M_n/\beta]\},$$

or

$$\mathrm{E}[M_n] \approx \beta \log(n+1) \approx \beta \log n.$$

It is reasonable then to use $d_n = \beta \log n$.

$$\lim_{n\to\infty} P\left[\frac{M_n - d_n}{c_n} \le y\right] = \lim_{n\to\infty} P\left[M_n - \beta \log n \le c_n y\right]$$
$$= \lim_{n\to\infty} \left[F(c_n y + \beta \log n)\right]^n$$
$$= \lim_{n\to\infty} (1 + e^{-(1/\beta)c_n y - \log n})^n$$
$$= \lim_{n\to\infty} (1 - (1/n)e^{-(1/\beta)c_n y})^n$$
$$= \exp\{-e^{-y}\} \quad \text{for } c_n \equiv \beta.$$

Therefore, if we choose $\{d_n\} = \{\beta \log n\}$ and $c_n = \beta$ the limiting distribution of $(M_n - d_n)/c_n$ is again the Gumbel distribution.

From examples 3 and 4 it is clear that the normalized maxima from the logistic distribution and the exponential distribution converge in distribution to the same limiting distribution, in this case the Gumbel. Therefore the logistic distribution and the exponential distribution both belong to the MDA of the Gumbel distribution.

The Generalized Extreme Value Distribution and the Generalized Pareto Distribution

Let us introduce a parametric family $(H_\xi)_{\xi \in R}$ of distribution functions that contains the three extreme value distributions given above,

$$H_\xi = \begin{cases} \Phi_{1/\xi} & \text{if } \xi > 0, \\ \Lambda & \text{if } \xi = 0, \\ \Psi_{-1/\xi} & \text{if } \xi < 0. \end{cases}$$

The distribution function H_ξ is called the generalized extreme value (GEV) distribution with parameter ξ. It is generalized in the sense that given the value of ξ it takes the form of one of the three standard extreme value distributions. The widely accepted standard representation of H_ξ is

$$H_\xi(x) = \begin{cases} \exp\{-(1+\xi x)^{-1/\xi}\} & \text{if } \xi \neq 0, \\ \exp\{-e^{-x}\} & \text{if } \xi = 0, \end{cases}$$

where $1+\xi x > 0$. A location-scale family $H_{\xi;\mu,\psi}$ is introduced by replacing x with $(x-\mu)/\psi$ for $\mu \in R$ and $\psi > 0$.

Another distribution that plays an important role in modeling extremal events is the generalized Pareto distribution (GPD). The GPD is defined by

$$G_\xi(x) = \begin{cases} 1-(1+\xi x)^{-1/\xi} & \text{if } \xi \neq 0, \\ 1\text{-}e^{-x} & \text{if } \xi = 0, \end{cases}$$

where $x \geq 0$ if $\xi \geq 0$, and, $0 \leq x \leq -1/\xi$ if $\xi < 0$.

Similarly we can introduce a location-scale family by replacing x with $(x-\nu)/\beta$, for, $\nu \in R$ and $\beta > 0$. The distribution function $G_{\xi;0,\beta}$ (with $\nu = 0$) plays an important role in modeling excesses over high thresholds. We denote it as

$$G_{\xi,\beta}(x) = 1-\left(1+\xi\frac{x}{\beta}\right)^{-1/\xi}, \quad \text{x} \in \text{D}(\xi,\beta),$$

where

$$x \in D(\xi,\beta) = \begin{cases} [0,\infty) & \text{if } \xi \geq 0, \\ [0,-\beta/\xi] & \text{if } \xi < 0. \end{cases}$$

If $\xi \geq 0,$ then the range of x is unbounded; otherwise there is a finite upper bound to the domain of the distribution. The GEV describes the limit distributions of normalized

maxima. The GPD describes the limit distribution of scaled excesses over high thresholds. Often when modeling extremal events we are concerned about excesses over some high value. For example, in a catastrophe bond structure that has a $1 billion attachment point, we are only concerned about loss claims greater than $1 billion. The following is a precise definition of the excess distribution function.

Definition 2

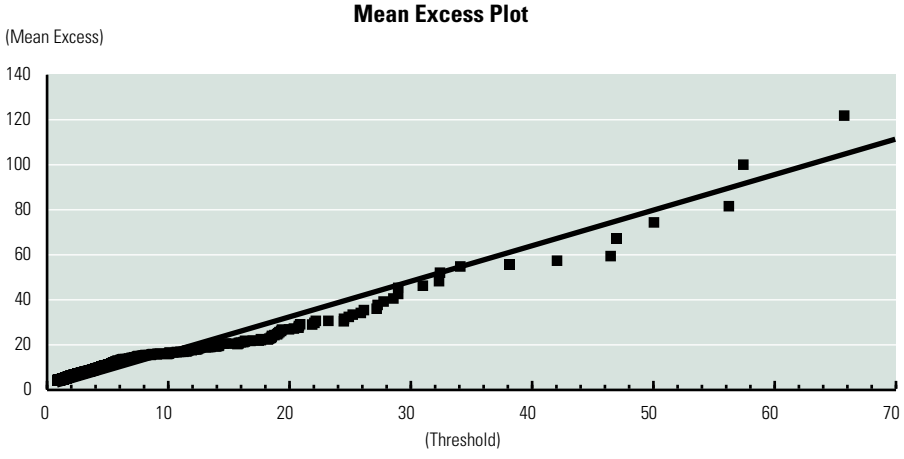
Let X be a random variable with distribution function F and right endpoint x_F . For some fixed $u < x_F$, $F_u(x) = P(X - u \le x \mid X > u), \quad x \ge 0,$ is the excess distribution function of the random variable X over the threshold u. Also, the function $e(u) = \mathrm{E}[X - u \mid X > u]$ is *the mean excess function of X*.

What definition 2 says is for any high threshold ***u***, we have the distribution of the amounts over ***u***, and $e(u)$ gives the average excess over ***u***. So in the catastrophe bond example above, $F_u(x)$ gives the conditional distribution of losses given the losses exceed $1 billion, and $e(u)$ is the average loss in excess of $1 billion.

The mean excess function provides a tool for finding an appropriate value of a high threshold. The mean excess function of a GPD is linear so we select a threshold from the region where the empirical mean-excess function is roughly linear (see Davison and Smith, 1990, and Embrechts et al., 1997).

Chart 3

Mean Excess Plot



Mean Excess Plot for Danish Fire Insurance Claims

Danish fire insurance claims data are used to demonstrate the use of the mean excess function. In chart 3, note that the mean-excess plot is roughly linear in the range up to about 10-15, indicating a threshold of 10 is appropriate (the claims are in millions of Danish Kroner, 1985 prices). This threshold could then be used to model the excess distribution. It can be shown that the number of exceedances of a high threshold is roughly a Poisson process. This is useful in modeling what hydrologists call a peaks-over-threshold (POT) model.

We conclude this section with the following summary:

- The number of exceedances of a high threshold follows a Poisson process.
- Excesses over high thresholds can be modeled by a GPD.
- Plotting the mean-excess function is useful in determining an appropriate value of the high threshold.
- The distribution of the maximum of a Poisson number of iid excesses over a high threshold is a GEV.

Peaks-Over-Threshold Models

Peaks-over-thresholds (POT) models have been used by hydrologists for many years in modeling such events as flood levels and dam heights. The basic formulation of the POT model is:

- The occurrence times of the excesses of an iid (or stationary) sequence over a high threshold follow a Poisson process;
- The corresponding excesses are independent and have a GPD; and,
- The excesses and the occurrence times of the excesses are independent of each other.

POT models are useful for modeling extremes when the occurrence times of the excesses are important. From a fitted model, one can estimate the return periods of extremal events. Additionally, one can determine the non-encounter probabilities of such events. The return period is the expected time to an event of a given threshold from some starting point. Since the occurrence times are independent, the return period is the average time to a given event. From this we can make statements such as "a loss of X is expected to occur every four years on average". The non-encounter probability of an event gives the likelihood that such an event will not occur in a given period, for example, one year. This modeling process offers an ideal method for simulation experiments and stress scenarios; it is the basis for the analysis done for catastrophe bonds.

Parisi and Lund (2000) fit a POT model to Atlantic basin hurricanes making landfall in the United States from 1935 to 1998, inclusive, then simulating from the fitted GPD estimate return periods and non-encounter probabilities for storms of various intensities. For example, they find that a storm with a wind speed of 240 km/h (150 mph) is expected to make landfall about once every 10.71 ± 0.104 years; the non-encounter probability is 0.912 ± 0.0028. That is, there is about a 91% chance that no such storm will make landfall in any given year; the complement is that there is about a 9% chance that this event will occur in any given year. The value of such an analysis is we find that such an event occurs about once every 11 years on average but there is a 9% chance it could occur in any year. Year-to-year storm counts and intensities are independent, so even though a 240 km/h storm makes landfall in one year there is still a 9% chance it could occur the following year. They include in their paper the results for a 320 km/h (200 mph) storm, although no such event occurs in their data set; this is the power of extreme value theory.

In a similar analysis, Parisi and Herlihy (1999) model catastrophic losses to the global reinsurance industry. Their paper gives the return periods and non-encounter probabilities for various levels of global catastrophic losses. Interestingly, losses of $1 billion or more are expected to occur about once a year.

For more details on extreme value theory and its applications, see for example Pickands, (1975), Leadbetter et al., (1983), Smith, (1989), Davison and Smith, (1990), and Embrechts et al., (1997).

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Structured Finance Key Contacts

Joanne W. Rose, Chief Rating Officer . . . . . (212) 438-6601

Research

Joseph Hu, Ph.D, Director of Research . . . . . (212) 438-2491
Peter P. Kozel, Ph.D, Director . . . . . (212) 438-2518
Francis Parisi, Director. . . . . (212) 438-2570
Erkan Erturk, Ph.D, Director . . . . . (212) 438-2450

Criteria & Chief Quality Officer
Thomas G. Gillis,
Managing Director . . . . . (212) 438-2468

Strategic Planning/
New Products Business Development
Chris Dalton, Managing Director . . (212) 438-2439

Global Asset-Backed Securities
Patrice Jordan,
Managing Director . . . . . (212) 438-2501

Global CBO/CLO/Derivatives/Market Value
Richard Gugliada,
Managing Director . . . . . (212) 438-2474

North America
Joseph F. Sheridan,
Managing Director . . . . . (212) 438-2605

Asia/Pacific

Melbourne
Edward Burbage, Director. . . . (61) 3-9-631-2050

Hong Kong
Calvin Wong,
Managing Director . . . . . (852) 2533-3501

Tokyo
Yu-Tsung Chang, Director . . . . (81) 3-3593-8724

Europe

London
Kurt Sampson,
Managing Director. . . . . (44) 171-826-3535

Paris (S&P-ADEF)
Alain Carron, Director . . . . . (33) 1-44-20-6664

Latin America

New York
Rosario Buendia, Director . . . . . (212) 438-2410

Buenos Aires
Jorge Solari,
Associate Director. . . (54) 11-4326-5686 Ext. 130

Real Estate Finance
Gale C. Scott,
Managing Director . . . . . (212) 438-2601

Residential Mortgages
Frank Raiter, Managing Director. . (212) 438-2579

Surveillance
Peter D'Erchia,
Managing Director . . . . . (212) 438-2438

Servicer Evaluations
Kim Betancourt, Director . . . . . (212) 438-2398

Legal Department
Petrina Dawson, Senior Managing
Director and General Counsel . . . (212) 438-8876

Subscriptions and Customer Service

Hong Kong, Michael Naylor . . . (852) 2533-3535
London, Steve Flaws. . . . . (44) 171-826-3510
Melbourne. . . . . (61) 3-9631-2000
New York . . . . . (1) 212-438-7280
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